EXPERIMENTAL STUDIES OF MULTIPASS BEAM BREAKUP AND ENERGY RECOVERY USING THE CEBAF INJECTOR LINAC

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THESIS

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Chapter 1

Introduction

Particle accelerators are now used in a multitude of applications in basic and applied science. Initially driven by the needs of nuclear physics, accelerators now are used in medicine, in the electronics industry for the development of fast microelectronic devices, in the oil industry, in heavy ion fusion and in nuclear and particle physics research (which use by far the largest, most powerful and therefore costliest accelerators). More recently they have also been used to drive high power free electron lasers (FELs). The unique features of high power and the ability to tune the wavelength of the light over a broad range make the FEL important to many fields in basic and applied research such as biology, chemistry, and materials science. It is not surprising in view of the many and varied applications of particle accelerators that accelerator physics as a discipline has grown out of the need to understand "the machine" itself.

Questions of prime importance to accelerator design concern the interaction of the particle beam with the electromagnetic fields used to accelerate, guide and focus it as it passes through the machine. These interactions can severely limit machine performance and are therefore important to understand both theoretically and experimentally. The experimental and theoretical study of a particular collective beam interaction known as multipass beam breakup in a superconducting linac is the subject of this thesis.

These experiments were performed at the Continuous Electron Beam Accelerator Facility (CEBAF) in Newport News, Virginia where a 4 GeV 200 μA continuous wave

(CW) superconducting linac is nearing completion. Understanding the multipass BBU instability is crucial for considering such a device for producing high energy and high current electron beams for nuclear physics research (CEBAF's main mission) or for driving a FEL where the energy of the beam after undergoing a lasing interaction is returned to the accelerator. The large amount of energy remaining in the beam after passing through the optical cavity can be recovered, thereby greatly increasing the efficiency of the FEL.

Before proceeding with the main discussion of the multipass BBU instability and energy recovery, it is useful to review some general aspects of particle motion in accelerators. Although particle beams vary widely in specific properties, two basic statements can be made that constitute a reasonable definition of a beam. First, the beam is made up of an ensemble of particles that have one momentum component much larger than the other two components so that each particle tends to follow the same trajectory through space. The common trajectory in a given machine is known as the "design" or "central" orbit. Particle motion at a given point along the design orbit is described naturally in terms of a curvlinear coordinate system anchored to the design orbit. Particle coordinates are expressed as functions of the arc length parameter along the design orbit. Second, the individual particles that constitute the beam remain "near" the design orbit. Using the curvlinear coordinate system the particle equation of motion is expanded in a series about the design orbit and then solved (see [Br84]). The first-order solution to the equation of motion shows that particle motion is harmonic (both longitudinally and transverse) about the design orbit (higher-order terms become important for particles that stray too far from the design orbit).

The most common acceleration technique employed is based on a resonance condition that must be satisfied in order for acceleration to occur. The particles obtain a small energy increment many times thereby gaining a large energy at the exit of the accelerator. The resonance condition guarantees that the acceleration field is applied when particles are present. The resonance condition implies that a particle beam must consist of pulses of charge (a bunched beam) separated equally in time such that the bunch separation is equal to (or some integral multiple of) the period of the acceleration field of the accelerator.

A common approach to particle acceleration is provided by the linear accelerator or linac. A linac uses the confined electromagnetic field of a microwave cavity to accelerate charged particles. To illustrate the idea, consider the simple cylindrical (pillbox) cavity excited at the frequency of its TM_{010} mode. This mode has an electric field that oscillates in the axial z direction and a magnetic field that circulates around in the ϕ direction. To first-order the electric field is constant for small values of r/a and the magnetic field is linear in r/a where r is the transverse coordinate and a is the cavity radius. If a charged particle beam made up of small bunches separated in time by the period of the TM_{010} cavity mode is made to pass through the cavity along its axis, the beam as a whole can be accelerated by the electric field and gain energy.

A simple linac consists of a chain of microwave cavities arranged along the axial coordinate. Each cavity imparts a small energy increment so that when the beam arrives at the end it has gained an energy equal to the number of cavities times the energy increment per cavity. The energy increment is usually expressed in terms of the energy gain per meter of acceleration structure or gradient. In practice a linac "cavity" is commonly made up of smaller units similar to the simple pillbox cavity coupled together and powered from a single radiofrequency (RF) power source. The CEBAF superconducting cavities are made of five elliptically-shaped cells coupled together to form a single structure. They are made of niobium and when cooled below 9.2° K are superconducting. As a result typical Q values for the fundamental accelerating mode (similar to the TM_{010} mode of the pillbox cavity) are very large and in the range 10^9-10^{10} [Su85] (as compared to similar room temperature Cu structures which have Qs typically around 10^4 [Ja83]) due to the very small BCS and other surface current losses.

Bunching of the particle beam is necessary for acceleration using a linac. Beam bunches are typically small in two ways: their transverse dimension is small enough so that the near-axis approximation for the cavity mode fields and other electromagnetic fields encountered in the accelerator is valid to first-order; and the axial bunch length is small compared to the temporal bunch spacing times the velocity of the particle beam (or equivalently the wavelength of the cavity acceleration mode). Electrons are so light that they reach relativistic speeds at low energy. At only 1.5 MeV kinetic energy electrons are already traveling at .967 c; so that, for a bunched electron beam, the bunch spacing remains fixed as the beam accelerates. The longitudinal motion of electrons within a bunch is also quickly frozen because of the relativistic motion. Electron linacs are therefore constructed using cavities of the same size distributed along the linac. In contrast, proton linacs are complicated by the fact that protons are not relativistic at low energies. Proton linac design must therefore take into account the velocity increase of the beam particles as the beam is accelerated. The largest electron linac is located at the Stanford Linear Accelerator Center (SLAC); it is 3 kilometers long and uses room temperature cavities. The SLAC linac operates as a pulsed accelerator so that power dissipation in the cavities is not excessive. It is used to accelerate electrons and positrons to 50 GeV.

Two accelerators that improve the efficiency of a linac are the microtron and the recirculating linac. In both of these machines a recirculation scheme using magnets is employed so that the beam passes through the linac more than once. The resulting beam energy after N passes is the same as if N linacs arranged in a straight line were used to accelerate the beam. The basic difference between the two types of machines is the magnetic recirculation method involved. Microtrons use two large 180° dipole bending magnets that bend all particle orbits back to the linac axis. The linac axis is a line of tangency for all orbits and the dipole magnets contain all orbits of the machine. Microtrons have been used at a number of laboratories such as the facilities at the Universities of Illinois and Mainz. The MUSL-2A microtron at Illinois employed a superconducting linac that was capable of accelerating an electron beam to 100 MeV after nine orbits. The recirculating linac employs separate beam lines for each orbit and has common dipole bending magnets on the high and low energy end of the linac that serve (respectively) to separate and combine each orbit according to its energy. The High Energy Physics Lab at Stanford (HEPL), the MIT-Bates facility, and CEBAF employ recirculating linacs. CEBAF will employ two 400 MeV superconducting linacs in a racetrack configuration that can pass the beam up to five times through each linac. Maximum electron beam energy will be 4 GeV at a CW current of up to 200 μA. For both types of machine the time for particles to traverse each orbit must be exactly equal to an integral number of RF periods of the cavity acceleration mode for acceleration to occur.

It is important to understand the concept of the duty factor because this parameter determines the average beam current. When bunches enter the cavity each cycle of the electromagnetic field the beam is said to be CW or continuous wave. When there are intervals when no bunches are present the beam is said to be pulsed. The duty factor, usually expressed as a percentage, describes the fraction of fundamental acceleration mode cycles beam is actually present. A CW beam therefore has a duty factor of 100%. A duty factor of 1% would describe a beam that, for example, was made up of a pulse of current containing 10 bunches repeated every 1000 acceleration mode cycles. A practical, high energy CW accelerator cannot employ room temperature cavities with large acceleration gradients because of the excessive power dissipation in the cavity walls. SLAC is therefore a pulsed, low duty factor machine because it uses room temperature cavities at high gradient whereas CEBAF uses high gradient superconducting cavities which experience very little dissipation and is a CW machine. Finally an important consideration for microtrons and recirculating linacs such as CEBAF arises because multipass BBU instabilities can become severe when these machines use superconducting cavities to accelerate high current CW beams.

1.1 Linac Beam Breakup Instabilities

The subject of BBU has generated much interest because these instabilities pose fundamental limitations on linac performance. The understanding of the instability (and of similar collective beam instabilities in synchrotrons and storage rings) has been the goal of a great deal of effort in accelerator physics since the first large scale experience at SLAC of beam loss due to BBU [Pa66]. The banishment of BBU at nominal operating conditions through clever machine design has been the ultimate quest of research into these types of beam instabilities. Linacs that use superconducting cavities to accelerate high average current (up to 200 μA at CEBAF) CW beams are especially prone to BBU. The instability is due to collective interactions of the beam, and involves the various acceleration, bending, and focussing electromagnetic fields used to guide the beam through the linac.

In linacs, and especially in linacs that use superconducting cavities, cavity modes other than the fundamental acceleration mode known as higher-order modes (HOMs) can adversely affect the motion of the beam. Of particular concern are modes similar to the TM_{110} mode in a pillbox cavity. These modes have constant transverse magnetic fields near the axis and longitudinal electric fields that increase linearly in the radial coordinate r near the axis. They are known as dipole modes because the fields also have a sinusoidal ϕ dependence (similar to a dipole radiation field) and hence act to deflect particle beams in one plane.

The basic cause of the instability is transverse beam deflection caused by the HOM magnetic field. The electric field can then couple to the charge of the beam bunches and kinetic energy can be transferred from the beam to the mode. It is the interaction of the beam with these HOMs that is the origin of collective instabilities in linacs. Other TM modes that have transverse magnetic fields that go as $(r/a)^{n-1}$ and longitudinal electric fields that go as $(r/a)^n$ [Ba89] where $n \geq 2$ (quadrupole, sextupole, etc. modes) near the axis can also cause instability but are not as important as the dipole modes because typical beam dimensions are small compared to the cavity radius a. This means that deflection of the beam due to the quadrupole mode ($n = 2$) can be expected to be a factor of r/a smaller than a deflection due to a dipole ($n = 1$) mode of the same quality factor for near axis beam bunches. In the rest of this thesis, unless otherwise stated, the term HOM will refer to a dipole mode because they are potentially the most destructive to the beam.

The first instability relevant to linacs, known as regenerative BBU, was first observed in a number of short, high-current commercial linacs in 1957 [He66]. Regenerative BBU occurs as the beam passes through a single acceleration structure or cavity. The regenerative interaction begins when an on-axis bunch is deflected by the magnetic field of a HOM within the cavity. The kick the bunch receives is then translated into a transverse displacement further down the structure. The displaced bunch interacts through the longitudinal electric field of the HOM resulting in a loss of kinetic energy of the bunch, leading to additional excitation of the mode (throughout the structure) which then kicks following bunches harder. The electromagnetic field of the HOM is enhanced because bunches receiving the stronger kicks are displaced a greater distance off axis. There is, therefore, closure of a feedback loop within the structure. The amount of beam kinetic energy loss depends upon the phase of the HOM field when the bunch arrives in the structure, with maximum energy loss occurring when the maximum electric field of the HOM opposes the bunch motion. There exists a maximum average beam current (the threshold current) for this interaction such that above threshold there is exponential growth of the HOM fields with time resulting in beam loss in a wall or aperture of the linac. For a standing wave linac the threshold current is given by [Wi82]

$$I_{th} = \frac{\pi^3 E}{2eRkL}, \tag{1.1}$$

where E is the energy of the beam, k is the wavenumber of the HOM, R is the shunt impedance of the HOM (which measures the efficiency with which energy is transferred from the beam to the mode), e is the electron charge, and L is the length of the acceleration structure.

The threshold for the regenerative BBU instability can be explained simply as that point where the energy the beam deposits in the mode (per unit time) due to the deflection just equals the energy dissipated by the mode through losses [La70]. In practice the regenerative interaction is suppressed in modern linacs by keeping individual acceleration structures short and the shunt impedance of typical HOMs low. For the CEBAF/Cornell cavities the Q of the HOM's are reduced relative to the Q of the fundamental acceleration mode through the use of waveguide couplers and HOM loads resulting in shunt impedances five orders of magnitude lower than for the fundamental acceleration mode. As a result, threshold currents for regenerative BBU at CEBAF are relatively high compared to other instabilities, and are calculated to be on the order of tens of amperes. Without the use of HOM damping at CEBAF the threshold currents could be on the order of 100 μA rendering the CEBAF/Cornell cavity useless for accelerating the design 200 μA beam.

The first large scale encounter with BBU occurred at SLAC in 1966 [Al66, Pa66] just after the linac was first turned on. The instability observed is known as cumulative BBU and results from the beam interacting with two or more cavities that make up the linac. Cumulative BBU begins when a bunch receives a transverse kick from a cavity HOM resulting in a transverse displacement in a given downstream cavity. The optical parameters that drive the instability are the angle to displacement transfer matrix elements in both transverse planes (M_{12} for the x-plane and M_{34} for the y-plane). The displaced bunches can drive the HOM at the second cavity coherently thereby transferring kinetic energy into the HOM. Following bunches arriving at the downstream cavity are then more strongly deflected because of the additional energy contained in the HOM, and the whole process continues as more beam bunches pass through the linac.

The important difference between cumulative BBU and regenerative BBU is that in cumulative BBU there is no feedback of the HOM energy of the driven cavity back to the cavity that initially deflects the beam. The cavities act to amplify the beam offsets due to the kick, and this amplification is strongly dependent on beam current. The worst case for the instability occurs when a harmonic of the bunching frequency is within a half width of the HOM frequency [Bi88a] so that the beam itself can excite cavity modes coherently. Another possibility for worst case beam deflection occurs when an upstream cavity HOM and a downstream cavity mode overlap in frequency which can occur in a long linac with many identical cavities.

The threshold current for cumulative BBU is the current where the offsets are amplified to the point where the beam hits the beam pipe. The offsets can, in principle, be suppressed by adjusting the optics between the cavities so that a beam which is deflected by the first cavity crosses the axis of the second cavity; this minimizes the coupling between the beam and the HOMs of the second cavity. This can be mathematically summarized by requiring that M_{12} and M_{34} equal zero between cavities. At SLAC, cumulative BBU was found to be due to the extreme length of the linac and the high Q of the HOMs. The cure at SLAC was to adjust the quadrupole focussing especially at the low energy injection end where HOM deflections are most severe and to detune the HOMs from section to section to prevent coherent excitation of HOMs along the linac [Ne68]. Cumulative BBU is expected to become important for currents on the order of one ampere [Kr86] for the first pass through the CEBAF main linac, a value that is still three orders of magnitude above the design current.

Multipass beam breakup is an instability that is of most concern in accelerators such as microtrons and recirculating linacs (like CEBAF) in which the beam is passed many times through the same linac structure. The important mechanism that causes the instability is the fact that the recirculated beam can be displaced at a given cavity due to a kick it receives from a HOM in the same cavity on a previous pass. The displaced beam can then interact with the fields of the HOM on subsequent passes and feed energy into it causing subsequent bunches to be kicked even harder, and so on. The M_{12} and M_{34} matrix elements (in this case for the recirculation) are important for the feedback aspect of the instability. There is also a cumulative aspect to multipass BBU [Bi88a] because the beam effectively sees N_p linacs in a row where N_p is the number of times the beam passes through the linac. Many cavities can therefore contribute coherently to the instability if their HOM frequencies overlap.

A definite average threshold beam current analogous to that for regenerative beam breakup exists where the power fed into the mode equals the mode power dissipation. The threshold current therefore depends on the various beam, transverse optical, and HOM parameters. Above threshold, HOM fields grow until the beam is deflected into the wall or an aperture of the machine. The first or low energy passes of the beam are most likely to exhibit this instability because deflections of the low energy beam result in the largest subsequent displacements in the machine. It is crucial for the CEBAF/Cornell superconducting cavities that they include HOM damping to reduce the HOM Q's otherwise the CEBAF linac would be limited by multipass BBU (as well as regenerative and cumulative BBU) to less than 200 μA CW current. Of the three types of BBU, multipass BBU is calculated via computer simulation using the beam breakup code TDBBU [Kr90] to be the limiting type at CEBAF with threshold currents in the range 11-24 mA [Kr90]–two orders of magnitude above design. A derivation of the threshold current is presented in the next section which illustrates how the theory is used to calculate BBU properties.

The CEBAF accelerator is an example of a linac where the combination superconducting cavities and the use of a recirculation scheme to accelerate the electron beam poses a unique set of potential multipass beam instability problems. The commissioning of the injector linac at CEBAF offered a unique opportunity to investigate experimentally multipass BBU problems basic to superconducting recirculating linacs. The experiments described in this thesis extended earlier beam instability measurements [Ly83] to the substantially higher beam currents, acceleration gradients, and parasitic mode damping used in the CEBAF/Cornell cavities. Another issue of current interest is the use of superconducting linacs in the construction of high-efficiency FEL's where the beam, after having gone through a lasing interaction, is returned through the linac 180 degrees out of RF phase with the electric field of the acceleration mode, thereby returning energy to the field. This process, called energy recovery, was performed at CEBAF and extends previous measurements [Ne89, Sm86] to the substantially higher accelerating gradients of the CEBAF/Cornell cavities. RF measurements of cavity HOMs were also performed while running the recirculator in the energy recovery mode.

A beam transport system was constructed at CEBAF [Ba90] that permitted the beam emerging from the CEBAF injector linac to be recirculated so that it passed twice through same linac cavities. The injected beam energy into the recirculator was 5.6 MeV. After passing once through the linac the energy was 42.8 MeV and after recirculation through the linac the beam was dumped at an energy of 80.1 MeV. The recirculated CW beam was used to directly measure HOM resonances in the CEBAF/Cornell superconducting cavities using an transverse RF stripline kicker and both a cavity as well as an RF stripline pickup. These are the first such measurements of HOM resonances using a recirculated CW beam in the CEBAF/Cornell cavities.

The main result of this experiment is that in the worst case location in the machine in terms of energy (5.6 MeV injection energy into the recirculator) and at the highest beam currents available from the injector (in excess of 200 μA CW beam current) the beam was not unstable due to multipass beam breakup. Adjustment of the transverse recirculation optics was made in an attempt to find a set of optics which would cause the beam to go unstable. These measurements confirm experimentally that the CEBAF/Cornell cavity HOM damping design is adequate to protect the machine against the multipass instability. The experimental results are compared to computer simulation along with a simple single cavity/HOM theoretical model of multipass BBU. Thresholds for multipass BBU computed using TDBBU were found to be between 5 and 20 mA depending on the tune of the recirculation optics. For comparison the main CEBAF recirculating linac is estimated to have threshold currents between 11 and 24 mA [Kr90].

Finally, there exist longitudinal multipass BBU effects that can occur due to longitudinal HOMs (longitudinal modes other than the fundamental acceleration mode). The relevant parameter that describes the optics central to the instability is the longitudinal momentum kick to longitudinal displacement matrix element (M_{56}) for the recirculation path. The feedback occurs analogous for transverse multipass BBU because a longitudinal momentum kick is translated to longitudinal displacement through non-isochronicity of the recirculation path. The longitudinal HOMs can therefore drive themselves coherently (analogous to transverse multipass BBU) because of recirculation. An estimate [Bi88b] of the threshold current for longitudinal multipass BBU for the CEBAF injector recirculator is about 200 mA, or about an order of magnitude higher than the expected transverse multipass BBU threshold currents.

1.2 Energy Recovery Using Superconducting Linacs

A promising aspect of using of superconducting linacs to drive FELs is that it is possible, in principle, to recover a large fraction of the energy in the accelerated beam [Ro88]. Since a superconducting cavity is very efficient at supplying energy to the beam and vice versa because of the low wall losses, a very efficient (in terms of wall plug power to laser power) FEL is possible. Since the wall losses and the cooling needed depend only on the operating gradient in the cavity, high average beam current implies the possibility of high overall efficiency provided almost all of the beam energy is recovered. Even if extremely high efficiency is not obtainable by this technique, the recovery of a significant fraction of the energy in the beam will reduce the operating costs of the FEL by greatly reducing the RF power requirements.

RF beam steering effects from transverse misalignment or from tilts of the symmetry axis of the cavities away from the nominal centerline scale as the gradient. As a consequence, recovering large fractions of the energy is expected to become increasingly difficult as the gradient is increased. Another problem is due to the fact that magnetic elements incorporated into the accelerator to correct steering errors for the accelerating beam will generally not work well for the decelerating beam. This is because at any point in the accelerator, the decelerated beam will generally be at a different energy than the accelerated beam, so the same correction will not work properly for both beams. It is important to know at what gradient steering errors become impossible to control, and understand which correction schemes utilizing the recirculation optics are best for minimizing this problem.

For the CEBAF energy recovery experiment reported here the main extensions beyond the SCA experience are the high accelerating gradient of 5 MV/m and the greater number of accelerating cavities (16) inside the recirculation path. The additional element needed to do this experiment is a way to vary the recirculation path length by one half an RF wavelength, or 10 cm. To accomplish the necessary path length adjustment, the first 180° bend was mounted on a stand so that a “trombone” pathlength adjustment was possible by translating the bend in the direction of the linac axis by at least 5 cm. This amount of path length adjustment is adequate for both acceleration and deceleration of the second pass beam.

The main result of the experiment is that full energy recovery (within the 1.8% energy measurement uncertainty) was achieved with the second pass beam at up to 30 μA CW beam current for the first time using the CEBAF/Cornell superconducting cavities. Specifically, the second pass beam, initially at 42.8 MeV at the entrance of the linac, was decelerated to the injection energy of 5.6 MeV in this experiment. This indicates a maximum of 1.1 kW of beam power delivered to the fundamental mode field of the linac cavities by the second pass beam at 30 μA.

The main limitation on achieving higher CW beam currents was that the second pass beam was large transversely and prone to scrape on small apertures in the system as it lost energy. Allowing for the fact that the recirculation optics were optimized for second pass acceleration, improvements in the recirculation optics would no doubt have allowed energy recovery for even higher currents. No evidence for multipass BBU was observed for the energy recovery run. Finally, an improvement to the experiment would be to incorporate a way to change the emittance and energy spread to mimic an FEL lasing interaction.

Chapter 2

Theory and Simulation of Multipass BBU

2.1 Overview

As the average current of a bunched particle beam passing through a linac is raised, interactions between beam bunches play an increasingly prominent role in determining the overall dynamics of the beam. The beam bunches can interact through the electromagnetic fields, known as wakefields, they excite in the linac structure through which they pass. The cavity wakefields induced by the bunch on previous passes couple to the bunch charge, resulting in feedback that can further enhance the wakefield. The wakefields manifest themselves as excitations of the various cavity HOMs which can deflect the beam so that it strikes a wall and is lost.

For fully relativistic beams such as the CW electron beam at CEBAF, calculation of the effect of the bunch-wakefield interaction is simplified by the fact that the bunches remain fixed longitudinally. The multipass BBU instability is due to the wakefields produced by bunches traversing the cavities off the linac axis. The excited HOMs can give a transverse momentum kick to subsequent bunches. The transverse deflection in general results in a transverse displacement on following passes through the linac causing further HOM excitation, and ultimately instability if the average beam current is high enough. The following sections present a formal approach for describing the transverse wakefield in terms of a transverse wake function. The transverse wake function is then used to compute the threshold conditions and response functions for the beam-HOM interaction for the simplest case of a linac–a single cavity containing a single HOM. The results derived from this simple model are used in Chapter 4 to determine bounds on the threshold current for multipass BBU for each optical setting from the RF measurement data.

2.2 Treatment of the Beam-Wakefield Interaction

2.2.1 Introduction

The physics of a charged particle beam interacting with an accelerating structure can be understood in terms of the wakefield. The charged particles that make up the beam interact with the various normal modes in the accelerating structure according to Maxwell's equations and the Lorentz force. The concept of the electromagnetic wakefield is a useful formulation of the solution to the Lorentz force equation which makes calculation of the beam dynamics straightforward. The formulation is quite general and can be used to describe the motion of charged particles traversing enclosed regions where electromagnetic fields are present. Coupling of the charge of the bunch to the electric field of the HOM is the basic energy exchange mechanism. For the case of multipass BBU, it is the transverse deflection of the beam by the cavity HOM field that drives the instability.

Early models described the interaction in a general fashion by treating the cavities as sets of coupled resonators [He66, Vi66]. The CEBAF cavities can be treated as uncoupled due to the fact that the beam pipe separating them has a cutoff frequency (about 6 GHz for a beam pipe of radius 1.9 cm) well above the frequency of the HOMs of greatest concern (that is, those that have the highest Q and/or shunt impedance) which lie below 2.5 GHz [Am84]. The theory outlined here [Bi87, Co41, Gl85, Kr87, Ve80] and on which the transverse BBU simulation code TDBBU is based, treats the cavity HOMs as high Q, uncoupled oscillators, where each HOM has two orthogonal polarizations that are furthermore assumed to be uncoupled.

In the discussion that follows we will focus on the transverse wake and its description in terms of the transverse HOMs of the cavities (the treatment of the longitudinal

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Figure 2.1: Definition of test and exciting charge coordinates.

wake is similar [Wi82]). In the analysis that follows, beam motion in the x plane will be considered in detail; the y plane is analogous for the typical case of no coupling of the x and y beam motions. Figure 2.1 shows the situation where a given exciting charge q_e induces a wakefield in a structure which then acts on a following test charge q_t. The electromagnetic wakefield induced by q_e displaced from the cavity axis a distance d in the x direction in the accelerator is:

$$E_x(\mathbf{r}, t; d) - cB_y(\mathbf{r}, t; d) = \int_{-\infty}^{\infty} j(\mathbf{r}', t'; d) G(\mathbf{r}, t; \mathbf{r}', t') d\mathbf{r}' \tag{2.1}$$

where $G(\mathbf{r}, t; \mathbf{r}', t')$ is the electromagnetic Green's function and,

$$\mathbf{r} = (x, y, z) \tag{2.2}$$

$$\mathbf{r}' = (x', y', z') \tag{2.3}$$

$$j(\mathbf{r}', t'; d) = q_e c \delta(x' - d) \delta(y') \delta(z' - ct'). \tag{2.4}$$

The transverse fields in equation 2.1 are related to the transverse momentum kick given to q_t by the Lorentz force,

$$\frac{c}{q_t} \frac{dp_x}{dz} = E_x(\mathbf{r}, z/c + \tau; d) - cB_y(\mathbf{r}, z/c + \tau; d) \tag{2.5}$$

where dp_x/dz is the transverse momentum kick per unit length of acceleration structure. Here c is the velocity of light (the charges are assumed to be relativistic), and τ is the time delay by which q_t follows q_e. The transverse wake function is defined as the total transverse momentum change of the test charge due to electromagnetic wakefield of the exciting charge divided by the magnitude of the test charge q_t, the magnitude of the exciting charge q_e, and the displacement d.

$$W(\tau) \equiv \left(\frac{c}{q_e q_t d} \right) \Delta p_x(\tau, d). \tag{2.6}$$

In terms of the electromagnetic wakefield given by equation 2.1 the wake function is given by,

$$W(\tau) \equiv \frac{1}{q_e d} \int [E_x(\mathbf{r}, z/c + \tau; d) - cB_y(\mathbf{r}, z/c + \tau; d)] dz. \tag{2.7}$$

Equation 2.6 most clearly shows that the wake function is an equivalent way of expressing the total transverse momentum change (kick) imparted to the test charge due to the wakefield of the the exciting charge. For fully relativistic particles, the electromagnetic fields are proportional to the displacement d of the exciting charge off the axis, and are independent of the transverse position, x, of the test charge, so that, in this limit, the transverse wake function is independent of d. The wake function given by equations 2.6 and 2.7 therefore depends only on the single time delay variable τ. To a good approximation, this is the situation for a superconducting electron linac such as CEBAF where the energy of the electrons after the first two superconducting cavities in the injector is 5.6 MeV corresponding to a velocity of .996 c. The wake function is expressed in MKS units as volts per coulomb per meter as may be seen upon inspection of equation 2.7.

It is useful to express the wake function in terms of HOM parameters (that can be computed and/or measured for a given cavity geometry) by using the definitions given by equations 2.6 and 2.7. The complete wake function that describes all the HOMs in the cavity is approximated as an expansion over all the HOMs in the structure [Co41]. The result is [Kr90, Wi82]

$$W(\tau) = \sum_m W_m(\tau) \tag{2.8}$$

$$W_m(\tau) = \left(\frac{(R/Q)_m k_m \omega_m}{2} \right) e^{-\frac{\omega_m \tau}{2Q_m}} \sin(\omega_m \tau), \tag{2.9}$$

where ω_m and k_m are the frequency and wavenumber of the HOM denoted by the subscript m. The quantity $(R/Q)_m$ is the shunt impedance divided by the HOM quality factor and is a purely geometric property of the accelerating structure. In particular, $(R/Q)_m$ is independent of the HOM quality factor and describes the strength of the excitation of the cavity due to the passage of short, single bunches. Q_m is the usual

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Figure 2.2: TM_{110} mode in a cylindrical "pillbox" cavity.

quality factor for the HOM which determines the time it takes for the HOM excitation to decay after the passage of a short bunch. The shunt impedance of the HOM $R_m = (R/Q)_m Q_m$ describes the average excitation of the HOM after many bunches have passed through the cavity and thus determines the average threshold current for BBU instabilities. The quantity $(R/Q)_m$ is sometimes called the "shunt impedance" in the literature resulting in confusion over terminology. For clarity, $(R/Q)_m$ will be denoted as the shunt impedance of the HOM throughout this work.

Equation 2.9 indicates that the functional form of $W(\tau)$ is that of a simple damped oscillator. The electromagnetic wakefield also has the same functional form which implies that $W(\tau)$ is simply proportional to the wakefield. Indeed the proportionality is evident from the wake function definition given by equation 2.7. For the low frequency regime of typical HOMs of most concern for the CEBAF superconducting cavities, the transverse wake function is completely described by an expansion in terms of the complete set of high Q HOM states represented by equation 2.8.

Figure 2.2 shows the field configuration of a typical dipole HOM. The figure shows a pillbox cavity and a TM_{110} mode which has an on-axis magnetic field and an electric field that varies linearly with distance off-axis. A charged particle passing through the cavity can be deflected by the magnetic field and excite the mode through the longitudinal electric field if it passes off-axis. The dipole modes in actual CEBAF cavities have this same TM_{110} like structure near the axis.

The transverse impedance is defined as proportional to the Fourier transform of the transverse wake function according to [Wi82],

$$Z(\omega) = i \int_{-\infty}^{\infty} W(\tau) e^{i\omega\tau} d\tau. \tag{2.10}$$

This impedance, multiplied by the current moment (the product of beam current and transverse beam offset) in the frequency domain, yields the voltage induced in the structure by the beam in the steady state. Using equations 2.8 and 2.9 an expression for the impedance in terms of HOM parameters is given by,

$$
\begin{aligned}
Z(\omega) &= iW(\omega) && (2.11)\\
W(\omega) &= \sum_m W_m(\omega) && (2.12)\\
W_m(\omega) &= \frac{\rho_m}{Q_m}\left\{1-\left(\frac{\omega}{\omega_m}-\frac{i}{2Q_m}\right)^2\right\}^{-1} && (2.13)\\
\rho_m &= \frac{(R/Q)_m Q_m k_m}{2}. && (2.14)
\end{aligned}
$$

Considering only one HOM term in equation 2.12 one can study each HOM by defining the normalized wake function and normalized frequency as,

$$
\begin{aligned}
\Upsilon_m(\Omega) &\equiv \frac{W_m(\Omega)}{\rho_m} && (2.15)\\
\Omega &\equiv \frac{\omega}{\omega_m}. && (2.16)
\end{aligned}
$$

The normalized wake function when explicitly written out depends only on the normalized frequency and the quality factor of the mode

$$
\begin{aligned}
\Upsilon_m(\Omega) &= A_m(\Omega)e^{-i\phi_m(\Omega)} && (2.17)\\
A_m(\Omega) &= \left\{Q_m^2(1-\Omega^2+1/4Q_m^2)^2+\Omega^2\right\}^{-\frac{1}{2}} && (2.18)\\
\phi_m(\Omega) &= \tan^{-1}\left\{\frac{\Omega}{Q_m(1-\Omega^2+1/4Q_m^2)}\right\}. && (2.19)
\end{aligned}
$$

Figure 2.3 shows plots of $A_m(\Omega)$ and $\phi_m(\Omega)$ for typical values of Q_m for CEBAF cavity HOMs. The resonance condition for each HOM is found by setting the derivative of $A_m(\Omega)$ equal to zero so that on resonance,

$$
\begin{aligned}
\Omega_r &= \sqrt{1-\frac{1}{4Q_m^2}} && (2.20)\\
A_m(\Omega_r) &= 1 && (2.21)\\
\phi_m(\Omega_r) &= \tan^{-1}\sqrt{4Q_m^2-1}. && (2.22)
\end{aligned}
$$

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Figure 2.3: Amplitude and phase of the normalized wake function for a HOM. The plots show the response for three different HOM Q values.

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Figure 2.4: CEBAF/Cornell superconducting cavity pair showing the HOM damping scheme.

The amplitude plot shows that on resonance $\Omega_r \sim 1$ or equivalently $\omega_r \sim \omega_m$ for the condition $Q_m \gg 1$. The amplitude of the oscillation on resonance is very large and equal to ρ_m for the HOM. This result is very important for a superconducting linac such as at CEBAF because in principle the HOM Q_m values might be as high as 10^{10}–the same order of magnitude as the fundamental accelerating mode Q. The CEBAF/Cornell cavity design shown in figure 2.4 incorporates special waveguide HOM couplers which act to absorb as much HOM energy as possible thereby lowering Q_m to values in the range $10^4 - 10^5$ [Am84]. The phase plot shows that on resonance $\phi(1) = 90°$ relative to the DC or $\Omega = 0$ situation. The impedance so defined by equation 2.10 is therefore a real quantity at each HOM resonance.

The wakefield concept is used in the following sections to derive expressions for threshold conditions and response functions for the simplest linac–a single cavity containing a single transverse HOM in addition to the fundamental accelerating mode. The simple linac, with the addition of a single recirculation, will serve to illustrate how the multipass BBU threshold depends on the various machine parameters such as energy, average current, and optics; in addition to HOM transverse impedance, frequency, and Q_m. The single cavity linac is studied in both the time and frequency domains. Finally, a simulation analysis of multipass BBU in the CEBAF injector using the code TDBBU is performed. The simulation takes into account the fact that the linac contains 16 cavities each containing more than one important HOM.

2.2.2 Time Domain Threshold Current Calculation for One Cavity with One Higher-Order Mode and One Recirculation

Figure 2.5 shows schematically a cavity where the (electron) beam enters from the left with momentum p_i, is recirculated along the dashed path with momentum p_r, and exits the cavity on the second pass with momentum p_f. The figure shows both the central trajectory (solid line) and the trajectory of the beam caused by the HOM deflection (dashed line). Assuming the beam enters the cavity on axis, the transverse deflecting voltage, known as the transverse wake potential, of the HOM in the time domain is given by:

$$V(t) = \int_{-\infty}^{t} W(t-t')I(t'-t_r)x_c^{(2)}(t')dt'. \tag{2.23}$$

Here the displacement of the beam at the cavity on the second pass at time t is denoted by $x_c^{(2)}(t)$. The wake potential is interpreted in terms of the transverse momentum kick $p_x(t)$ given to the beam at time t according to,

$$p_x(t) = \frac{eV(t)}{c}. \tag{2.24}$$

The angular deflection of the beam at the exit of the cavity on the first pass is the ratio:

$$\theta_c^{(1)}(t) = \frac{p_x(t)}{p_r} \tag{2.25}$$

where p_r is the longitudinal momentum of the beam after the cavity.

The displacement of the beam on the second pass can be written as:

$$x_c^{(2)}(t) = M_{12}^{(r)}\theta_c^{(1)}(t-t_r) \tag{2.26}$$

where $M_{12}^{(r)}$ represents the angle to displacement recirculation transfer matrix element. The bunch passing the cavity on the second pass at time t was kicked by the HOM at time $t-t_r$ where t_r is the recirculation time (the time it takes a bunch to travel from a point in the cavity on the first pass to the same point on the second

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Figure 2.5: Single cavity linac with a single recirculation path. The cavity is assumed to contain a single HOM in addition to the fundamental acceleration mode.

pass). Substituting equations 2.24, 2.25, and 2.26 into equation 2.23 results in an integral equation for the wake potential $V(t)$.

$$V(t) = \frac{eM_{12}}{p_r c} \int_{-\infty}^{t} W(t-t')I(t'-t_r)V(t'-t_r)dt'. \tag{2.27}$$

The beam current is approximated by:

$$I(t) = \sum_{n=-\infty}^{\infty} I_\circ t_\circ \delta(t - nt_\circ) \tag{2.28}$$

where the delta function structure approximates the bunched nature of the beam, $I_\circ$ is the average current, and $t_\circ = 2\pi/\omega_\circ$ is the bunch period. For the case of a single HOM, $W(\tau)$ as given by equation 2.8 consists of only one term.

A normal mode solution for the wake potential is assumed and substituted into equation 2.27 where

$$V(t) = V_\circ e^{-i\omega t}. \tag{2.29}$$

In this expression, the imaginary part of ω describes exponential growth above threshold. Using equations 2.28 and 2.9 for the beam current and the wake function, equation 2.27 becomes:

$$\begin{aligned} V_\circ e^{-i\omega t} &= \kappa V_\circ \sum_{n=-\infty}^{\infty} \int_{-\infty}^{t} e^{-\frac{\omega_m(t-t')}{2Q_m}} \sin(\omega_m(t-t')) \times \\ &\qquad \delta(t' - nt_\circ - t_r) e^{-i\omega(t'-t_r)} dt' \\ \kappa &= \frac{eM_{12}^{(r)} I_\circ t_\circ (R/Q)_m k_m \omega_m}{2p_r c}. \end{aligned} \tag{2.30}$$

Performing the integration over the delta function results in:

$$e^{-i\omega t} = \frac{\kappa}{2i} e^{-\frac{\omega_m(t-t_r)}{2Q_m}} \sum_{n=-\infty}^{l} \left\{ e^{i\omega_m(t-t_r)} e^{n\left\{\frac{\omega_m}{2Q_m} - i(\omega+\omega_m)\right\} t_\circ} - \right.$$

$$e^{-i\omega_m(t-t_r)}e^{n\left\{\frac{\omega_m}{2Q_m}-i(\omega-\omega_m)\right\}t_\circ}\Bigg\} \tag{2.31}$$

$$l = \frac{t-t_r}{t_\circ} \tag{2.32}$$

where l is the number of bunches that have passed through the cavity on the second pass at time t (here t is given as an integer number of bunch spacings $t_\circ$). The sum in equation 2.31 is of the form:

$$\sum_{n=-\infty}^{l} e^{nz_\pm} = \frac{e^{(l+1)z_\pm}}{e^{z_\pm}-1} \tag{2.33}$$

$$z_\pm = \left\{\frac{\omega_m}{2Q_m} - i(\omega \pm \omega_m)\right\} t_\circ \tag{2.34}$$

which converges to the value given when $\mathrm{Re}(z) > 0$. Substitution of equations 2.33 and 2.34 into equation 2.31 results after simplification in the equation for the complex frequency ω,

$$1 = \kappa e^{i\omega t_r}\left\{\frac{\xi \sin(\omega_m t_\circ)}{1-2\xi\cos(\omega_m t_\circ)+\xi^2}\right\} \tag{2.35}$$

$$\xi = e^{\frac{\omega_m t_\circ}{2Q_m}} e^{-i\omega t_\circ}.$$

Equation 2.35 is exact as it stands. For a given set of HOM parameters, the threshold current can be found numerically by treating ω as a real quantity and solving for the current $I_\circ$. The smallest real value of $I_\circ$ is the threshold current. In practice one would only need to search around the HOM resonance peak ω_m for the threshold current since the HOM impedance is maximum near resonance.

By treating κ as a small quantity ($\kappa \ll 1$), a perturbative solution to equation 2.35 is obtained. The complex frequency is approximated to first-order in κ by:

$$\omega \approx a + b\kappa \tag{2.36}$$

where a and b are parameters to be determined. Substitution of equation 2.36 into equation 2.35 and collecting only first-order terms in κ results in the desired expression for ω:

$$\omega \approx \pm\omega_m - \frac{i\omega_m}{2Q_m} \mp \frac{e^{\pm i\omega_m t_r} e^{\frac{\omega_m t_r}{2Q_m}}}{2t_\circ}\kappa. \tag{2.37}$$

The imaginary part of equation 2.37 can be written as:

$$\mathrm{Im}(\omega) = -\frac{\omega_m}{2Q_m}\left\{1 - \frac{I_\circ}{I_\mathrm{t}}\right\}, \tag{2.38}$$

so that, $\mathrm{Im}(\omega) = 0$ at the threshold current as required. Using equations 2.37 and 2.38, the expression for the threshold current is to first-order:

$$I_\mathrm{t} = \frac{-2p_r c}{e(R/Q)_m Q_m k_m M_{12}^{(r)} \sin(\omega_m t_r) e^{\frac{\omega_m t_r}{2Q_m}}}. \tag{2.39}$$

For beam currents slightly larger than the threshold current, the growth rate of the HOM amplitude from equation 2.38 is

$$\mathrm{Im}(\omega) = \frac{\omega_m}{2Q_m} \times \frac{\delta I_\circ}{I_\mathrm{t}} \tag{2.40}$$

$$I_\circ = I_\mathrm{t} + \delta I_\circ \tag{2.41}$$

The momentum dependence (p_r) in Equation 2.39 shows that the threshold current increases as the energy of the beam is increased. This is to be expected because at higher energies the kick given by a HOM to the beam gets translated on the next pass to a smaller displacement. It is therefore expected that the first few passes of recirculating linacs will be most prone to exhibit multipass BBU. The phase factor $\sin(\omega_m t_r)e^{\frac{\omega_m t_r}{2Q_m}}$ is present because the bunch can arrive back at the cavity at a time when the HOM fields are not at their optimum phase for excitation. The matrix element determines the transverse displacement of the second pass beam after having been kicked on the first pass. The threshold current therefore goes down as $M_{12}^{(r)}$ gets larger because the beam couples more strongly to the HOM field the farther off axis it is displaced on the second pass. The quality factor Q_m determines how fast the mode can dissipate energy put into it by the beam. Large values of Q_m therefore imply small threshold currents because of the long time it takes for the energy in the mode to be dissipated by losses. Thought of in terms of energy exchange, the threshold current is the average beam current where the energy given up by the beam to the HOM fields equals the energy dissipated by the HOM due to losses. Above threshold

Parameter	Value
$(R/Q)_m$	21.925Ω
Q_m	90,000
ω_m	1.194×10^{10} rad/s
k_m	39.812 m^{-1}

Table 2.1: Parameters for a CEBAF cavity HOM at $f_m = 1899.54$ MHz.

there is exponential growth and below threshold a steady state is eventually reached. Equation 2.39 is only valid for $I_\mathrm{t} > 0$ which occurs when $M_{12}^{(r)} \sin(\omega_m t_r) < 0$. When this condition is not satisfied the assumptions made in deriving equation 2.39 are not valid and a numerical solution of equation 2.35 must be made.

Table 2.1 lists parameters for a relatively high Q HOM in the CEBAF cavities. Taking a typical beam energy in the CEBAF injector of 23.7 MeV, a typical matrix element from a DIMAD [Se85] simulation of the recirculator $M_{12}^{(r)}$ of -6.779 m, and a the recirculation time $t_r = 320\, t_\circ$ where $t_\circ = 668$ ps, the threshold current from equation 2.39 is .287 Amperes. Without HOM damping as in the CEBAF/Cornell cavity design, Q_m could be a factor of 10^5 larger and the threshold current would be 2.87 μA according to equation 2.39. HOM damping is an essential requirement for superconducting cavities such as CEBAF where ~ 200 μA beam currents are required. Equation 2.39 was checked by using TDBBU to calculate the threshold current for this numerical example using the parameters of table 2.1. The threshold current was found to be .260 Amperes, in good agreement with the approximate formula. For comparison, a numerical solution of the exact formula given by equation 2.35 was performed by treating ω as a real quantity and searching for the value of the frequency that caused the imaginary part of equation 2.35 to vanish. In principle there are many values of the frequency that make the equation real and the search was narrowed by only looking very close to the HOM frequency. The calculated threshold current was found to be .25975 Amperes at a normalized frequency $\Omega = 1.0000152438$, in excellent agreement with TDBBU.

It is useful to estimate the time it takes for the beam loss to occur when the current is near (but above) threshold. It is reasonable to assume that beam loss begins to occur when the amplitude of the HOM angular deflection becomes the same as typical angular deviations in the beam. Using equations 2.24, 2.25 and 2.29, the time it takes for the HOM amplitude to grow large enough for beam loss is given by:

$$\left| \frac{eV_\circ e^{\mathrm{Im}(\omega)t_g}}{p_r c} \right| \sim 10^{-3} \tag{2.42}$$

where the imaginary part of ω is given by equation 2.40 and t_g is the time it takes for the HOM amplitude to grow to the point of beam loss. A reasonable value for the initial HOM amplitude $V_\circ$ at threshold is obtained from the definition of the wake function as:

$$V_\circ \sim \frac{(R/Q)_m k_m \omega_m I_\mathrm{t} t_\circ x_{tp}}{2} \tag{2.43}$$

where x_{tp} is a typical beam offset taken to be 10^{-3} m. Solving equation 2.42 using the previous HOM parameters for the case where the threshold current is 1% above threshold ($\delta I_\circ / I_\mathrm{t} = .01$) results in a growth time of 15 ms for $V_\circ \sim 1$ volt.

The generalization of this simple example of multipass BBU must include the possibility of many cavities, recirculation arcs, and many cavity HOM's. Matters are complicated by the fact that at each cavity site bunches from every pass of the machine are present. The transfer matrices must also be generalized to include HOM kicks between various passes of the machine [Bi87, Gl86, Kr90]. The analysis of real machines therefore is only possible through computer simulation.

2.2.3 Frequency Domain Calculation of the Higher-Order Mode Deflection, Pickup Current Moment and Threshold Current

The wakefield formalism is now used to analyze two situations depicted schematically in figures 2.6 and 2.7 where the wake potential and pickup current moment are analyzed in the frequency domain. The calculations involve determining the effect of

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Figure 2.6: Kicker-Cavity-Pickup single pass case for the frequency domain response function calculations.

signals placed on the beam due to the HOM along with signals placed on the beam in a controlled way by deflecting it transversely similar to a HOM deflection. By calculating response functions for the cavity and beam, threshold properties for multipass BBU identical to those from the previous time domain analysis are obtained. The relative ease of making RF measurements in the frequency domain makes them an attractive way to measure HOM properties as a function of beam energy, current and recirculation transfer matrix elements. The simple analytic results derived here are used in Chapter 4 to analyze the RF measurements that were taken during the course of these experiments.

Figure 2.6 shows a single cavity containing a single HOM preceeded by a kicker device used to deflect the beam transversely and followed by a similar pickup device used to detect transverse position of the beam. For this situation, called the "single pass case," the beam proceeds from left to right passing once through the kicker, cavity, and pickup. Figure 2.7 shows the second situation, the "recirculation case," where the beam passes first through the kicker and cavity, then follows the recirculation path and passes through the cavity a second time, and finally passes through the pickup. For each situation the HOM deflection (wake potential) and current moment at the pickup are computed in the frequency domain and are shown to depend on the various beam, HOM, and optical parameters for the two situations.

2.2.4 Single Pass Case

Considering first the single pass case, the beam is accelerated from an initial momentum p_i before the cavity to a final momentum p_f after passing once through the cavity. The transfer matrices $M^{(k)}$ and $M^{(p)}$ are first-order transfer matrices from kicker to cavity and cavity to pickup respectively. The kick due to the HOM is first calculated and the result used to compute the current moment at the pickup. The current moment calculation shows that the shunt impedance $(R/Q)_m$ of the HOM can be determined from a measurement of the current moment at the pickup.

In the time domain, the kicker used in these experiments produces a time dependent angular kick of the beam:

$$\theta_k(t) = \theta_\circ \cos(\omega_k t), \tag{2.44}$$

where $\theta_\circ$ is the maximum kick produced (which depends on beam energy at the kicker, kicker feed power, etc.) and ω_k is the angular frequency of the kicker drive source. The beam current is modeled as an infinite series of pulses according to equation 2.28.

In the time domain the wake potential is given by,

$$V(t) = \int_{-\infty}^{t} W(t-t')I(t')x_c(t')dt', \tag{2.45}$$

where $I(t)$ is the beam current given by equation 2.28, and $x_c(t)$ is the displacement coordinate of the beam at the cavity. Both beam coordinates can be obtained by defining a matrix that includes the HOM deflection. They are given by,

$$\begin{pmatrix} x_c(t) \\ \theta_c(t) \\ p_f \end{pmatrix} = M^{(k)} \begin{pmatrix} x_k(t-t_k) \\ \theta_k(t-t_k) \\ p_i \end{pmatrix} \tag{2.46}$$

$$M^{(k)} = \begin{pmatrix} M_{11}^{(k)} & M_{12}^{(k)} & 0 \\ M_{21}^{(k)} & M_{22}^{(k)} & \frac{p_x(t)}{p_i p_f} \\ 0 & 0 & \frac{p_f}{p_i} \end{pmatrix} \tag{2.47}$$

where the column vector on the right has components that are the initial beam coordinates at the kicker. Each column vector also includes as a third component the value of the beam momentum at the axial position of the first two components x and θ. The matrix $M^{(k)}$ includes not only the normal first-order transfer from kicker to cavity but also the cavity HOM kick as the $M_{23}^{(k)}$ matrix element. The HOM kick is modeled as an angular impulse given to the beam at the exit of the cavity. The HOM kick $p_x(t)$ is given in terms of $V(t)$ according to equation 2.24. From now on the initial transverse position of the beam at the kicker $x_k(t-t_k)$ will be assumed constant and will be denoted as x_i.

Equation 2.45 shows that the kick received by a bunch that enters the cavity at time t is due to the wakefield generated by all bunches that have already passed the cavity. Equation 2.46 indicates that a bunch that enters the cavity at time t was deflected by the kicker at the earlier time $t-t_k$ where t_k is the time it takes for a bunch to drift the distance from kicker to cavity. A change of variables in equation 2.45 yields

$$V(t) = \int_{-\infty}^{\infty} W(\tau) I(t-\tau) x_c(t-\tau) d\tau \tag{2.48}$$

where $\tau = t - t'$ and $W(\tau) = 0$ for $\tau < 0$. Taking the Fourier transform and using the convolution theorem results in the frequency domain expression

$$V(\omega) = W(\omega) I x_c(\omega). \tag{2.49}$$

This is the desired result for the cavity wake potential in the frequency domain. An antenna suitably located in the cavity would detect a signal proportional to $V(\omega)$ depending on the effective coupling between the HOM and antenna. The current moment at the cavity is given as,

$$I x_c(\omega) = I x_c(\omega)_b + I x_c(\omega)_k \tag{2.50}$$

where

$$I x_c(\omega)_b \equiv M_{11}^{(k)} I(\omega) x_i \tag{2.51}$$

$$I x_c(\omega)_k \equiv M_{12}^{(k)} I \theta_k(\omega) \tag{2.52}$$

$$I(\omega) = 2\pi I_\circ \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_\circ) \tag{2.53}$$

$$I\theta_k(\omega) = \pi I_\circ \theta_\circ \sum_{n=-\infty}^{\infty} \left\{ e^{-i\omega_k t_k} \delta(\omega_k - \omega - n\omega_\circ) + e^{i\omega_k t_k} \delta(\omega_k + \omega + n\omega_\circ) \right\}, \tag{2.54}$$

and use was made of the Fourier identities:

$$\sum_{n=-\infty}^{\infty} e^{in\omega_\circ t} = t_\circ \sum_{n=-\infty}^{\infty} \delta(t - nt_\circ) \tag{2.55}$$

$$\sum_{n=-\infty}^{\infty} e^{in\omega t_\circ} = \omega_\circ \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_\circ). \tag{2.56}$$

In equation 2.50 the current moment spectrum at the cavity $Ix_c(\omega)$ consists of a term due only to the beam current $I(\omega)$ denoted by the subscript b and a term due to the product of the beam current and kicker angular deflection $I\theta_k(\omega)$ denoted by the subscript k.

The spectrum given by 2.50 is composed of signals at all harmonics of the bunching frequency due to the beam (equation 2.53) as well as sidebands separated from each harmonic by $\pm\omega_k$ due to the kicker (equation 2.54). The real beam spectrum does not have all harmonics present but begins to roll off at frequencies of the order of the inverse temporal bunch length. At CEBAF where the bunch length is approximately 2 ps rolloff occurs at 500 GHz. The most important HOMs at CEBAF have frequencies less than about 2.5 GHz, so that as far as CEBAF HOMs are concerned the beam spectrum extends so high in frequency that the summation index n in equations 2.53 and 2.54 effectively runs to infinity. The beam model given by equation 2.28 is therefore a valid approximation for the situation at CEBAF where the HOMs of most concern have widths on the order of 50 kHz and are low enough in frequency that they have no appreciable amplitude at the rolloff frequency. In contrast, for the class of instabilities known as the single-bunch wakefield effects, the short time structure of the bunch is crucial to the analysis and understanding of the instability [Kr90]. For these effects where times on the order of the bunch length are important in describing the resultant instability the beam model 2.28 is clearly not valid.

For the final part of the single pass analysis the current moment at the pickup is calculated. The coordinates at the pickup are given by the previous matrix formalism in terms of the coordinates at the cavity:

$$\begin{pmatrix} x_p(t) \\ \theta_p(t) \\ p_f \end{pmatrix} = M^{(p)} \begin{pmatrix} x_c(t-t_p) \\ \theta_c(t-t_p) \\ p_f \end{pmatrix} \tag{2.57}$$

$$M^{(p)} = \begin{pmatrix} M_{11}^{(p)} & M_{12}^{(p)} & 0 \\ M_{21}^{(p)} & M_{22}^{(p)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.58}$$

where $M^{(p)}$ is defined to have the same dimensionality as $M^{(k)}$. Next the current moment at the pickup is written in the time domain as

$$I_p(t)x_p(t) \equiv I(t-t_p)x_p(t) \tag{2.59}$$

where this definition is due to the fact that a bunch arriving at the pickup at time t passed the cavity at time $t - t_p$ where t_p is the time it takes a bunch to travel from the cavity to the pickup. Using equations 2.46 and 2.57 to obtain the expression for the coordinate $x_p(t)$, equation 2.59 is transformed to the frequency domain resulting in,

$$I_px_p(\omega) = I_px_p(\omega)_b + I_px_p(\omega)_k + \frac{eM_{12}^{(p)}e^{-i\omega t_p}}{2\pi p_f c} I(\omega) * V(\omega) \tag{2.60}$$

$$I_px_p(\omega)_b \equiv e^{-i\omega t_p}\left(M^{(p)}M^{(k)}\right)_{11} I(\omega)x_i \tag{2.61}$$

$$I_px_p(\omega)_k \equiv e^{-i\omega t_p}\left(M^{(p)}M^{(k)}\right)_{12} I\theta_k(\omega), \tag{2.62}$$

where the convolution is performed in the frequency domain and $V(\omega)$ is given by equation 2.49.

The current moment at the pickup splits naturally into a beam term and a kicker term analogous to the current moment at the cavity given by equation 2.50 with an additional term due to the HOM kick. Evaluating the convolution yields:

$$I(\omega) * V(\omega) = Ix_c(\omega) \sum_{m=-\infty}^{\infty} 2\pi I_\circ W(\omega - m\omega_\circ) \tag{2.63}$$

where the property

$$Ix_c(\omega - m\omega_\circ) = Ix_c(\omega), \tag{2.64}$$

has been used. Using equations 2.60 and 2.63 the current moment at the pickup is,

$$\begin{aligned} I_p x_p(\omega) &= I_p x_p(\omega)_b + I_p x_p(\omega)_k + \\ & \frac{eI_\circ M_{12}^{(p)} e^{-i\omega t_p}}{p_f c} Ix_c(\omega) \sum_{m=-\infty}^{\infty} W(\omega - m\omega_\circ) \end{aligned} \tag{2.65}$$

where $Ix_c(\omega)$ is the current moment at the cavity defined in terms of a beam and kicker term according to equation 2.50. Equations 2.49 and 2.65 represent the main results of this section. It is useful to express these equations in terms of a ratio where the denominator is the current moment in the absence of any HOM interaction and is due simply to the beam or the kicker. This results in the equations,

$$\frac{I_p x_p(\omega)}{I_p x_p(\omega)_b} = 1 + \frac{M_{11}^{(k)} M_{12}^{(p)}}{(M^{(p)} M^{(k)})_{11}} G(\omega) \tag{2.66}$$

$$\frac{I_p x_p(\omega)}{I_p x_p(\omega)_k} = 1 + \frac{M_{12}^{(k)} M_{12}^{(p)}}{(M^{(p)} M^{(k)})_{12}} G(\omega) \tag{2.67}$$

$$G(\omega) = \frac{eI_\circ}{p_f c} \sum_{m=-\infty}^{\infty} W(\omega - m\omega_\circ) \tag{2.68}$$

where the first is due to the beam term and the second is due to the kicker term. Similarly the HOM wake potential in the frequency domain as given by equation 2.49 can be written,

$$\frac{V(\omega)}{Ix_c(\omega)} = W(\omega) \tag{2.69}$$

These equations show that both the kicker and cavity signals can be expressed conveniently in terms of the wake function or the HOM impedance divided by i.

The quantity $G(\omega)$ is a strongly peaked function of ω with peaks corresponding to the HOM resonance. From equation 2.68, each term in the sum is maximum only if

$$\omega - m\omega_\circ = \omega_r \sim \omega_m \tag{2.70}$$

and is simply the statement that the wake function is strongly peaked only when its argument is equal to the HOM resonance frequency given by equation 2.20. From equation 2.53 appreciable response due to the bunched beam occurs when

$$\omega - n\omega_\circ = 0. \tag{2.71}$$

Combining equations 2.70 and 2.71 and eliminating ω gives:

$$(n-m)\,\omega_\circ = \omega_r \sim \omega_m \tag{2.72}$$

or when the HOM frequency is some harmonic of the bunching frequency appreciable excitation of the HOM occurs. This situation can be troublesome especially if the HOM in question is not heavily damped, because the beam itself can drive the mode. On the other hand, for the experiments discussed here it is desirable to be able to drive a particular HOM selectively using the kicker. From equation 2.54 appreciable response due to the kicker occurs occurs when

$$\omega_k \pm (\omega + n\omega_\circ) = 0. \tag{2.73}$$

Combining equations 2.73 and 2.70 yields,

$$\omega_k = \pm\left(\omega_r + (n+m)\,\omega_\circ\right) \tag{2.74}$$

and is the desired result for the kicker-HOM excitation condition. We can thus adjust the kicker frequency to drive any mode we wish and in addition we can have the kicker operate at a low frequency and use the bunched beam to “alias up” the kicker signal to a HOM much higher in frequency.

Using previous results for the wake function (impedance) at resonance, equation 2.69 can be written as

$$\left|\frac{V(\omega_r)}{Ix_c(\omega_r)}\right| = \frac{(R/Q)_m k_m Q_m}{2}. \tag{2.75}$$

Equation 2.75 implies that by measuring the wake potential at a particular HOM resonance, along with a measurement of the frequency and width of the HOM peak (to obtain Q_m and k_m) the transverse impedance $(R/Q)_m$ can be obtained experimentally. Using the HOM parameter values from table 2.1 along with equation 2.75 one obtains $3.02 \times 10^7\ \Omega/\mathrm{m}$ for the magnitude of the impedance on resonance. Similarly, the magnitude of the current moment at the pickup from equations 2.66 and 2.67 can be written as:

$$\left|\frac{I_p x_p(\omega_r)}{I_p x_p(\omega_r)_b}\right| = \sqrt{1+g_b^2} \tag{2.76}$$

$$\left|\frac{I_p x_p(\omega_r)}{I_p x_p(\omega_r)_k}\right| = \sqrt{1+g_k^2} \tag{2.77}$$

$$g_b = \frac{M_{11}^{(k)} M_{12}^{(p)}}{(M^{(p)} M^{(k)})_{11}} |G(\omega_r)| \tag{2.78}$$

$$g_k = \frac{M_{12}^{(k)} M_{12}^{(p)}}{(M^{(p)} M^{(k)})_{12}} |G(\omega_r)| \tag{2.79}$$

where,

$$|G(\omega_r)| = \frac{eI_\circ}{p_f c}\left|\frac{V(\omega_r)}{Ix_c(\omega_r)}\right| \tag{2.80}$$

is written in terms of the magnitude of the HOM impedance. $|G(\omega_r)|$ can be regarded as a measure of the strength of the HOM kick for a given average beam current $I_\circ$ and final beam momentum p_f. These results indicate that a measurement of the current moment at the pickup can in principle determine the HOM impedance and thus the shunt impedance of the mode $(R/Q)_m$. Using the HOM parameters from table 2.1, assuming a final beam momentum of 5 MeV/c and a beam current of 200 μA equation 2.80 becomes

$$|G(\omega_r)| = 1.21 \times 10^{-3}\ \mathrm{m}^{-1} \tag{2.81}$$

For CEBAF HOMs one sees that $|G(\omega_r)|$ is relatively small compared with that of typical undamped HOMs in superconducting cavities where the Q_m values (and hence the strength factor $|G(\omega_r)|$) can be up to a factor of 10^5 higher than the value in table 2.1. For these experiments one would like to use the signal detected at the pickup to obtain a measurement of $(R/Q)_m$. The factor g_k should therefore be on the order of 1 so that the HOM signal is clearly distinguished from that of the kicker

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Figure 2.7: Kicker-Cavity-Pickup recirculation case for the frequency domain response function calculations.

alone. This implies that the ratio of matrix elements in equation 2.79 should be on the order of 1000 m to detect a HOM kick signal with $|G(\omega_r)|$ given by equation 2.81. The first-order matrix elements in the denominator of equations 2.78 and 2.79 can in principle be made zero which means that the division in equation 2.66 and 2.67 is therefore not valid. This implies that the only signal at the pickup would be due to the HOM. If the deflections produced by the HOM are relatively small, the HOM signal at the pickup will be in the noise and therefore impossible to detect. Finally, the ratio of the matrix elements in equations 2.78 and 2.79 can be viewed as a gain parameters that can be used to increase the HOM signal at the pickup. In practice, the matrix element $(M^{(p)}M^{(k)})_{12}$ in the denominator of equations 2.67 and 2.79 is mimimized to maximize the sensitivity to HOM deflections. This means that the optics from kicker to pickup are point to point so that only HOM deflections can produce net displacement of the beam off axis at the pickup.

2.2.5 Recirculation Case

The case where the beam traverses the recirculation path shown in figure 2.7 is now considered. The beam with momentum p_i is deflected by the kicker and has coordinates at the cavity that are determined by the transfer matrix $M^{(k)}(t)$. The beam then passes through the cavity, is accelerated to momentum p_r, and has coordinates at the cavity on the second pass determined by the transfer matrix $M^{(r)}(t)$. Finally, the beam is accelerated to momentum p_f and has coordinates at the pickup determined by the transfer matrix between the cavity on the second pass and the pickup $M^{(p)}(t)$. Determination of the HOM kick and pickup current moment proceeds along the same line of arguement as that for the single pass case and begins in the time domain.

The wake potential for the recirculation case including the kicker and pickup is written as,

$$V(t) = \int_{-\infty}^{t} W(t-t')I(t')x_c^{(1)}(t')dt' + \int_{-\infty}^{t} W(t-t')I(t'-t_r)x_c^{(2)}(t')dt' \qquad (2.82)$$

where the first and second term are due to the first and second pass beam respectively, and the principle of superposition has been used. The current moment at the cavity on the first pass at time t is $I(t)x_c^{(1)}(t)$ and that on the second pass is $I(t-t_r)x_c^{(2)}(t)$. Time is measured relative to when the beam passes the cavity on the first pass. The second pass beam therefore enters the cavity on the first pass at time $t - t_r$ where t_r is the recirculation time. Changing variables in equation 2.82 by letting $\tau = t - t'$ and requiring that $W(\tau) = 0$ for $\tau < 0$, $V(t)$ is written as:

$$\begin{aligned} V(t) &= \int_{-\infty}^{\infty} W(\tau)I(t-\tau)x_c^{(1)}(t-\tau)d\tau + \\ & \int_{-\infty}^{\infty} W(\tau)I(t-\tau-t_r)x_c^{(2)}(t-\tau)d\tau. \end{aligned} \qquad (2.83)$$

The coordinates $x_c^{(1)}(t)$ and $x_c^{(2)}(t)$ are given in terms of a matrix equation, where

$$\begin{pmatrix} x_c^{(1)}(t) \\ \theta_c^{(1)}(t) \\ p_r \end{pmatrix} = M^{(k)}(t) \begin{pmatrix} x_i \\ \theta_k(t-t_k) \\ p_i \end{pmatrix} \qquad (2.84)$$

$$\begin{pmatrix} x_c^{(2)}(t) \\ \theta_c^{(2)}(t) \\ p_f \end{pmatrix} = M^{(r)}(t) \begin{pmatrix} x_c^{(1)}(t-t_r) \\ \theta_c^{(1)}(t-t_r) \\ p_r \end{pmatrix} \qquad (2.85)$$

$$M^{(k)}(t) = \begin{pmatrix} M_{11}^{(k)} & M_{12}^{(k)} & 0 \\ M_{21}^{(k)} & M_{22}^{(k)} & \frac{p_x(t)}{p_i p_r} \\ 0 & 0 & \frac{p_r}{p_i} \end{pmatrix} \qquad (2.86)$$

$$M^{(r)}(t) = \begin{pmatrix} M_{11}^{(r)} & M_{12}^{(r)} & 0 \\ M_{21}^{(r)} & M_{22}^{(r)} & \frac{p_x(t)}{p_r p_f} \\ 0 & 0 & \frac{p_f}{p_r} \end{pmatrix} \qquad (2.87)$$

and t_k is the time it takes for the beam to travel from the kicker to the cavity. The above transfer matrices include not only the usual first-order optical transfer matrix elements, but also include the time dependent cavity kick as the M_{23} matrix element where the transverse momentum kick $p_x(t)$ is given in terms of $V(t)$ according to equation 2.24. Performing the matrix multiplication yields the correct time domain expression for the coordinates of the beam at the cavity on the first and second passes. Taking the Fourier transform of equation 2.83 and using equations 2.84-2.87 results in,

$$V(\omega) = W(\omega)\left\{Ix_c^{(1)}(\omega) + e^{-i\omega t_r} Ix_c^{(2)}(\omega) + \frac{eM_{12}^{(r)} e^{-i\omega t_r}}{2\pi p_r c} I(\omega) * V(\omega)\right\} \tag{2.88}$$

$$Ix_c^{(1)}(\omega) = M_{11}^{(k)} I(\omega) x_i + M_{12}^{(k)} I\theta_k(\omega) \tag{2.89}$$

$$Ix_c^{(2)}(\omega) = (M^{(r)}M^{(k)})_{11} I(\omega) x_i + (M^{(r)}M^{(k)})_{12} I\theta_k(\omega). \tag{2.90}$$

The current moment in brackets in equation 2.88 can be written in terms of a beam and kicker piece:

$$Ix_c(\omega)_b + Ix_c(\omega)_k \equiv Ix_c^{(1)}(\omega) + e^{-i\omega t_r} Ix_c^{(2)}(\omega) \tag{2.91}$$

$$Ix_c(\omega)_b = \left\{M_{11}^{(k)} + e^{-i\omega t_r}(M^{(r)}M^{(k)})_{11}\right\} I(\omega) x_i \tag{2.92}$$

$$Ix_c(\omega)_k = \left\{M_{12}^{(k)} + e^{-i\omega t_r}(M^{(r)}M^{(k)})_{12}\right\} I\theta_k(\omega), \tag{2.93}$$

where the subscripts b and k indicate the beam and kicker current moments. Evaluation of the convolution yields:

$$I(\omega) * V(\omega) = 2\pi I_\circ \sum_{n'=-\infty}^{\infty} V(\omega - n'\omega_\circ), \tag{2.94}$$

so that equation 2.88 becomes:

$$V(\omega) = W(\omega)\left\{Ix_c(\omega)_b + Ix_c(\omega)_k + \frac{eI_\circ M_{12}^{(r)} e^{-i\omega t_r}}{p_r c} \sum_{n'=-\infty}^{\infty} V(\omega - n'\omega_\circ)\right\} \tag{2.95}$$

where $V(\omega)$ is given in terms of itself evaluated at all other harmonics of the bunching frequency. The sum in equation 2.95 can be expressed in closed form by summing the left hand side of the equation $V(\omega - m\omega_\circ)$ over all integers m and noting that,

$$Ix_c(\omega - m\omega_\circ)_b = Ix_c(\omega)_b \tag{2.96}$$

$$Ix_c(\omega - m\omega_\circ)_k = Ix_c(\omega)_k \tag{2.97}$$

$$\sum_{n'=-\infty}^{\infty} V(\omega - m\omega_\circ - n'\omega_\circ) = \sum_{n'=-\infty}^{\infty} V(\omega - n'\omega_\circ) \tag{2.98}$$

$$e^{-i(\omega - m\omega_\circ)t_r} = e^{-i\omega t_r} \tag{2.99}$$

where the final equality results from the fact that t_r is an integer number of bunch periods. The final result for the sum in 2.95 becomes

$$\sum_{n'=-\infty}^{\infty} V(\omega - n'\omega_\circ) = \frac{\{Ix_c(\omega)_b + Ix_c(\omega)_k\}}{D(\omega)} \sum_{m=-\infty}^{\infty} W(\omega - m\omega_\circ) \tag{2.100}$$

$$D(\omega) = 1 - \frac{eI_\circ M_{12}^{(r)} e^{-i\omega t_r}}{p_r c} \sum_{m=-\infty}^{\infty} W(\omega - m\omega_\circ) \tag{2.101}$$

and upon substitution into equation 2.95 yields

$$V(\omega) = \frac{W(\omega) Ix_c(\omega)}{D(\omega)} \tag{2.102}$$

$$Ix_c(\omega) = Ix_c(\omega)_b + Ix_c(\omega)_k \tag{2.103}$$

where the beam and kicker terms are given according to equation 2.91. Equations 2.102 and 2.103 have the same form as equations 2.49 and 2.50 for the single pass case except for the denominator term. Equation 2.102 shows that when $D(\omega) = 0$ instability results because of feedback due to beam recirculation. The threshold current will be derived shortly from a zero analysis of the complex function $D(\omega)$.

The previous results are now used to calculate the current moment at the pickup. The beam coordinates at the pickup are given according to the matrix formalism as:

$$\begin{pmatrix} x_p(t) \\ \theta_p(t) \\ p_f \end{pmatrix} = M^{(p)} \begin{pmatrix} x_c^{(2)}(t - t_p) \\ \theta_c^{(2)}(t - t_p) \\ p_f \end{pmatrix} \tag{2.104}$$

$$M^{(p)} = \begin{pmatrix} M_{11}^{(p)} & M_{12}^{(p)} & 0 \\ M_{21}^{(p)} & M_{22}^{(p)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.105)$$

where the coordinates $x_c^{(2)}(t)$ and $\theta_c^{(2)}(t)$ are given by equations 2.84-2.87. The 3×3 dimension of the transfer matrix from the cavity to the pickup ($M^{(p)}$) is retained for consistency with previously defined matrices that include the cavity HOM kick. The current moment at the pickup in the time domain is defined similar to that for the single pass case (equation 2.59):

$$I_p(t)x_p(t) \equiv I(t - t_{pr})x_p(t), \quad (2.106)$$

and

$$\begin{aligned} I(t - t_{pr})x_p(t) = I(t - t_{pr})x_p(t)_b + I(t - t_{pr})x_p(t)_k + \\ \frac{e(M^{(p)}M^{(r)})_{12}}{p_r c} I(t - t_{pr})V(t - t_{pr}) + \\ \frac{eM_{12}^{(p)}}{p_f c} I(t - t_{pr})V(t - t_p) \end{aligned} \quad (2.107)$$

$$I(t - t_{pr})x_p(t)_b = (M^{(p)}M^{(r)}M^{(k)})_{11}I(t - t_{pr})x_i \quad (2.108)$$

$$I(t - t_{pr})x_p(t)_k = (M^{(p)}M^{(r)}M^{(k)})_{12}I(t - t_{pr})\theta_k(t - t_{pr} - t_k) \quad (2.109)$$

$$t_{pr} = t_p + t_r \quad (2.110)$$

where the current moment is written in terms of a beam, kicker, and two HOM terms and t_p is the time it takes for the second pass beam to travel from the cavity to the pickup. Following the usual procedure the Fourier transform yields the frequency domain expressions:

$$\begin{aligned} Ix_p(\omega) &= Ix_p(\omega)_b + Ix_p(\omega)_k + \frac{e(M^{(p)}M^{(r)})_{12}}{2\pi p_r c} e^{-i\omega t_{pr}} I(\omega) * V(\omega) + \\ &\quad \frac{eM_{12}^{(p)}}{2\pi p_f c} e^{-i\omega t_p} \left\{ e^{-i\omega t_r} I(\omega) \right\} * V(\omega) && (2.111) \\ Ix_p(\omega)_b &= e^{-i\omega t_{pr}} (M^{(p)}M^{(r)}M^{(k)})_{11} I(\omega) x_i && (2.112) \\ Ix_p(\omega)_k &= e^{-i\omega t_{pr}} (M^{(p)}M^{(r)}M^{(k)})_{12} I\theta_k(\omega) && (2.113) \end{aligned}$$

where the first convolution is given by equations 2.94. Evaluating the second convolution results in

$$\left\{e^{-i\omega t_r} I(\omega)\right\} * V(\omega) = 2\pi I_\circ \sum_{n'=-\infty}^{\infty} e^{-in'\omega_\circ t_r} V(\omega - n'\omega_\circ). \tag{2.114}$$

The phase factor $e^{-in'\omega_\circ t_r} = 1$ for t_r equal to an integer number of bunch periods $t_\circ$ so that equation 2.114 is identical to equation 2.94. Both single pass and recirculation pickup current moment expressions can be written similarly in terms of beam, kicker and HOM terms except that instability occurs when

$$D(\omega) = 0 \tag{2.115}$$

for the recirculation case.

For completeness, the above results are slightly modified when energy recovery is analyzed. For energy recovery the recirculation time t_r is some odd integer number of half bunch periods $t_\circ/2$. Equations 2.96 and 2.97 are modified because the phase factor $e^{-i(\omega - m\omega_\circ)t_r} = (-1)^m e^{-i\omega t_r}$. The phase factor in equation 2.114 becomes $e^{-in'\omega_\circ t_r} = (-1)^{n'}$. The equivalent of equation 2.102 for energy recovery has exactly the same instability denominator so that energy recovery presents nothing essentially new other than a slightly different recirculation time factor t_r. The equivalent energy recovery expression for the pickup current moment is modified in that the final momentum p_f is smaller than the recirculated beam momentum p_r–again, the instability condition is the same except for the recirculation time phase factor.

The perturbative result for the threshold current (equation 2.39) derived while working in the time domain can also be obtained from an analysis of $D(\omega)$. This is expected due to the complete equivalence between time and frequency in the Fourier transform. One further observes that a zero can only occur when $D(\omega)$ is real and the threshold current is readily obtained by solving equation 2.115. In general a numerical solution of equation 2.115 is performed and the current is computed for each zero with the smallest current being the threshold current.

The sum in equation 2.101 has an appreciable contribution from only a single term when the kicker frequency is set according to equation 2.74. For this situation equation 2.115 becomes:

$$1 - \frac{eI_\circ M_{12}^{(r)} \rho_m A_m(\Omega) e^{-i(\omega t_r + \phi_m(\Omega))}}{p_r c} = 0. \tag{2.116}$$

For this equation to be real, Ω and therefore ω must satisfy,

$$\begin{aligned} \omega t_r + \phi_m(\Omega) &= k\pi \\ k &= 0, \ \pm 1, \ \pm 2... \end{aligned} \tag{2.117}$$

Equation 2.117 is a transcendental expression which must be solved numerically. An approximate solution is obtained by observing that the threshold current occurs when $A_m(\Omega)$ is a maximum implying that $\Omega \sim 1$ or the oscillation frequency is very close to the HOM frequency. We may thus write,

$$\Omega = 1 + \Delta \tag{2.118}$$

where $\Delta \ll 1$. Substitution of equation 2.118 into equation 2.117 while keeping terms first order in both Δ and $1/Q_m$ results in a first order expression for Δ,

$$\Delta \sim \frac{1}{(2Q_m + \omega_m t_r)\tan(\omega_m t_r)}. \tag{2.119}$$

Substituting equations 2.119 and 2.118 into equation 2.18 and keeping only first order terms results in the first order expression,

$$A_m(\Omega) \sim \sin(\omega_m t_r) e^{\frac{\omega_m t_r}{2Q_m}}. \tag{2.120}$$

Using equation 2.120 in equation 2.116 and solving for $I_\circ$ results in the same perturbative expression for the threshold current as given by equation 2.39.

This analysis indicates that the approximate result given by equation 2.39 is only valid for solutions of equation 2.116 where $\Delta \sim 1/Q_m$ so that the first order expression for $A_m(\Omega)$ is valid. A numerical solution of equation 2.116 using the various parameters of section 2.2.2 resulted in a threshold current of .25976 Amperes at a normalized frequency of $\Omega = 1.0000152440$ yielding a phase factor given by equation 2.117 of 813π, in excellent agreement with the TDBBU result of .260 Amperes and in exact agreement (within roundoff error) with the numerical solution of equation 2.35 given in section 2.2.2. Equation 2.116 is therefore seen to be the equivalent frequency domain expression to equation 2.35 derived in the time domain.

The approximate and numerical results are based on the observation that the threshold current occurs when $A_m(\Omega)$ is maximum (at the peak of the resonance) consistent with Ω being a solution of equation 2.117. The feedback mechanism is the displacement modulation of the beam on the second pass due to recirculation optics resulting from a HOM kick of the beam on the first pass. The displaced second pass beam then deposits kinetic energy into the HOM coherently at the HOM frequency. The simple example discussed in these sections serves to illustrate the basic physics of the multipass beam breakup instability. The analysis of the instability for accelerators such as CEBAF with many recirculations, cavities, and HOMs per cavity is accomplished through computer simulation. In the next section the simulation program TDBBU is used to analyze the behavior of the recirculator built around the main linac of the CEBAF injector. The simulation results are then compared to those for the full five pass CEBAF recirculating linac.

2.3 Beam Breakup Simulations Using the Code TDBBU

The BBU simulations were performed using the code TDBBU for all optical settings including energy recovery described in Chapter 3. The various optical settings were implemented in an attempt to increase sensitivity to multipass BBU (low threshold currents) by adjustment of the transverse recirculation optics (transverse matrix elements) in a controlled way. TDBBU treats the cavity HOMs as high-Q, uncoupled resonators with a strength given by the shunt impedance R/Q. These modes act to deflect the beam in either the x or y plane and both planes are treated as uncoupled.

Frequency (MHz)	R/Q (Ω)	Q
1899.6	21.9	$*90,000$
1969.6	48.1	$4,000$
2086.9	13.1	$10,000$
2110.5	25.6	$*30,000$

Table 2.2: HOM parameters used in the TDBBU calculations. These modes had the highest Q and R/Q for the CEBAF superconducting cavities. The asterisk ($*$) indicates that the HOM Q was determined from the RF measurements.

Table 2.2 lists parameters of the HOMs used in the simulations; the asterisk indicates the Q values for these modes come from RF measurements described in Chapter 4. Reference [Am84] lists the shunt impedance in terms of the parameter Z'', which is given by

$$R/Q = (Z''/Q)\left(\frac{l_e}{k^2}\right) \tag{2.121}$$

where k is the HOM wavenumber and l_e is the effective length of the HOM in the cavity. The code uses the first-order transfer matrix describing the recirculation path as computed using DIMAD. In addition, the superconducting cavity model used in DIMAD is incorporated into TDBBU to take into account cavity focussing at low energy.

Table 2.3 lists the threshold current $I_{\rm t}$ computed for each optics setting along with the maximum CW current $I_{\rm m}$ achieved in the experiment. Similar BBU calculations for the full CEBAF linac indicate threshold currents in the range 11–24 mA [Kr90] so that the injector recirculator is calculated to be more sensitive to multipass BBU by a factor of 2 in the threshold current. For setting 1, over 200 μA CW was recirculated. This is the CEBAF maximum design current, but is still an order of magnitude below the calculated threshold current for setting 1. The energy recovery setting is seen to have the lowest calculated threshold current due to large recirculation transfer matrix elements. Beam current was limited primarily by large beam sizes on the second pass (with energy recovery having the largest) that resulted in scraping. The beam loss

Setting	I_{t} (mA)	I_{m} (μA)
1	5.3	215
2	6.3	68
3	19.5	120
4	13.2	95
5	15.5	64
6	5.0	67
Energy recovery	.4	30

Table 2.3: TDBBU threshold current and maximum beam current attained for each optical setting.

monitoring system shut the beam off when approximately 1 μA of scraping occurred.

Chapter 3

Recirculator Design, Modeling, and Measurements

3.1 Overview

Both multipass BBU and energy recovery experiments were accomplished using the CEBAF injector recirculator shown in figure 3.1. The beam was recirculated once around the injector linac so that it passed twice through each of the sixteen superconducting cavities in cryomodules 1 and 2. The experiments required that the recirculator satisfy three criteria. First, it was necessary for the BBU experiment to be able to adjust the transfer matrix for the recirculated beam. The theory outlined in Chapter 2 indicated that the threshold current for multipass BBU depends strongly on the transfer matrix elements that determine the displacement of the beam at a cavity on the second pass resulting from a HOM kick received by the beam at the cavity on the first pass. Adjustment of the transfer matrix was accomplished by changing the strengths of quadrupole magnets in the straight return path of the recirculator. The return path was adjusted to provide six optical settings (including energy recovery) and a recirculation transfer matrix was determined for each setting by DIMAD. These matrices were used in TDBBU to calculate the multipass BBU threshold current as described in section 2.3.

The second criterion for the recirculation system was that each optical setting provide small transverse beam spot sizes for loss-free transport through the recirculator. Finally, the need for both acceleration and energy recovery of the second pass beam implies that the recirculation path length must be easily adjustable. This was

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Figure 3.1:

accomplished by mounting the first bend, B1, on a carriage and making a "trombone" adjustment by translating the entire magnet array along the direction of the linac axis by 7.3 cm for a total path length adjustment of 14.6 cm (nearly 75% of an RF wavelength).

Sections 3.2 and 3.3 describe the basic design of the recirculator including the DIMAD modeling of the first order optics. Appendix B goes into detail about the sextupole fringe field model used to describe the bend dipoles. This model accounts for the low dispersion and dispersion asymmetry measured for B1. The last three sections of the chapter describe the dispersion, dispersion suppression, and recirculation tranfer matrix element measurements made during the course of the experiment.

3.2 Recirculator Beam Orbit Geometry

Figure 3.1 shows that the recirculation path is made up of six main optical elements: the injection chicane; the linac, which consists of two cryomodules each containing eight superconducting cavities; the energy recovery chicane; two 180° bends (B1 and B2); and the straight return path that contains only quadrupoles. The beam orbit geometry in the optical units that contain dipoles determines the beam energies required for transport around the recirculator. All dipoles and quadrupoles used in the recirculator were measured to determine their integrated multipole strength as a function of current using the rotating coil technique [Ka92]. The effective lengths of the dipoles were inferred from these measurments. The quadrupole effective length was taken to be the pole length plus the aperture.

The beam sizes, matrix elements and lattice functions are determined by the first-order transport properties of each optical element. These elements will be described in the order an electron in the beam encounters them, starting at the injection point where the beam energy is 5.6 MeV. For both acceleration and energy recovery of the second pass beam, the beam is accelerated to 42.8 MeV on the first pass and recirculated. Acceleration to 80.1 MeV or deceleration to 5.6 MeV is performed for

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Figure 3.2: Injection chicane beam orbit geometry.

acceleration and energy recovery respectively of the second pass beam.

The first pass beam at 5.6 MeV initially arrives at the injection chicane shown in figure 3.2. This chicane is used to bring both the first pass beam at 5.6 MeV and the recirculated beam at 42.8 MeV (second pass beam) onto the axis of the cryomodules and consists of seven small dipoles. The first four dipoles (DC1-DC4) each bend the 5.6 MeV first pass beam by 12° according to case 1 beam orbit geometry as described in Appendix A. The injected beam is thereby translated off the linac axis to make room for the 42.8 MeV second pass beam and returned to the linac axis. DC5-DC7 bring the second pass beam onto the linac axis by making the small corrections necessary so that the dipole DC4 common to both beams brings both on axis. The first pass beam momentum and the bend angle in DC1-DC4 determine the magnetic field of all injection chicane dipoles and thereby determine the second-pass (case 1) beam orbit in DC4-DC7. Dipole D10 is considered part of the 180° bend B2 that brings the recirculated beam to the linac axis for re-injection into the linac by the injection chicane. It should be noted that DC1 is also used for energy measurment of the first pass beam by bending it 30° to a beam dump.

Analysis of the beam orbit in DC4 starts with the equation of motion for an electron in a dipole field,

$$B\rho = \frac{p}{e}, \tag{3.1}$$

which relates the magnetic field B, the radius of curvature of the orbit ρ, and the particle momentum p. In dipole DC4 both beams traverse the same magnetic field so

$$\frac{\rho_i}{\rho_r} = \frac{p_i}{p_r} \tag{3.2}$$

where ρ_i and p_i (ρ_r and p_r) are the radius of curvature and momentum of the first pass (second pass) beam. Using equation A.11 for case 1 beam orbit geometry and equation 3.2 results in,

$$\frac{\sin(\theta_r)}{\sin(\theta_i)} = \frac{p_i}{p_r} \tag{3.3}$$

where θ_i (θ_r) is the orbit bend angle of the first pass (second pass) beam. For this case $\theta_r = 12°$, $p_i = 5.5$ MeV/c, $p_r = 42.8$ MeV/c, and $\theta_r = 1.5°$. The effective length of the chicane dipoles is 12.2 cm resulting in a radius of curvature and sagitta (see equations A.11 and A.12 in Appendix A) of 58.6 cm and 1.3 cm respectively for the injected beam and 456.3 cm and .16 cm for the recirculated beam. Using equations 3.1 and A.11, the orbit parameters require that all seven dipoles are powered so that they each have a field of .313 kG.

After traversing the injection chicane, the first pass beam travels to the first two cryomodules that comprise the principal component of injector linac. The superconducting cavities accelerate the first pass beam to 42.8 MeV after the second cryomodule. When the recirculator is in the accelerating mode the second pass beam is accelerated by these same cavities to 80.1 MeV. For energy recovery the second pass beam is decelerated back to 5.6 MeV. The cavity gradients that were used in the experiments are listed in table 3.1. Each table lists the cavites in order from the low energy end of the cryomodule to the high energy end. The last cavity of the second cryomodule was not powered due to a frozen tuning mechanism that prevented the cavity from operating at the accelerating mode resonance of 1497 MHz. This cavity was used in the RF measurements as a pickup device to detect HOM resonances.

After acceleration in the linac, the first pass beam arrives at the energy recovery chicane shown in figure 3.3. This chicane is designed to transmit the first and second pass beams undisturbed when the recirculator is operated in the acceleration mode, and to recover the second pass beam (which is back at the injection energy of 5.6 MeV) when the recirculator is operated in the energy recovery mode. It consists of the three dipoles DE1, DE2 and DE3. Both first and second pass beam orbits follow the specifications of case 1 for DE1 and DE3 and case 2 for DE2 (see Appendix A). The magnetic field of dipole DE1 is adjusted to bend the 5.6 MeV beam by 20° into the

Cryomodule 1

Cavity #	Energy gain (MeV)
1	2.236
2	2.290
3	2.797
4	2.184
5	1.498
6	1.852
7	2.510
8	2.754

Cryomodule 2

Cavity #	Energy gain (MeV)
1	2.375
2	2.824
3	2.851
4	2.817
5	2.627
6	2.754
7	2.848
8	0.000

Table 3.1: Linac cryomodule cavity energy gain.

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Figure 3.3: Energy recovery chicane beam orbit geometry for the energy recovery mode.

dump. Dipoles DE2 and DE3 are adjusted so that the net field integral for the first pass beam at 42.8 MeV is zero, consequently the first pass beam continues straight along the linac axis after DE3. Using equation 3.3 in an analysis identical to that of the injection chicane, the bend angle of the first pass beam is found to be 2.5°. Using equations A.11 and A.12 and the effective length of 18.2 cm for DE1 from magnet measurements, the radius of curvature and sagitta for the first pass beam are 414.1 cm and .4 cm; the same values are obtained for DE3 by symmetry. Similarly, the radius of curvature and sagitta of the second pass beam in DE1 are 53.2 cm and 3.2 cm. Equations 3.1 and A.11 require that each dipole be powered so that the field is .345 kG. DE2 is twice the effective length of DE1 and DE3 so that it has the same field, radius of curvature and orbit saggitta as the two short dipoles by symmetry. For the case of acceleration of the second pass beam, the three dipoles of the energy recovery chicane are simply left unpowered letting both beams pass straight along the linac axis.

After passing the energy recovery chicane the first pass beam arrives at the entrance to the first 180° bend (B1) shown in figure 3.4. The first pass beam at 42.8 MeV is bent 45° by each dipole (D1-D4) and enters each dipole symetrically at 22.5° to the pole face normal so that case 2 beam orbit geometry applies. The first pass orbit passes through the centers of quadrupoles Q1 and Q2, which are used to eliminate dispersion after D4. Both first and second pass beams are common to dipole B1 and therefore their momenta must satisfy,

$$\frac{\rho_r}{\rho_f} = \frac{p_r}{p_f} \tag{3.4}$$

where the ρ_r and p_r (ρ_f and p_f) are the radius of curvature and momentum of the first pass (second pass) beam. The second pass beam at 80.1 MeV is bent by dipole B1 by

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Figure 3.4: Bend B1 beam orbit geometry.

24.8° and enters the dipole at 22.5° (as with the first pass beam) and exits the dipole at 2.3°. Using the effective length of 19.1 cm along with equations A.14 and A.15, the radius of curvature and sagitta of the first pass beam in D1-D4 are 25.0 cm and 1.9 cm. Using equations 3.1 and A.14 this requires a magnetic field of 5.717 kG for a first pass beam momentum of 42.8 MeV/c. The sagitta of the second pass beam in D1 is complicated by the fact that the beam orbit is neither case 1 or case 2. The radius of curvature for a second pass beam of momentum 80.1 MeV/c is 46.7 cm. Tracing the orbit through D1 indicates a sagitta of 3.2 cm which is the distance from the first (and second) pass beam orbit at the entrance of the magnet from the second pass beam orbit at the exit of the magnet. D1 was therefore positioned according to the sagitta of the second pass beam since this orbit represents the largest orbit excursion in the dipole.

Extraction of the second pass beam is accomplished by dipoles D5 and D6. D5 is a small corrector dipole used to adjust the orbit trajectory to compensate for small horizontal steering errors of the second pass beam at the entrance to D1. D6 is used to bend the beam parallel to the linac axis into the beam dump. D6 is placed so that the beam orbit follows case 2 with entry and exit angles of 12.4°. For an effective length of 19.1 cm, the radius of curvature and sagitta for the orbit in D6 are 44.5 cm and 1.0 cm. The magnetic field required is slightly higher (6.004 kG) than that for D1-D4 due to the slightly different orbit geometry.

The beam continues down the straight return path containing quadrupoles QR1-QR10 to bend B2. The return path is the key element that is used to adjust the first-order optics described in the next section. Bend B2, which consists of dipoles D7, D8, D9, and D10 and dispersion suppression quadrupoles Q3 and Q4 (see figure 3.5), has the same orbit properties as the first pass beam in B1 (case 2 for the

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Figure 3.5: Bend B2 beam orbit geometry.

dipoles) and returns the beam to the linac axis for re-injection into the linac via the injection chicane. The beam is then either accelerated and dumped after D6 of B1 or decelerated and dumped at the energy recovery chicane beam dump. The total path length for the recirculated beam can be adjusted by moving the entire B1 array of magnets along the direction parallel to the linac axis. Bellows in the vacuum line accomodate this motion. The total motion from stop to stop is 7.3 cm, which corresponds to 14.6 cm of path length adjustment or 262.8° of accelerating mode RF phase shift of the second pass beam relative to the first pass beam.

3.3 Optical Modeling Using DIMAD

Optics calculations for the injector recirculator were performed using the general purpose optics code DIMAD. The main optical requirement for the BBU experiments is the ability to vary the transverse matrix elements (M_{12} and M_{34}) of the recirculation arc. In addition, dispersion free beam transport is desired so as to avoid beam offsets due to small energy drifts. Matrix element adjustment is accomplished using the quadrupoles in the straight return path of the recirculation arc. The chicanes and bends are all designed to be doubly acromatic, thereby satisfying the dispersion requirement. The DIMAD code also includes the capability to model the RF focussing of the fundamental mode of the CEBAF cavities. The first-order optical properties of the six optical elements are described in the same order the beam encounters them, as in the last section. The DIMAD calculations provide transfer matrices that describe the recirculation optics. These matrices are used by the code TDBBU to compute threshold currents as described in Chapter 2.

The main requirement for the injection chicane is that it pass both the injected and recirculated beams dispersion-free onto the linac axis. The top plot of figure 3.6 shows the behavior of the dispersion as the beam passes through the chicane. The slope of the dispersion in the horizontal (x) plane is zero at the center of each leg of the chicane by symmetry (between DC2 and DC3 for the first pass beam and DC6 and DC7 for the second pass beam). The dispersion η and its slope η' are both zero at the end of the chicane as a result of symmetry. Dispersion-free transport is therefore accomplished in practice by setting the net field integral to zero as the beam is transported or, equivalently, by setting the magnets so both beams travel along the linac axis downstream of the chicane. The bottom plot of figure 3.6 shows the principle cosine and sine-like rays for both (x and y) planes (M_{11}, M_{12}, M_{33}, and M_{34}) for the first pass beam.

The horizontal optics of the chicane is simply transport through a drift equivalent to the arc length of the trajectory. The vertical optics includes focussing due to the non-zero entry or exit angles of the beam orbit in the dipoles. The optics of the second pass beam through dipoles DC4, DC5, DC6 and DC7 is virtually identical to a drift in both planes. This is due to the fact that the bend angle of the second pass beam is relatively small and therefore there is little y-plane edge focussing in the thb-error.c=

Figure 3.6: Dispersion and matrix elements for the injection chicane. The matrix elements shown are for the first pass only.

cryomodule 1 where the beam energy on the first pass at the entrance is 12.9 MeV and at the exit is 15.1 MeV is calculated to have a focal length of 53.9 m. It is seen that only in the first few cavities, where the energy is below about 10 MeV, is the focussing of any consequence. For energy recovery of the second pass beam the focussing due to the final few cavities of the second cryomodule also is of consequence. As a final note the linac quadrupoles at locations HVQL1-HVQL6 were left unpowered and all focussing of the first pass beam was accomplished by backphasing the first cavity of the cryounit and adibatic damping of the emittance as the beam accelerates.

Another non-ideal feature of the cavities that occurs at low energy is x-y coupling between planes [Ti93]. The DIMAD model of the cavities does not take into account the coupling between planes and assumes the planes to be completely uncoupled. The uncoupled case represents the worst case scenario (results in the lowest threshold currents) for multipass BBU.

For the energy recovery chicane, the first pass beam has very similar optical properties to that in the injection chicane. Figure 3.7 shows the dispersion function for first and second pass beams in the energy recovery chicane. Specifically the slope of the dispersion is zero at the geometric center of dipole DE2. By setting the magnets so the net field integral experienced by the first pass beam is zero, both the dispersion and its slope are zero after dipole DE3 by symmetry. The second pass beam is simply bent and dumped approximately one meter after DE1 so it has a nonzero dispersion at the beam dump. As with the second pass beam in the injection chicane, the principle ray optics for the first pass beam are essentially identical to a drift due to the small bend angle. The second pass is simply a drift horizontally with dipole edge focussing vertically at the exit edge of DE1.

Bend B1 is designed to transport the first pass beam dispersion-free to quadrupole doublet QR1-QR2 on the return path. In addition, by leaving quadrupoles Q1 and Q2 unpowered B1 was also used to measure the energy spread by first measuring the dispersion at the end of the bend. The dispersion calculated using a simple model for the bend dipoles was roughly a factor of two above the value measured with quadrupoles Q1 and Q2 unpowered. In addition, the dispersion was different depending on whether the measurement was performed by increasing the magnetic field (in which case the beam follows the trajectory of a lower momentum particle for a higher central momentum setting for the bend) and recording the resulting beam displacement or decreasing the magnetic field (in which case the beam follows the trajectory of a higher momentum particle for a lower central momentum setting for the bend). A more realistic model for the bend dipoles was constructed which added a sextupole field component to the dipole end fringe fields. This model reproduced the reduction in the dispersion as well as the measured asymmetry. The details of the dipole model are discussed in Appendix B and the dispersion measurement is described in section 3.4

Each bend is designed so that the dispersion η as well as its slope η' is zero after the bend. This is achieved by powering quadrupoles Q1-Q2 for B1 and Q3-Q4 for B2. The quadrupole strength (each quadrupole pair is powered in series from the same supply) that achieved dispersion suppression after B1 was determined by a measurement described in section 3.5. A DIMAD computation of the dispersion with the quadrupoles powered to the value determined by measurement indicated a small residual linear dispersion of .77 m. An additional fit to the fringe field sextupole strength was required to fully reduce the dispersion (and slope) to zero. The top plot of figure 3.8 shows the dispersed ray for B1 which has a maximum at the center of symmetry of the bend and is zero (as well as slope) after the bend. Appendix B gives a discussion of how well the model accounts for dispersion suppression of the bend.

The bottom plot of figure 3.8 shows the principal rays for the first pass beam in B1 (and therefore B2). The main feature of the bends shown by the principle rays is the substantial focussing horizontally due to the large Q1 and Q2 strengths (the beam executes a full betatron oscillation horizontally for a tune of 1.0) and edge focussing in the vertical plane by the dipoles. The large focussing made matching out of B2 especially difficult. Ideally a quadrupole doublet would be placed after (downstream) of B2 as in B1 for matching except that there was not enough space available on the injection chicane table. Finally, the extraction path which includes dipoles D1, D5, and D6 was not dispersion-suppressed and had a dispersion value of .44 m at the rhb-errors=

Figure 3.8: B1 and B2 first pass linear dispersion and matrix elements. The quantities shown are for the dispersion suppressed mode of operation.

Setting #	k_F^2 (m^{-2})	k_D^2 (m^{-2})	ν_x	ν_y
1	1.16	-2.07	.5	.5
2	1.09	-1.57	.5	.25
3	1.22	-2.53	.5	.75
4	1.24	-2.71	.5	1.0
5	-1.57	1.09	.25	.5
6	-2.53	1.22	.75	.5

Table 3.2: FODO quadrupole strength and tune advance for each setting.

beam dump. As with the energy recovery chicane this small dispersion was not a problem as the beam was dumped a short distance ($\sim$ 2.5 m) after the first dipole that introduced dispersion in the second pass beam.

The main optical feature of the return path is the six quadrupoles, QR3-QR8, that are used to adjust the recirculation optics in a controlled manner. These quadrupoles formed a FODO array consisting of QR3, QR4, QR5 as the first cell and QR6, QR7, and QR8 as the second, where each cell begins and ends with a drift equal to half the distance between any two quadrupoles. The first and third quadrupoles in each cell (QR3, QR5, QR6, and QR8) are horizontally (x plane) focussing and of strength k_F and the central one at the center of symmetry in each cell (QR3, and QR7) is vertically focussing (y plane) and of strength k_D. Table 3.2 lists the quad values for each optical setting as well as the betatron tune advance of the beam across all six quads for each setting. The first optical setting consisted a -I (I being the identity matrix) transformation in both planes, or, equivalently, a tune advance of .5 in both planes. Settings 2-4 consisted of varying the y plane optics while keeping the x plane optics fixed at -I. Settings 5 and 6 were optained by simply reversing the polarity of the quadrupole strengths for settings 2 and 3 resulting in the x plane being varied and the y plane kept at -I. Figures 3.9 and 3.10 show the principle rays and beta functions

-bb-error =

Figure 3.9:

-bb-error =

Figure 3.10:

for each optical setting. The initial beta functions used for all settings were those for the "matched" case (initial and final beta functions equal) for setting 1. The energy recovery setting was identical to setting 1.

The following matrices were generated using DIMAD for all six optical settings for the FODO array (units are meters for length and radians for angles).

$$\text{Setting 1} \implies \begin{pmatrix} -1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.0 \end{pmatrix}$$

$$\text{Setting 2} \implies \begin{pmatrix} -1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 6.33 \\ 0.0 & 0.0 & -.158 & 0.0 \end{pmatrix}$$

$$\text{Setting 3} \implies \begin{pmatrix} -1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.06 \\ 0.0 & 0.0 & .944 & 0.0 \end{pmatrix}$$

$$\text{Setting 4} \implies \begin{pmatrix} -1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.57 & 1.0 \end{pmatrix}$$

$$\text{Setting 5} \implies \begin{pmatrix} 0.0 & 6.33 & 0.0 & 0.0 \\ -.158 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.0 \end{pmatrix}$$

$$\text{Setting 6} \implies \begin{pmatrix} 0.0 & -1.06 & 0.0 & 0.0 \\ .944 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.0 \end{pmatrix}$$

The final optical consideration in the model of the recirculation path is the setting of the quadrupole doublets QR1-QR2 and QR9-QR10 for acceleration of the beam on the second pass as well as energy recovery. These doublets were used to match the beam out of B1 and into B2 respectively. They are necessary because of the relatively large horizontal and vertical focussing of the bends which causes the beam envelope to diverge rapidly after emerging from each bend. The situation was particularly troublesome when matching out of B2 as there is no focussing other than emittance damping available for the second pass beam. In the case of energy recovery the emittance actually grows as the energy decreases, causing additional difficulties with beam scraping.

The quadrupole doublets were set up to produce the minimum spot size possible at the B1 beam dump for setting 1 of the quadrupoles QR3-QR8. When changing to the other settings the doublet strengths were not changed (energy recovery being the sole exception). To realistically model the complete recirculation path, the doublet strengths were fit to measurements (described in section 3.6) of the M_{12} and M_{34} matrix elements that were measured using the correctors and viewscreens at the positions of HVQL4, HVQL5, and HVQL6 for each optical setting (except energy recovery) because it is these matrix elements that are most important for multipass BBU. For energy recovery the main goal of the fitting was to mimimize the matrix elements for recirculation at the positions of quadrupoles HVQL4, HVQL5, and HVQL6. The difficulty in the matrix element minimization underscores the fact that in practice the energy recovery mode was the most difficult of all the optical settings to set up and operate. Table 3.3 lists these matrix element values computed with DIMAD for each optical setting. The result is that the calculations reproduce the order of magnitude of

Setting #	Location	M_{12} (m)	M_{34} (m)
1	HVQL4	2.00	1.60
	HVQL5	7.39	-8.16
	HVQL6	16.1	-28.5
2	HVQL4	7.80	.500
	HVQL5	18.3	-6.02
	HVQL6	10.2	-42.8
3	HVQL4	1.40	-.400
	HVQL5	6.72	21.4
	HVQL6	18.7	33.2
4	HVQL4	-.533	1.99
	HVQL5	-3.70	-4.80
	HVQL6	-3.48	-16.8
5	HVQL4	-3.47	2.06
	HVQL5	-.625	13.7
	HVQL6	-2.50	23.0
6	HVQL4	-.300	3.30
	HVQL5	-5.64	-7.77
	HVQL6	-29.4	-41.4
Energy Recovery	HVQL4	1.87	.582
	HVQL5	55.5	29.9
	HVQL6	50.0	50.0

Table 3.3: Recirculation transfer matrix elements calculated from DIMAD.

the transfer matrix elements determined by the measurements. This was most likely due to unaccounted for non-ideal (such as non-linearities other than sextupole and coupling between planes) optical behavior in B1 and B2, the strongly focussing bend quadrupoles Q1-Q2 and Q3-Q4 and similarly strongly focussing matching doublets QR1-QR2 and QR9-QR10 all of which had focal lengths of about 1 m on average for all optical settings.

Figure 3.11 shows beta functions for transport from the end of the cryounit to the beam dump for all six optical settings and figure 3.12 shows these functions for the energy recovery setting. The initial beta functions used in the calculation ($\beta_x = 40$ m, $\beta_y = 27$ m, $\alpha_x = \alpha_y = 4$ rad, and $\epsilon_x = \epsilon_y = \epsilon = 5 \times 10^{-8}$ m $\cdot$ rad) are in agreement with emittance measurements at 5.6 MeV. Furthermore they reproduce a waist qualitatively observed at the position of the injection chicane on the first pass. The difficulties in matching out of the bends manifest themselves in rather large beta functions (in the kilometer range for two settings) and therefore beam envelopes. The beta functions in general become large after bend B1 due to the strong quadrupole focussing of the bend. The beta functions are also large (primarily the y plane) after B2 through the second pass through the linac for the same reason. This limited the beam current because scraping occurred due to 60 Hz motion on the beam and the relatively large beam envelopes of .5 cm compared to typical apertures in the system of 2.5 cm (the beam loss monitors were set to turn the beam off when a beam current loss of 1 μA occurred)

Beam envelopes are given by

$$\sigma_{x,y} = \sqrt{\beta_{x,y}\epsilon(p)} \tag{3.5}$$

where $\beta_{x,y}$ is the beta function and $\epsilon(p)$ is the emittance as a function of momentum. Table 3.4 gives the 4σ emittance as a function of the beam momentum over the region of the arc length parameter s where the beam momentum is constant. For acceleration in the linac (or deceleration of the second pass beam for energy recovery) one can infer the emittance by linear interpolation in the arc length region of the

Setting #	Energy (MeV)	Arc Length s (m)	Emittance (ϵ) $\times 10^{-8}$ m $\cdot$ rad
1–6	5.6	$0.0 < s < 13.8$	5.00
	42.8	$30.8 < s < 77.9$	0.64
	80.1	$s > 94.9$	0.34
Energy Recovery	5.6	$0.0 < s < 13.8$	5.00
	42.8	$30.8 < s < 77.9$	0.64
	5.6	$s > 94.9$	5.00

Table 3.4: Beam emittance (4σ) in the recirculator.

linac for the first pass (13.8 m $< s <$ 30.8 m) and second pass (77.9 m $< s <$ 94.9 m) beams.

Finally, the first-order transfer matrix describing the recirculation arc from the exit of cryomodule 2 to the entrance of cryomodule 1 at 42.8 MeV is computed for each optical setting. The matrices were calculated after the doublet strengths (QR1-QR2 and QR9-QR10) were fit to the measured (using the correctors and viewscreens at locations HVQL4, HVQL5, and HVQL6) angle to displacement matrix elements described in section 3.6. These matrices were used in TDBBU to calculate the threshold current for multipass BBU and were generated using DIMAD for all six optical settings for acceleration as well as the single energy recovery setting.

$$\text{Setting 1} \quad \Longrightarrow \begin{pmatrix} .562 & 6.69 & 0.0 & 0.0 \\ -.0705 & .940 & 0.0 & 0.0 \\ 0.0 & 0.0 & .838 & -22.1 \\ 0.0 & 0.0 & .0914 & -1.22 \end{pmatrix}$$

$$\text{Setting 2} \implies \begin{pmatrix} 2.40 & 10.6 & 0.0 & 0.0 \\ -.0235 & .313 & 0.0 & 0.0 \\ 0.0 & 0.0 & .268 & -8.15 \\ 0.0 & 0.0 & .219 & -2.92 \end{pmatrix}$$

$$\text{Setting 3} \implies \begin{pmatrix} .362 & 7.03 & 0.0 & 0.0 \\ -.0844 & 1.12 & 0.0 & 0.0 \\ 0.0 & 0.0 & -.859 & 50.5 \\ 0.0 & 0.0 & -.0256 & .342 \end{pmatrix}$$

$$\text{Setting 4} \implies \begin{pmatrix} .0986 & -19.2 & 0.0 & 0.0 \\ .0492 & .563 & 0.0 & 0.0 \\ 0.0 & 0.0 & .405 & 17.7 \\ 0.0 & 0.0 & -.103 & -2.04 \end{pmatrix}$$

$$\text{Setting 5} \implies \begin{pmatrix} -1.04 & -7.54 & 0.0 & 0.0 \\ .150 & .128 & 0.0 & 0.0 \\ 0.0 & 0.0 & .634 & 2.52 \\ 0.0 & 0.0 & .0148 & 1.64 \end{pmatrix}$$

$$\text{Setting 6} \implies \begin{pmatrix} -.0109 & -6.66 & 0.0 & 0.0 \\ .147 & -1.96 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.44 & -25.8 \\ 0.0 & 0.0 & .154 & -2.05 \end{pmatrix}$$

$$\text{EnergyRecovery} \implies \begin{pmatrix} .247 & 24.9 & 0.0 & 0.0 \\ .00435 & 4.49 & 0.0 & 0.0 \\ 0.0 & 0.0 & .247 & -4.24 \\ 0.0 & 0.0 & .00435 & 3.97 \end{pmatrix}$$

3.4 B1 Dispersion Measurement

In preparation for this measurement, the beam was set up so that the beam energy was 45 MeV (cavity 8 of cryomodule 2 was operational at the time of this measurement). The beam current was set to 5 μA pulsed at .5% duty factor. This resulted in an adequate average current such that the viewscreens did not saturate when the beam struck them. The quadrupoles Q1 and Q2 were left unpowered to maximize the dispersion, and the beam was set up going straight into D1. The magnets were then cycled around their hysteresis curves from 0 to 100 Amperes three times. The dipoles in each bend were chosen based on magnet measurements to be as "identical" as possible. Each had a field integral that varied less than .5% from magnet to magnet on the same point on the hysteresis curve [Ka92, Se91]. A Hall probe was placed in dipole D1 so that by measurement of its central field the field integral could be found based on an effective length of 19.1 cm from the magnet measurements.

The relative momentum spread of a beam of particles of central momentum p that deviate by a momentum difference Δp traveling in a dipole field can be found by using equation 3.1 resulting in

$$\frac{\Delta p}{p} = \frac{\Delta B}{B} \tag{3.6}$$

$$\Delta p = p - p' \tag{3.7}$$

$$\Delta B = B - B', \tag{3.8}$$

where p' is the momentum of the highest (or lowest) momentum particle in the beam and B' is the magnetic field required so that the extremal particle orbit moves to the orbit of a particle at the central momentum p. Different values of the magnetic field can therefore be used to simulate the trajectory of off momentum particles by changing the beam orbit.

This approach was taken in the B1 dispersion measurement. The four dipoles were ramped (after proper hysteresis conditioning) from 0 to 100 A and then back to 0 A while the beam position on the viewscreen after D4 was recorded as a function of the central field of D1. Two sets of measurements were made; they are displayed in figure 3.13. Figure 3.13 shows both the beam position on the viewscreen vs the recorded magnetic field in D1 while going from 0 to 100 A or low to high field (high to low simulated momentum) on the hysteresis curve and the beam position on the viewscreen while going from 100 to 0 A or high to low field (low to high simulated momentum) on the hysteresis curve. The viewscreen position of 0 m represents the position of the central ray of momentum 45 MeV/c for a dipole field of 6.005 kG.

Both curves clearly show that a difference in slope exists depending on whether one is at high field (low simulated momentum) or low field (high simulated momentum) and this difference is not due to some unaccounted for hysteresis effect. This implies that the dispersion, which is related to the slope of the displacement vs. field curve, is different depending on whether the particle is higher or lower in momentum than the central particle. The dispersion η, is defined as

$$\eta \equiv \frac{\Delta x}{\Delta p/p} \tag{3.9}$$

where Δx is the relative displacement of an off momentum particle of momentum offset Δp from the position of the central particle of momentum p. Using equation 3.6, equation 3.9 becomes

$$\eta \equiv \frac{\Delta x}{\Delta B} \times B \tag{3.10}$$

for a relative field offset $\Delta B/B$ of the dipoles, resulting in a relative displacement of the beam Δx which simulates a momentum offset. The dispersion can be written in terms of the slope of the displacement vs field curve as

$$\eta = \frac{dx}{dB} \times B \tag{3.11}$$

$$\frac{\Delta x}{\Delta B} \approx \frac{dx}{dB} \tag{3.12}$$

where dx/dB is the slope of a best fit curve to the data.

Figure 3.13 shows the best-fit straight line to the high field $>$ 6.005 kG (low simulated momentum) and low field $<$ 6.005 kG (high simulated momentum) data.

In addition, the best linear fit to all the data for both curves is also shown. The range of field values that these data span represents a simulated momentum bite of $\Delta B/B = \pm.35\%$ from the central momentum of 45 MeV/c at $B \sim 6.005$ kG. The vertical error bars represent the statistical error of $\sigma = .0003$ m in determining the centroid of the beam using the camera/viewscreen diagnostic [Bo93].

The statistical error in the slope of the best-fit line, which was driven by the position measurement error, was obtained using the error matrix formalism of Bevington [Be69] where the diagonal terms of the error matrix ϵ_{jj} are equal to the square of the statistical error of the coefficients σ_{a_j} of the fitting function, in this case a line with slope a_1 and intercept $a_\circ$ $(j = 0, 1)$. Table 3.5, which summarizes the dispersion measurement results, clearly shows the asymmetry in the dispersion where low momentum particles effectively experience more horizontal focussing and hence lower dispersion than do high momentum particles. This result can be understood in terms of a sextupole fringe field component for the bend dipoles as described in Appendix B.

3.5 B1 Dispersion Suppression Measurement

This measurement was performed to determine the quadrupole current of Q1 and Q2 (both powered in series from the same supply) that was required to reduce the dispersion, η, and its slope, η', after B1 to zero. Both sets of quadrupoles Q1-Q2 in B1 and Q3-Q4 in B2 were chosen to be as similar in field properties as possible and each pair differed by at most .5% in integrated field gradient at each point on their respective hysteresis curves [Ka92, Se91]. The beam energy was set to 42.8 MeV/c and cavity 8 of the second cryomodule was unpowered. As with the B1 dispersion measurment, the beam current was 5 μA pulsed at .5% duty factor. Initially, the quadrupoles were cycled from 0 to 10 A three times around the hysteresis training cycle. As a result, the integrated quadrupole gradient can be determined from a measurement of the quadrupole current. The quadrupole multipole constant is given

Hysteresis Ramping Cycle	Beam Orbit (Field)	$\frac{dx}{dB}$ (m/kG)	η (m)
$0 \rightarrow 100$ A (Low to high field)	> 6 kG	$.382 \pm .018$	$2.30 \pm .11$
	< 6 kG	$.478 \pm .023$	$2.87 \pm .14$
	All Data	$.439 \pm .009$	$2.64 \pm .05$
$100 \rightarrow 0$ A (High to low field)	> 6 kG	$.353 \pm .017$	$2.12 \pm .10$
	< 6 kG	$.502 \pm .023$	$3.01 \pm .14$
	All Data	$.419 \pm .008$	$2.51 \pm .05$
Average over both cycles	> 6 kG	$.367 \pm .025$	$2.21 \pm .15$
	< 6 kG	$.490 \pm .032$	$2.94 \pm .20$
	All Data	$.429 \pm .012$	$2.57 \pm .07$

Table 3.5: Dispersion measurement summary.

by

$$k^2 = \frac{e}{p}\left(\frac{B'}{l_q}\right) \tag{3.13}$$

where B' is the integrated gradient (that is integrated along the z or beam axis), p is the beam momentum, and l_q is the effective length of the quadrupole. The second downstream viewscreen on the return path after B1 was used to record the displacement of the beam (see figures 3.1 and 3.4). Quadrupole doublet QR1-QR2 was kept unpowered during the measurement so that the beam traversed a drift of 5.83 m from the exit of D4 to the viewscreen.

The measurement consisted of recording the displacement of the beam from the central momentum position on the viewscreen as the beam momentum was varied as a function of quadrupole current in Q1 and Q2. Displacements were measured using the camera/viewscreen diagnostic. The beam momentum was varied by changing the gradient of cavity 7 of cryomodule 2. The cavity gradient was both lowered and raised to achieve an energy shift of $\Delta E/E \cong \Delta p/p = \pm.047\%$ at each quadrupole current where E and p are the beam energy and momentum. Figure 3.14 shows the resulting net displacement for both the high and low momentum ray from the central ray as a function of Q1 and Q2 current.

The intersection of the best fit curves in figure 3.14 with the vertical axis indicates the quadrupole current at which B1 transports the beam dispersion free. The high momentum curve intersects the vertical axis at a quadrupole current of 2.496 A, and the low momentum curve at 2.367 A. The quadrupole current was set midway between these two values at 2.432 A as being the dispersion suppressed current where $\eta = 0$. Using the measured hysteresis curves, this current yielded a quadrupole constant k of 2.37/m for a quadrupole effective length of .236 m and beam momentum of 42.8 MeV/c.

The vertical error bars result from the determination of the position difference of the centroid of the central momentum from either the high or low momentum ray. Each centroid determination has a $\sigma = .0003$ m error [Bo93] (as with the B1 dispersion measurement) resulting in a $\sigma_{\Delta x} = .0004$ m error for the position difference when each centroid position error is added in quadrature. An estimate of the error in the dispersion as well as its slope is given in terms of the (dominant) displacement error as:

$$\sigma_\eta \sim \frac{\sigma_{\Delta x}}{\Delta p/p} \tag{3.14}$$

$$\sigma_{\eta'} \sim \frac{\sigma_{\Delta x}}{d_v} \tag{3.15}$$

where in this case $d_v = 5.83$ m is the drift length from the viewscreen to dipole D4 of B1. Using the numbers previously given for the parameters in equations 3.14 and 3.15 yields:

$$|\eta| < .86 \text{ m} \tag{3.16}$$

$$|\eta'| < .07 \text{ mrad} \tag{3.17}$$

indicating the degree to which the dispersion was "suppressed."

3.6 Recirculation M_{12} and M_{34} Measurement

The results of this measurement were used in Chapter 3 to model the recirculation optics by fitting the quadrupole strength of doublets QR1-QR2 and QR9-QR10 to the measured recirculation transfer matrix elements. The horizontal (M_{12}) and vertical (M_{34}) deflection to displacement matrix elements were measured using the camera/viewscreen diagnostic and the horizontal and vertical corrector magnets at locations HVQL4, HVQL5, and HVQL6. Ideally one would have liked to measure the full transfer matrix at these points. This was, however not possible due to lack of space on the beamline to produce anything but the sine-like ray using the correctors.

Each linac quadrupole at positions HVQL4, HVQL5, and HVQL6 is placed on a girder containing the viewscreen and correctors. Starting upstream the component placement on each girder, in order, is a viewscreen, horizontal corrector, vertical corrector and finally the quadrupole. The horizontal corrector is placed 12 cm and the vertical corrector 34 cm downstream from the viewscreen. The measurement procedure consisted of horizontally (vertically) deflecting the beam by a known angle θ (ϕ) using the horizontal (vertical) corrector and measuring the net deflection Δx (Δy) from the undeflected "central" trajectory. The viewscreen has a 2 mm hole at its geometric center to allow the first pass beam to pass undisturbed. The ratio of the displacement from the central trajectory to the deflection angle gives the matrix element. The actual displacement needed is that of the second pass beam at the position of the corrector and not the viewscreen. The effect of the small displacement of the viewscreens from the correctors on the net actual displacement at the position of the corrector is small compared to the orbit length ($\sim$ 60 m) and is therefore neglected. The matrix element measured is therefore that for nearly the full orbit.

In terms of the measured displacements and angles, the first order angle to displacement matrix elements are given by,

$$M_{12} = \frac{\Delta x}{\Delta \theta} \tag{3.18}$$

$$M_{34} = \frac{\Delta y}{\Delta \phi}. \quad (3.19)$$

The corrector dipole geometry is that of case 1 from Appendix A so that the net angular displacements are given by,

$$\Delta\theta = \frac{e\Delta B l_h}{p} \quad (3.20)$$

$$\Delta\phi = \frac{e\Delta B l_v}{p} \quad (3.21)$$

where ΔBl_h (ΔBl_v) is the net change in the field integral for the horizontal (vertical) corrector and p is the (first pass) beam momentum at the corrector. The Δ notation is adopted because the quantity that matters for the matrix element measurement is the angular change that the corrector produces relative to its nominal setting (which is determined by making the first pass beam travel along the linac axis). Equations 3.20 and 3.21 were derived for the case of a small deflection angle where the sine of the angle is approximated as the angle in radians. This is valid as typical deflection angles are ~ 1 mrad. The change in the field integral of the correctors is given in terms of the current change in the corrector as

$$\Delta Bl_h = K_c \Delta I_h \quad (3.22)$$

$$\Delta Bl_v = K_c \Delta I_v \quad (3.23)$$

where K_c is the field integral constant for the corrector and ΔI_h and ΔI_v are the current change for the displaced ray in the horizontal and vertical corrector from the current corresponding to the central ray. Table 3.6 shows the corrector constant for the three sets of correctors used as well as the first pass beam momentum at the corrector.

The error in the matrix element is found by adding in quadrature the errors in the net position and angular displacement as

$$\left(\frac{\sigma_{M_{12}}}{M_{12}}\right)^2 = \left(\frac{\sigma_{\Delta x}}{\Delta x}\right)^2 + \left(\frac{\sigma_{\Delta\theta}}{\Delta\theta}\right)^2 \quad (3.24)$$

$$\left(\frac{\sigma_{M_{34}}}{M_{34}}\right)^2 = \left(\frac{\sigma_{\Delta y}}{\Delta y}\right)^2 + \left(\frac{\sigma_{\Delta\phi}}{\Delta\phi}\right)^2. \quad (3.25)$$

Figure 3.12: Energy recovery beta functions.

Corrector Location	Corrector	K_c (kG·cm)/Amp	p_{fp} MeV/c
HVQL4	Horizontal	.43	5.6
	Vertical	.43	
HVQL5	Horizontal	.43	23.7
	Vertical	.43	
HVQL6	Horizontal	1.72	42.8
	Vertical	1.72	

-bb-error =

Table 3.6: Corrector field integral constant for each linac location. The first pass beam momentum p_{fp} is also shown.

-bb-error =

Figure 3.11:

The error in the angular displacement is given by,

$$\frac{\sigma_{\Delta\theta}}{\Delta\theta} = \frac{\sigma_{\Delta I_h}}{\Delta I_h} \qquad (3.26)$$

$$\frac{\sigma_{\Delta\phi}}{\Delta\phi} = \frac{\sigma_{\Delta I_v}}{\Delta I_v} \qquad (3.27)$$

after combining equations 3.20-3.23. The net error in the position displacements, $\sigma_{\Delta x}$ and $\sigma_{\Delta y}$, is .0004 m; it is found by adding in quadrature the .0003 m error position determination of the central and deflected ray centroids on the viewscreen. The dominant error in the angular displacement is due to the corrector field integral error due to the error in the corrector current. The error in each corrector current reading is .005 Amperes corresponding to the central and deflected ray. This error when added in quadrature results in .007 Amperes for $\sigma_{\Delta I_h}$ and $\sigma_{\Delta I_v}$. This relative error in the current differences is on the order of 5 to 10% whereas the error in the momentum is 1.8% [Ka92]. The relative error in the momentum is therefore neglected. Table 3.7 lists the matrix elements measured at each position for each optical setting.

The quadrupole doublet strengths for QR1-QR2 and QR9-QR10 were determined by adjusting their strengths in a DIMAD fit to the matrix elements measured for nearly the complete recirculation. The matrix elements for the full recirculator calculated using these inferred quadrupole strengths are listed in table 3.3; they agree only in order of magnitude with the measured matrix elements listed in table 3.7 for all optical settings and locations. For settings 1, 2, 3, and 6 the fitted matrix elements at the location of QL4 agree exactly with the measured values; for setting 4 the matrix elements at the location of QL5 agree exactly, and for setting 5 the matrix elements at location QL6 agree exactly. The fitting procedure was to try and minimize for each setting the differences between the measured and fitted matrix elements at each location in the linac. This minimum occurred when the matrix elements at one of the locations was fit exactly to the measured values at that location consistent with maximum beta functions for the second pass beam being as small as possible (no larger than 10^3 m in order of magnitude).

The relatively poor agreement can be traced to non-ideal optical behavior in the

Optics Setting #	Corrector position	M_{12} m/rad	M_{34} m/rad
1	HVQL4	$2.0 \pm .7$	$1.6 \pm .3$
	HVQL5	24 ± 6	4.6 ± 1.3
	HVQL6	-6.9 ± 2.0	–
2	HVQL4	7.8 ± 1.4	$.5 \pm .4$
	HVQL5	-2.1 ± 1.1	15 ± 4
	HVQL6	16 ± 3	-6.6 ± 1.4
3	HVQL4	$1.4 \pm .4$	$-.4 \pm .3$
	HVQL5	$2.3 \pm .9$	-2.2 ± 1.5
	HVQL6	-3.9 ± 1.4	$6.2 \pm .8$
4	HVQL4	$1.6 \pm .7$	$-.7 \pm .6$
	HVQL5	-3.7 ± 1.6	-4.8 ± 1.0
	HVQL6	-5.4 ± 2.5	-7.5 ± 6.2
5	HVQL4	-4.5 ± 1.8	$1.6 \pm .7$
	HVQL5	-9.8 ± 3.3	-5.5 ± 1.2
	HVQL6	$-2.5 \pm .9$	23 ± 5
6	HVQL4	$-.3 \pm .4$	3.3 ± 1.0
	HVQL5	17 ± 4	27 ± 10
	HVQL6	34 ± 12	21 ± 4

Table 3.7: Measured matrix elements for each optical setting at each linac location using correctors and viewscreens.

bends B1 and B2 and the difficulties this caused in matching the beam through the linac on the second pass. The TDBBU calculations were performed using the computed transfer matrices resulting from inclusion of the fitted doublet strengths in a full DIMAD calculation of the recirculator optics for each optical setting.

3.7 Energy Recovery Measurement

This measurement determined the final energy of the second pass beam in the energy recovery mode. This was done by referring to the magnet measurements of dipole DE1 of the energy recovery chicane [Ka92]. DE1 is designed to bend the second pass beam through an angle of 20° as shown in figure 3.3, and the beam orbit is given by case 1 beam orbit geometry described in Appendix A. Using equations 3.1 and A.11 the beam momentum of the second pass beam is given by,

$$p = \frac{eBl_e}{\sin(\theta)} \tag{3.28}$$

in terms of the dipole field, effective length, and bend angle.

The procedure was to first produce a second pass accelerated beam at 80.1 MeV with the energy recovery chicane powered to bend a beam of approximately 5.6 MeV into the dump. The first pass beam was accelerated to the standard energy of 42.8 MeV. The value of the starting field for dipoles DE1, DE2, and DE3 was set to approximately .35 kG based on a magnet effective length of roughly 18 cm for DE1 and DE3 and 36 cm for DE2. B1 was then moved 1/4 the fundamental mode wavelength (5 cm) toward the linac thereby decelerating the second pass beam. Minor adjustments were then made to the powering of the dipoles until the beam was found on the viewscreen after DE1. Once the beam was found, the position of bend B1 was moved slightly so as to move the beam spot as far to the low energy end of the viewscreen as possible. When this minimum position was found, the spot was centered on the viewscreen and the value of the magnetic field of DE1 was measured and found to be .364 kG at a magnet current of 1.214 Amperes. This magnet current corresponds to an effective length of 17.49 cm interpolated from the magnet measurement table for DE1. Using equation 3.28, the result for the minimum second pass beam momentum turns out to be 5.59 MeV/c for a total energy of 5.61 MeV. Owing to the 1.8 % measurement uncertainty in the dipole field integral [Ka92], the final energy is recorded as $5.6 \pm .1$ MeV. The injection beam momentum was set to its nominal value of 5.55 MeV/c for an injection beam energy of $5.6 \pm .1$ MeV. The conclusion is that full energy recovery was achieved within experimental uncertainty and this was done at up to 30 μA CW beam current.

The energy measured for the first pass beam was $42.8 \pm .8$ MeV as determined from the magnet measurements. Using this value, a limit on the minimum percentage of the energy recovered can be inferred from the measurement uncertainties. The minimum is defined as the ratio

$$\frac{42.0 - 5.7}{43.6 - 5.5} \times 100 = 95.3\% \tag{3.29}$$

and is interpreted as the minimum possible energy recovered for the second pass beam divided by the maximum possible energy gain of the first pass beam. This ratio quantifies the statement that full energy recovery was achieved.

The linac viewscreens with $\sim$ 2 mm diameter holes at their centers located at the positions of the linac quadrupoles HVQL3, HVQL4, HVQL5, and HVQL6 greatly aided the procedure of finding the second pass beam for both acceleration and energy recovery. The first pass beam was threaded through the hole and the second pass beam could be seen by slightly steering it off axis using the injection chicane dipoles DC5, DC6, and DC7. Indeed, for energy recovery it was hard to keep the second beam on axis as it was decelerated. A major difficulty with the energy recovery experiment was the large second pass spot sizes (on the order of a centimeter in diameter) at the exit of the second cryomodule.

Figure 3.14: Beam centroid displacement vs Q1 and Q2 current.

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Figure 3.13: Viewscreen position vs D1 magnetic field. The top (bottom) plot shows data taken while ramping upward (downward) on the hysteresis curve.

-bb-error =

Chapter 4

RF Measurements

4.1 Overview

This chapter describes the RF measurements performed using the recirculator. The measurements were performed under single-pass and recirculating conditions using a CW beam. The primary set of RF measurements were used to investigate the multi-pass BBU instability properties of the HOMs under the conditions of a recirculated CW beam for all six optical settings including energy recovery. In these measurements an RF stripline kicker was used to impress a known transverse modulation on the CW beam, and cavity 8 of cryomodule 2 was used to detect the HOM signal. From an analysis of the HOM resonance measured as a function of CW beam current, an experimental lower limit can be set for the multipass BBU threshold current for that particular HOM. The theoretical analysis of the data was based on the simple single cavity recirculating linac discussed in Chapter 2. An additional RF measurement was performed with a single pass CW beam using the RF kicker and a second, identical device located after cryomodule 2 used as a pickup. This experiment investigated the practicality of measuring the impedance of the HOMs using the RF pickup. Appendix C describes the operation of the RF stripline kickers and pickups and their design. The first topic discussed is the injector setup procedure which was common to all the measurements.

4.2 Injector Operating Parameters and Setup Procedure

The injector is designed to provide a $\sim$ 45 MeV CW beam for injection into the first CEBAF (North) linac by accelerating the beam from 100 keV (see figure 3.1). The setup procedure for the recirculation experiments is described in this section. Setup of the 100 keV gun, choppers, buncher, capture section and first two superconducting cavities (or up to an energy of 5.6 MeV) is identical to that for the North linac. In addition, cryomodule 1 and 2 cavity gradients are for the most part set to the same operating point in both setups resulting in nominal 45 MeV operation. The main difference between North linac setup and setup for these experiments is the addition of the injection chicane which is necessary for recirculating the beam.

The electron beam originates in a 100 keV gun, and the beam is transported to the entrance of a short, room-temperature, graded-beta capture section after chopping and bunching at 1497 MHz. The choppers produce 60° bunches that are compressed to 11° by the buncher and finally to 4.5° after acceleration by the capture section. The capture section boosts the beam energy to 500 keV before the entrance of the first superconducting cavity pair (cryounit). The buncher and capture section phase was initially set based on a PARMELA [Kr89] calculation that gave the required bunch length (4.5°) at the entrance to the cryounit. The cryounit boosts the energy to 5.6 MeV and further compresses the bunch to $< 1°$.

The beam is then transported to the main injector linac consisting of two cryomodules each containing four superconducting cavity pairs. The bunch length at injection into the first cryomodule is checked to be $< 1°$ of RF phase or $< .5$ mm using the techniques developed by Yao [Ya89], Jackson, and Krafft [Ja92]. Minor adjustments of buncher amplitude and capture section phase are made to achieve this good bunch length since the buncher and capture section perform 93% of the bunching (the first two superconducting cavities perform the rest). The beam is then accelerated to 45 MeV by the two cryomodules for injection into the CEBAF north linac. The bunch length remains the same as the beam is accelerated from 5.6 to 45

MeV and no further bunching occurs because the electrons are relativistic. Finally, the first superconducting cavity of the cryounit was backphased by 7.5° to provide transverse focussing so that the beam required no quadrupole focussing through the entire injector. All focussing was accomplished through the backphasing technique and adiabatic damping of the emittance as the beam accelerates. Resulting spot sizes through 45 MeV were less than or equal to 1 mm in radius.

For the recirculation experiments the first pass beam energy was set slightly below the nominal 45 MeV to 42.8 MeV because cavity 8 of the second cryomodule was left unpowered. The cavity gradients used to achieve the 42.8 MeV first pass beam energy were given in Chapter 3 in table 3.1. The injection chicane dipoles were powered so that both first and second pass beams traveled on the linac axis. Each bend B1 and B2 was set up in dispersion suppressed mode and then the six quadrupoles of the FODO array were set to the values calculated by DIMAD for setting 1. Quadrupole doublets QR1-QR2 and QR9-QR10 were adjusted so that the second pass beam spot size at the beam dump after B1 was minimum for setting 1. Setting 1 was considered the nominal setting and the other 5 settings for acceleration of the second pass beam were accomplished by adjustment of QR3-QR8 strengths. All other magnetic and acceleration elements were left unchanged when going to a different setting. Energy recovery was performed by using setting 1 for QR3-QR8 and readjusting doublets QR1-QR2 and QR9-QR10 to minimize the spot size at the dump after the energy recovery chicane.

The next sections describe in detail an analysis of the RF measurements that were performed during the experiment based on the simple examples given in Chapter 2. The RF measurements used stripline kickers and pickups along with a superconducting cavity to detect cavity HOM signals. An analysis of the HOM signal strength as a function of beam current was performed for all recirculation optical settings as well as energy recovery in order to set experimental limits on the BBU threshold current. In addition, a single pass RF measurement was performed on a HOM and the data were used to determine its shunt impedance R/Q.

4.3 RF Measurements of Cavity Higher-Order Modes

4.3.1 Introduction

The first RF measurement consisted of using a broadband RF stripline kicker to excite the beam and cavity 8 of cryomodule 2 to detect the HOM signal under the conditions of a CW recirculated beam. The second consisted of using a kicker to excite the beam and a broadband stripline pickup to detect HOM signals placed on the beam under the conditions of a single pass CW beam. These measurements are sometimes referred to as a "beam transfer function" measurement.

The basic physics of both of these measurements was discussed in terms of the simple single cavity containing a single HOM "linac" in Chapter 2. This simple case is relevant to these measurements because of the nature of the cavity impedance, which is strongly peaked at each HOM frequency. Near a particular HOM in frequency, the impedance is dominated by the mode at that frequency and all other modes do not contribute significantly. We can therefore use the results of Chapter 2 as a guide when analyzing the RF measurement data taken over the range of frequencies where the mode impedance is peaked (roughly ω_m/Q_m). For the case of recirculation, the wake potential was found to be proportional to the HOM impedance divided by a denominator term that is zero at the threshold current (see equations 2.101 and 2.102). For the single pass case there was no such denominator term and the pickup current moment was simply proportional to the impedance (see equation 2.67 and 2.68).

These expressions can be written normalized to the kicker drive signal. This is useful because the ratio of the cavity (or pickup) signal to the kicker drive signal (the beam transfer function) can be directly measured using a network analyzer. The basic measurement setup consists of driving the kicker (through a suitable amplifier) using port A of the network analyzer, detecting the cavity or pickup signal using port B and performing an $S_{21}(\omega)$ measurement (see [Go84] for the definition of the scattering

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Figure 4.1: Recirculation HOM measurement setup schematic using a stripline kicker and a superconducting cavity.

parameters of a two-port network). This can be thought of in terms of the kicker-recirculated beam-cavity HOM or kicker-single pass beam-cavity HOM-pickup "black box" two-port equivalent network on which an $S_{21}(\omega)$ measurement is conducted.

4.3.2 Cavity Higher-Order Mode Measurement Using a Recirculated CW Beam

Figure 4.1 (and figure 2.7) shows schematically the setup for the cavity HOM measurement using a CW recirculated beam. The stripline kicker was located at the entrance to dipole DC1 of the injection chicane and was oriented to deflect the beam in the vertical (y) optics plane. As mentioned earlier, cavity 8 of cryomodule 2 was used as the pickup that detected the HOM signal for this measurement. Port A of the network analyzer drives a 25 W broadband amplifier with a center frequency of 500 MHz with a bandwidth of 900 MHz through a mixer. The stripline kicker is driven by the amplifier through a 180° hybrid coupler that splits the drive signal into two signals 180° out of phase and is inherently broadband (see Appendix C). It is constructed with the same center frequency of 500 MHz and has bandwidth from D.C. to 640 MHz. Port B of the network analyzer was connected to the cavity probe antenna attached to the HOM load waveguide. The probe is usually used to regulate cavity gradient and phase but also couples to the HOM fields.

The HOMs with the highest Q were found to lie in the neighborhood of 2.0 GHz. The two modes that were measured using the setup of figure 4.1 have a frequency of 1899 and 2110 MHz and have Q values of $90,000$ and $30,000$ respectively. These modes were used in the BBU simulations and are listed in table 2.2. As a result, the port B detector of the network analyzer required a frequency sweep near this frequency when searching for the HOMs. The mixer at the output of port A is used to reduce the frequency of the sweep signal down to frequencies around the 500 MHz operating range of the kicker (and kicker amplifier) by mixing the sweep signal with the 1497 MHz bunching frequency (ω_b). Via aliasing, the transverse modulation of the beam at 500 MHz can excite the HOMs around 2 GHz in the cavity. The scattering parameter $S_{21}(\omega)$ is the ratio of the signal amplitude detected at port B [the cavity signal $V_c(\omega)$] to the sweep signal amplitude of port A [the kicker drive signal $V_k(\omega)$] expressed in dB.

These measurements were performed for a variety of CW beam currents for each recirculator optics setting. The measured HOMs (at 1899 and 2110 MHz) were found to be the highest Q objects near 2.0 GHz. These modes were also among the strongest modes (large Q and R/Q) found in the cavity bench measurements [Am84]. The kicker exitation frequencies corresponding to these modes were 402 and 613 MHz because of aliasing. The kicker operating in the vertical plane was chosen because the cavity signal was found to be maximum for vertical beam deflections.

In Chapter 2 the instability condition for multipass BBU was found in terms of the wake potential and current moment due to the stripline kicker. Data analysis of the BBU instability requires that the measured signal $V_c(\omega)$ and drive signal $V_k(\omega)$ be expressed in terms of these quantities. Equations 2.6 and 2.7 define the wake function in terms of the momentum kick imparted to the beam by the electromagnetic wakefield and can be expressed as an integral over the wakefield. As stated in Chapter 2, the wake function is simply proportional to the wakefield in the relativistic limit.

The wake potential is expressed as an integral over the wake function and beam current and is the accumulated exitation of the wakefield at a given time t by the beam that has passed through the structure. The wake potential, $V(t)$, is defined to be proportional to the kick imparted to the beam at time t by the wakefield. The wake potential is therefore also proportional to the wakefield at time t and the proportionality holds in the frequency domain. The electromagnetic wakefield in the frequency domain is also proportional to the cavity signal $V_c(\omega)$ so that the cavity signal can be written directly in terms of the wake potential as

$$V_c(\omega) = \alpha_c V(\omega), \tag{4.1}$$

where α_c is a constant of proportionality which takes into account the effective coupling to the wakefield by the cavity probe antenna and any other cable attenuation factors. In Chapter 2 it was shown that the wake potential is proportional to the impedance. Therefore at a given beam current below threshold the cavity signal at a HOM resonance has the same form as the HOM impedance (shown in figure 2.3 scaled by a constant.

The kicker drive signal can be expressed in terms of the current moment due to the kicker deflection at the cavity. The kicker current moment is given in terms of a matrix element term multiplied by $I\theta_k(\omega)$ (see equation 2.93). From Appendix C the angular kick as a function of frequency is simply proportional to the drive signal $V_k(\omega)$, so that

$$IV_k(\omega) \propto I\theta_k(\omega) \tag{4.2}$$

$$IV_k(\omega) = 2\pi I_\circ \sum_{n=-\infty}^{\infty} V_k(\omega - n\omega_\circ) \tag{4.3}$$

where $IV_k(\omega)$ is the Fourier transform of $I(t)V_k(t)$. The sum in equation 4.3 collapses to the $n = 0$ term because the drive signal has appreciable components at frequency $\omega = \omega_k$ (there is no aliasing of the drive signal emerging from the network analyzer). The drive signal can be written as

$$V_k(\omega) \propto \frac{I\theta_k(\omega)}{I_\circ} \tag{4.4}$$

where $I_\circ$ is the average beam current. Equations 4.4 and 2.54 indicate that the kicker drive signal is proportional to all the aliased signals placed on the beam by the kicker scaled by the average beam current. Expressed in terms of the current moment due to the kicker at the cavity,

$$V_k(\omega) = \alpha_k \frac{Ix_c(\omega)_k}{I_\circ} \tag{4.5}$$

where the constant α_k is a constant of proportionality that takes into account the gain of the amplifier and the optical transfer matrix elements from kicker to cavity (see equation 2.93).

Using equations 4.1 and 4.5, equation 2.102 can now be written in terms of the cavity and kicker drive signals as,

$$\frac{V_c(\omega)}{V_k(\omega)} = \frac{\alpha_c W(\omega)}{\alpha_k}\left(\frac{I_\circ}{1 - I_\circ/I_t}\right) \tag{4.6}$$

$$S_{21}(\omega) \equiv \frac{V_c(\omega)}{V_k(\omega)} \tag{4.7}$$

where I_t denotes the (real) threshold current. The instability denominator $D(\omega) = 1 - I_\circ/I_t$ in equation 4.6 has been written as a real quantity and is zero at the threshold current. The $S_{21}(\omega)$ measurement near a HOM resonance is thus seen to scale like the impedance. Taking the modulus and the logarithm of both sides of equation 4.6 yields

$$\log|S_{21}(\omega)| = \log\left|\frac{\alpha_c W(\omega)}{\alpha_k}\right| + \log(I_\circ) - \log\left(1 - \frac{I_\circ}{I_t}\right). \tag{4.8}$$

The network analyzer was set up to measure the amplitude (modulus) of $S_{21}(\omega)$ in dB as a function of average CW beam current $I_\circ$. Equation 4.8 is the most useful way to express the measurement quantities in terms of the threshold condition.

Equation 4.8 has the form:

$$\log|S_{21}(\omega)| = a_\circ + \log(I_\circ) - \log(1 - a_1 I_\circ) \tag{4.9}$$

where $a_\circ$ and a_1 are parameters that can be fit to the data. The parameter $a_\circ$ contains information about the HOM impedance and a_1 is simply the inverse of the threshold current. The fitting function given by equation 4.9 is non-linear in the parameter a_1 (linear in $a_\circ$). Furthermore the parameter a_1 is constrained to be ≥ 0 because a negative threshold current is not physically real. Therefore a constrained non-linear fit of the data was performed which stepped the parameter a_1 and found the value of $a_\circ$ that minimized χ^2 for each value of a_1. The parameter a_1 was stepped to a value one step size less than the inverse of the data point representing the largest current value (to avoid taking the logarithm of a negative number). The miminum value of χ^2 that corresponds to the best-fit values of $a_\circ$ and a_1 was thereby determined numerically.

A similar procedure was used to determine the threshold current of cavity HOMs in the experiment of Lyneis *et al* [Ly83]. Instead of using a kicker to excite the beam which then excited cavity HOMs, the cavity was excited directly using a microwave source at the frequency of the HOM using an input antenna coupled to the HOMs. The HOM signal was then detected using another pickup antenna located in the cavity. The power output of the pickup antenna for a given input power at the exitation probe is given by

$$P(I_\circ) \propto \frac{1}{(1 - I_\circ/I_t)^2} \tag{4.10}$$

when a beam is recirculated through the cavity. By measuring the output power for two average currents $I_\circ = 0$ and $I_\circ = I$ (where in practice I is as large as possible to get a good signal) the threshold current is given by,

$$I_t = \frac{I}{1 - \sqrt{P(0)/P(I)}}. \tag{4.11}$$

The measurement in [Ly83] was done in the time domain but the same instability denominator results as it must. For the measurements described here, equation 4.6 was fit to the data to extract I_t. The difference between measurement techniques is the extra factor of $I_\circ$ in equation 4.6 due to the kicker.

Figure 4.2 shows the two HOMs that were measured using the setup of figure 4.1. The top plot is the network analyzer scan of the 1899 MHz mode at a CW beam current of 67 μA and the bottom plot is the scan for the 2110 MHz mode at 46 μA for optical setting 1. Each figure shows the resonant lineshape of a HOM with a width given by its Q. The measured Q of these modes was found to be a factor of three higher than the values obtained in bench measurements [Am84].

Figures 4.3, 4.4, 4.5, and 4.6 show plots of the height of the peak, $\log|S_{21}|$, as recorded by marker 1 shown in figure 4.2 as a function of $\log(I_\circ)$ for each optical

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Figure 4.2: Network analyzer frequency scan of the 1899 and 2110 MHz modes at 67 μA and 46 μA respectively. The vertical axis is $|S_{21}|$ in dB $(20\,\mathrm{Log}\,|S_{21}|)$ and horizontal axis is the frequency $f = \omega/2\pi$. The center and span frequencies determine the frequency range for the horizontal axis.

setting along with fitted curves. The data for each optical setting shows the data for the 1899 MHz mode at a larger signal amplitude (upper curve) than that for the 2110 MHz mode (lower curve). The vertical error bars represent an estimation of the noise evident on the resonance curves shown in figure 4.2. The jitter is a consequence of the noise signal incident on the network analyzer receiver at port 2 where the cavity signal enters. The solid curve corresponds to the best-fit values of parameters $a_\circ$ and a_1 which minimize χ^2. The dashed curve is a plot for the maximum value of a_1 as implied by its statistical error σ_{a_1}, or the curve that corresponds to to the quantity $a_1 + \sigma_{a_1}$ where σ_{a_1} was determined using the error matrix formalism of Bevington [Be69]. The dashed curve therefore represents the minimum value of the threshold current I_t as implied by the data.

Table 4.1 summarizes the fits and minimum threshold currents obtained from the data for the two HOMs at each optical setting. A remarkable feature of the data is the fact that for setting 4 no 1899 MHz peak was observed! The mode with orthogonal polarization and much lower Q (~ 7000) to this mode at 1901 MHz was measured instead. An explanation of the vanishing 1899 MHz peak is that the effective matrix element as given by equation 2.93 was zero (or very small) for setting 4 thereby reducing the cavity signal and S_{21} to the same order of magnitude as the noise.

In general the data show no significant evidence of multipass BBU for these modes. There are a few exceptions but the data for the most part are best-fit when the parameter $a_1 = 0$ or equivalently when $I_t = \infty$. The best-fit occurs when the dependence on S_{21} varies in a simple linear fashion with the threshold current. This experimental evidence supports the conclusion of the TDBBU simulations which indicated threshold currents on the order of a few to tens of milliamperes. Data for the energy recovery case also shows no significant non-linear trend due to a low threshold current. The large error results from some points which lie well away from the best-fit line. A common problem with these measurements was that the beam was not stable and would activate the machine loss protection system before the network analyzer could

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Figure 4.3: Data and fits for settings 1 and 2. The solid curve is the best-fit and the dashed curve represents the maximum value of a_1 implied by its error (minimum I_t).

-bs-error =

Figure 4.4: Data and fits for settings 3 and 4. The solid curve is the best-fit and the dashed curve represents the maximum value of a_1 implied by its error (minimum I_t).

-bs-error =

Figure 4.5: Data and fits for settings 5 and 6. The solid curve is the best-fit and the dashed curve represents the maximum value of a_1 implied by its error (minimum I_t).

-bb-error =

Figure 4.6: Data and fits for energy recovery and single pass settings. The solid curve is the best-fit and the dashed curve represents the maximum value of a_1 implied by its error (minimum I_t).

Setting #	HOM (MHz)	$a_\circ$	a_1 ($\times 10^{-3} \mu A^{-1}$)	χ^2	I_t (μA)
1	1899	$-4.08 \pm .02$	$0 + 1$	.321	> 875
	2110	$-5.09 \pm .03$	3 ± 2	3.53	> 207
2	1899	$-4.153 \pm .009$	$0.0 + 0.5$	29.4	> 2183
	2110	$-4.87 \pm .02$	$0 + 1$	14.1	> 819
3	1899	$-4.66 \pm .01$	$0.0 + .8$	5.40	> 1247
	2110	$-5.21 \pm .02$	$0 + 1$	8.30	> 897
4	1901	$-5.36 \pm .02$	$0.0 + 1.7$	14.3	> 605
	2110	$-5.53 \pm .03$	$0.0 + 2.5$	14.8	> 403
5	1899	$-4.10 \pm .02$	$5.4 \pm .8$	9.64	> 161
	2110	$-5.11 \pm .02$	$2.3 \pm .7$	1.30	> 334
6	1899	$-3.76 \pm .01$	$.5 + .6$	2.29	> 987
	2110	$-4.72 \pm .02$	$0 + 1$	2.18	> 784
Energy Recovery	1899	$-4.17 \pm .02$	$0.0 + 2.5$	92.0	> 401
Single pass	1899	$-4.490 \pm .003$	$0.00 + .04$	73.6	> 28210
	2110	$-5.448 \pm .009$	$0.00 + .09$	40.5	> 11578

Table 4.1: Fitted parameters and threshold current estimates for the 1899 and 2110 MHz modes for all optical settings.

complete its sweep in frequency over the HOM resonance.

For comparison, a measurement of both HOMs was performed under the conditions of a single pass CW beam after the recirculation measurements were performed. The first pass beam was therefore identical to that for the recirculation measurements and dumped after B1 by simply bending it using D1 and D6 powered to bend the first pass 42.8 MeV beam by 24°. The rest of the recirculation arc was left unpowered except for the injection chicane. The chicane was kept powered to bend the first pass beam through the 12° dogleg as in the recirculation measurements.

For this case, the cavity signal is given by equation 2.49. In particular, there is no instability denominator term because of the lack of beam recirculation. Therefore the cavity signal $V(\omega)$ should simply vary linearly with current. Referring to equation 4.8, this means that the impedance (first) term and second terms appear and the third term does not appear. For this case the effective threshold current is in principle infinite for multipass BBU because there is no recirculation and therefore no feedback mechanism (note that cumulative BBU can possibly occur in this case however "threshold" currents for this type of BBU are well above the maximum CW beam current available from the injector). In the interest of comparison with the recirculation case, the functional form represented by equations 4.8 and 4.9 was fit to the data shown in the bottom plot of figure 4.6. Table 4.1 does indeed show that the best-fit to the data occurs when a_1 is zero or $I_t = \infty$. The error of a_1 is also an order of magnitude less for the straight through case than the other recirculation cases resulting in minimum mathematical "threshold" currents an order of magnitude above the maximum value obtained for the recirculation settings. The data as expected is well fit by a simple linear dependence on the average CW current $I_\circ$. Appendix D lists the raw data (in dB) plotted in figures 4.3, 4.4, 4.5, and 4.6.

4.3.3 Cavity Higher-Order Mode Measurement Using a Single Pass CW Beam

In Chapter 2 section 2.2.4 a theoretical discussion was presented of a HOM interaction with a single pass beam. The beam is deflected upstream of the cavity by a kicker and the resultant displacement is detected by a downstream pickup as shown in figure 2.6. The measurement setup based on this physical situation is shown schematically in figure 4.7. In this measurement the stripline pickup (see Appendix C) is used as a HOM signal detector instead of the cavity. The pickup is broadband and has the same characteristics as the kicker and is used to detect the aliased HOM signals placed on the beam. The kicker circuit is the same as that for the recirculation measurements except that no mixing down of the kicker sweep signal is required. This is because

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Figure 4.7: Single pass HOM measurement setup schematic using a stripline kicker and pickup.

the network analyzer port B detector can be set up to detect the aliased HOM signals around 500 MHz from the pickup. The pickup electronics consists of a broadband (10 to 1000 MHz) 66 dB preamp and a broadband hybrid combiner used to subtract the signals from each stripline in the pickup to determine the transverse position of the beam.

Since there is no recirculation, there is no instability denominator that becomes zero at the threshold current. The cavity HOM acts as a driven oscillator where the driving signal is provided by the transverse modulation of the beam due to the kicker. The resulting transverse beam modulation due to the HOM is then detected at the pickup. The goal of the measurement is to extract from the data an estimate of the shunt impedance R/Q of a HOM.

The usefulness of this measurement scheme is that the pickup measures directly the transverse displacement due to the deflection of the beam caused by the HOM. HOMs that are not adequately damped (and therefore have a large Q) will cause the largest beam deflections and are therefore potentially most destructive. The pickup produces a relatively large signal for HOMs that have the greatest coupling (shunt impedance R/Q and Q) to the beam because these modes produce the largest deflection. The relative size of the HOM signal detected by the pickup therefore determines the modes that are potentially most destructive for multipass (or cumulative) BBU. Since both kicker and pickup are broadband devices centered at 500 MHz with 350 MHz bandwidth, a search in frequency was performed to find the HOMs that produced the largest signal.

The single pass RF measurements were performed after all the recirculation magnets were removed from the accelerator tunnel except for dipole DC1 of the injection chicane. It was retained for energy measurement purposes. The kicker was located immediately upstream of injection chicane dipole DC1. The pickup was located at the former position of energy recovery chicane dipole DE2. Two sets of kickers and pickups were located at each of these positions and were oriented to deflect and detect in the horizontal and vertical planes in an attempt to detect the strongest dipole modes acting with either polarization.

The 100 keV gun, choppers, buncher, capture section, and first two superconducting cavities of the cryounit were set up to produce a beam with the same characteristics as described in section 4.2 (ie. beam energy of 5.6 MeV, bunch length $< 1°$). The two cryomodules were set to a lower gradient so that the first pass beam emerged from the second cryomodule at an energy of 24.6 MeV as measured by a dipole that bent the beam by 40° downstream of the pickup. The lower beam energy (and hence momentum) was used in an attempt to maximize the HOM deflection and therefore overall measurement sensitivity. Linac quadrupoles at locations HVQL4 and HVQL6 were adjusted to make the sine-like ray zero from kicker to pickup to maximize the sensitivity to HOM deflections as discussed earlier in Chapter 2.

Referring to equation 2.67, the ratio of current moments must first be related to the kicker drive signal V_k and the pickup signal V_p. As with the recirculation case the kicker current moment $I_p x_p(\omega)_k$ is related to the kicker drive signal according to equation 4.5 where $Ix_c(\omega)$ replaced with $I_p x_p(\omega)_k$. From Appendix C, the current moment at the pickup is given in terms of the pickup signal as,

$$V_p = \alpha_p I_p x_p(\omega) \tag{4.12}$$

where α_p takes into account the shunt impedance of the pickup and the gain factor of the pickup amplifier. Using equations 4.5 (inserting the pickup current moment instead of the cavity current moment) and 4.12, the current moment ratio in equation 2.67 is written in terms of the kicker drive and pickup signals as,

$$\frac{V_p(\omega)}{V_k(\omega)} = \left(\frac{\alpha_p}{\alpha_k}\right) \frac{I_p x_p(\omega)}{I_p x_p(\omega)_k} \times I_\circ \tag{4.13}$$

where

$$S_{21} \equiv \frac{V_p(\omega)}{V_k(\omega)} \tag{4.14}$$

is defined analogous to that for the recirculation measurements that used the cavity as the pickup device.

Equation 2.77 expresses the ratio of the pickup to kicker current moments at the resonance peak (ω_r) in terms of the impedance through the term g_k. Combining equations 2.77, 4.13, and 4.14 yields

$$|S_{21}(\omega_r)| = \left(\frac{\alpha_p}{\alpha_k}\right) I_\circ \sqrt{1 + g_k(I_\circ)^2} \tag{4.15}$$

for the scattering parameter at $\omega = \omega_r$ as a function of the average current. Taking the logarithm of both sides of 4.15 yields,

$$\log|S_{21}| = \log\left(\frac{\alpha_p}{\alpha_k}\right) + \log(I_\circ) + \frac{1}{2}\log(1 + g_k(I_\circ)^2) \tag{4.16}$$

which has the form

$$\log|S_{21}| = a_\circ + \log(I_\circ) + \frac{1}{2}\log(1 + a_1 I_\circ^2) \tag{4.17}$$

$$a_1 \equiv \left(\frac{e\mathcal{M}(R/Q)_m k_m Q_m}{2p_f c}\right)^2 \tag{4.18}$$

$$\mathcal{M} = \frac{M_{34}^{(k)} M_{34}^{(p)}}{(M^{(p)} M^{(k)})_{34}} \tag{4.19}$$

where $\mathcal{M}$ denotes the ratio of matrix elements in equation 2.79 and the (12) matrix elements were replaced with (34) matrix elements because the HOM resonance measured occurred in the vertical plane for this measurement. Equation 4.18 defines the fitting parameter a_1 in terms of the parameters of g_k. Determination of the fitting parameter a_1 thus yields a value for the shunt impedance (R/Q) provided the various other parameters can be determined.

The measurement consisted of setting up a beam as described previously and searching over the frequency range centered at 500 MHz for HOM resonances using both horizontal and vertical kicker-pickup pair. Quadrupoles at HVQL4 and HVQL6 were set from a DIMAD computation so that the sine-like ray [$(M^{(p)}M^{(k)})_{34}$, the denominator of equation 2.78] from the kicker passes through zero at the location of the pickup. This is equivalent to minimizing the kicker current moment $(I_p x_p(\omega)_k)$ and hence the pickup signal V_p so as to maximize the sensitivity to HOM deflections. When kicker pair was changed from horizontal to vertical the quadrupole polarity was simply reversed to maintain the same optical condition in the vertical plane. By observing the pickup signal on a spectrum analyzer, the quadrupole strengths were adjusted so as to minimize the peak at the center frequency of 500 MHz. The quadrupole at HVQL1 was then used to minimize the beam envelope at the position of the beam dump, resulting in single pass beam envelopes similar to those for the first pass for the recirculation measurements.

A search in frequency was performed for both horizontal and vertical kicker-pickup pairs. A HOM resonance was found at the aliased frequency of 401.849 MHz corresponding to the mode at $401.849 + 1497.000 = 1898.849$ MHz using the vertical kicker-pickup pair. This mode therefore most closely corresponds to the 1899.5 MHz mode measured during the recirculation experiment because frequency shifts of a few MHz are observed for the HOMs from cavity to cavity. This was the only definitive resonance seen over the bandwidth of the kickers and pickups. No other resonance was definitively seen using the horizontal kicker-pickup pair over the bandwidth due to the noise present on the signal. Of course a higher Q pickup would have helped in observing the small HOM signals at the expense of bandwidth.

The top plot of figure 4.8 shows the HOM resonance observed as a function of frequency and average beam current. The bottom plot shows the resonance on a finer scale for $I_\circ = 195.2$ μA. One observes from the top plot the simple logarithmic dependence of the resonance tail as the average beam current is increased. Figure 4.9 shows a plot of the peak of the resonance as a function of beam current. The error bars result from the noise observed at the resonance peak and generate an error in

Figure 4.8: Network analyzer frequency scan of the HOM resonance measured using an RF stripline kicker and pickup. The plots show frequency scans at various single pass CW beam currents.

the fitted parameters. The parameters for the best-fit are:

$$a_\circ = -.98 \pm .01 \quad (4.20)$$

$$a_1 = (5.6 \pm 1.9) \times 10^{-6}\ \mu\mathrm{A}^{-2} \quad (4.21)$$

$$\chi^2 = 0.25. \quad (4.22)$$

The value of χ^2 indicates a good fit to the data and the errors in the parameters are calculated from the error matrix formalism [Be69]. The parameter $a_\circ$ was set via a calibration of the network analyzer so that at 9.9 μA $|S_{21}| = 0$. The calibration is simply a matter of convenience so that frequency response of no interest in the measurement is defined to be zero. In this case the low current response is not interesting because the HOM signal is weakest and the response is expected to be flat.

The most difficult parameter to estimate in equation 4.18 is the ratio of matrix elements. It is not known which cavity HOM produced this resonance. In the absence of this information only an estimate of the matrix element ratio can be done. The sine-like ray (denominator term) from kicker to pickup was estimated using the vertical corrector upstream from the kicker. It produced an approximately 10 mrad kick resulting in no observable displacement at the viewscreen immediately upstream of the pickup where the spot size was approximately 1 mm. This results in

$$(M^{(p)}M^{(k)})_{34} \sim \frac{.001\ \mathrm{m}}{.01\ \mathrm{rad}} = .1\ \mathrm{m/rad} \quad (4.23)$$

as an estimate for the maximum value of the sine like ray.

An estimate of the matrix elements for the sine-like rays from kicker to cavity and cavity to pickup was calculated with DIMAD using the values of the quadrupole strengths determined empirically using the spectrum analyzer. The effective cavity "positions" were taken to be before cryomodule 1 at the location HVQL4, between the cryomodules at the location HVQL5, and after cryomodule 2 at the location HVQL6. The matrix element ratio values at each of these effective cavity position will serve to bound the shunt impedance R/Q. Table 4.2 lists the values of these matrix elements along with the matrix element ratio $\mathcal{M}$ for each effective position where the beam momentum is p_f.

Using table 4.2 R/Q is calculated using the other known quantities in equation 4.18. The Q of the mode is taken to be $90,000$ as determined in the recirculation

Effective location	a_1 ($\times 10^{-6}$ μA^{-2})	R/Q Ω
HVQL4	7.5	93.7
	5.6	80.8
	3.7	65.5
HVQL5	7.5	58.7
	5.6	50.7
	3.7	41.0
HVQL6	7.5	84.8
	5.6	73.2
	3.7	59.2

Table 4.3: Estimated HOM shunt impedance determined from the single pass RF measurement using a kicker and pickup.

experiments. The wavenumber is found as usual from the measured frequency of 1898.849 MHz. The result is summarized in table 4.3 for the best-fit value of the parameter a_1 and its maximum and minimum possible values given by its statistical error. The average value of the shunt impedance from table 4.3 for the three effective cavity positions is $R/Q = 67.5\ \Omega$. This is about a factor of three larger than that given in table 2.2 from the cavity HOM bench measurements in reference [Am84] (21.9 Ω). The most difficult parameter to estimate in this measurement is the matrix element ratio. In particular, the (nominally zero) matrix element $(M^{(p)}M^{(k)})_{34}$ is the most difficult to estimate and can easily by off by a factor of three.

The shape of the resonance in figure 4.8 is interesting in that it does not have the resonant lineshape measured in the recirculation measurements. One possible explanation is that two similar modes but perhaps different in polarization or cavity location were interfering with each other. S_{21} measurements using a single cavity driven through its fundamental coupler and using the HOM load antenna indicate interference effects of the dipole modes in a single cavity [Bi91]. These modes were not seen to be isolated resonances but had "notches" which were approximate zeros in the response due to the interference of two nearby modes. At the 1899 MHz frequency corresponding to the measured HOM resonance, the single cavity data exhibited complicated interference-like features that could be the cause of the measured lineshape. Appendix D lists the raw data (in dB) plotted in figure 4.9.

4.4 Comparison with Theory and Simulation

The RF HOM measurement performed with a recirculated CW beam measured actual HOM resonances present in cavity 8 of cryomodule 2. An estimate of the multipass BBU threshold current one could expect from this mode acting on the beam was made from a measurement of the height of the resonance peak as a function of the average CW beam current. The form of the fitting function used in the data analysis was based on a theory describing a simple single cavity containing a single HOM where the beam passes twice through the cavity due to recirculation. The threshold currents deduced from this analysis of the recirculation measurements are generally consistent with the thresholds calculated in a full TDBBU simulation of the recirculator.

Table 4.4 summarizes results shown in tables 4.1 and 2.3. The data for the 6 optical settings indicate that for the most part the minimum threshold current inferred from an analysis of the RF measurements is on the order of 1 mA to within a factor of about two. In particular, the data for optical settings 1 (1899 MHz mode only), 2, 3, 4, 6 (2110 MHz mode only), and energy recovery follow this pattern. The best-fit value for a_1 turns out to be zero, indicating a very large (infinite) threshold current provides the best description of the data; a lower bound on the threshold current can be inferred from the upper bound on a_1 given by its uncertainty. The uncertainty in the fitting parameters relates directly to the error bars of the raw data. An improvement of the noise characteristics of the measurement circuit would result in an improvement in our ability to set limits on the threshold current by this method.

Setting #	I_m (μA) (Actual Max. Rec. Current)	HOM (MHz)	I_t (μA) (Inferred from Meas.)	I_t (μA) (From TDBBU)
1	215	1899 2110	> 875 > 207	5300
2	68	1899 2110	> 2183 > 819	6300
3	120	1899 2110	> 1247 > 897	19500
4	95	1901 2110	> 605 > 403	13200
5	64	1899 2110	> 161 > 334	15500
6	67	1899 2110	> 987 > 784	5000
Energy Recovery	30	1899	> 401	400

Table 4.4: Threshold current comparison between measurement, calculation and maximum recirculated current.

Two notable exceptions to the general pattern occur, as is evident from table 4.4. They are the 2110 MHz mode of setting 1 and both the 1899 and 2110 MHz modes of setting 5. For these cases the best-fit value of a_1 is not zero so the threshold currents inferred are finite. Mathematically this means that there is some non-linear curvature to the best-fit curve. For setting 1 we were able to recirculate a 215 μA beam without observing multipass BBU consistent with the 207 μA lower limit set in the RF measurement for the 2110 MHz mode. For all settings the main limitation to our measurements is the absence of data at high average CW beam currents compared to those for the single pass case (up to 275 μA). It is therefore difficult to say that these settings (except for the energy recovery setting) were more prone to exhibit multipass BBU because the best-fit value of a_1 was not zero. This could simply be due to statistical fluctuations at low average current that would not continue at higher currents. Another possibility is that a systematic error crept into the measurements for these settings. For the energy recovery setting the best-fit value of a_1 was indeed zero and the low minimum threshold current was due to the relatively large error of a_1 for this case (which, in turn, is probably due to the relatively low current we were able to recirculate).

The results of these measurements using the injector recirculator can be used to estimate the threshold current for the full CEBAF recirculating linac using the calculated TDBBU threshold currents as a figure of merit. The computed threshold current takes into account differences such as optics and beam energy of the of the two machines. Two approaches are possible. In the first, we simply note that we were able to recirculate a current of 215 μA for optical setting one of our injector recirculator. Since the TDBBU calculations indicate that the threshold for multipass BBU in the full CEBAF accelerator is roughly twice the value of that for the injector recirculator in this optics mode, then the fact that we could recirculate 215 μA in the injector recirculator is good evidence that we can recirculate at least that current (and probably at least twice that current) in the full CEBAF recirculator.

Alternately, we can use the lower bounds of the threshold currents inferred from this experiment to estimate the lower bound of the threshold current for the full CEBAF recirculator. Two steps are necessary. First we note that the different optical settings have different sensitivities to multipass BBU (as evidenced by the varying threshold currents calculated using TDBBU). For the 1899 MHz mode our measurements were able to set experimental bounds on the threshold current that ranged from 1% of the theoretical current (for optical setting 5) to about 35% of the theoretical current (for optical setting 2) and even the full current for the energy recovery mode. For the 2110 MHz mode, the limits ranged from 3.9% (for optical setting 5) to 15.7% (for optical setting 6) of the calculated threshold current. Since these numbers are all lower bounds, we can infer that the actual threshold is at least 35% of the theoretical value for the 1899 MHz mode and at least 15.7% of the theoretical value for the 2110 MHz mode by taking the bounds inferred from the most sensitive optical setting for each mode (we exclude the energy recovery mode from this estimate as the beam is running under very different conditions in this mode). The experimentally determined threshold for BBU in the injector recirculator would be whichever of these currents is lower for a particular optical setting. Because $(R/Q) \cdot Q$ for the worst-case 1899 MHz mode is a factor of 2.6 larger than that for the 2110 MHz modes the threshold limit set by our measurements of the 1899 MHz mode is, in general, slightly lower than that set for the 2110 MHz mode. To estimate the breakup current for the full CEBAF recirculator, we can therefore estimate that it should occur at a current that is at least 35% of the value of the TDBBU-calculated threshold for the 1899 MHz mode in the full recirculator. Because the calculated [Kr90] threshold current for the full recirculator is 11 mA, our experiment implies that we should be able to recirculate at least 35% of that current, or 3.8 mA. This is roughly an order of magnitude above the maximum CEBAF design current of 200 μA.

It must be noted, of course, that all of these estimates are based on the assumption that the theoretical description of BBU in the two recirculating accelerators by TDBBU is valid. Since we were unable to observe the beam breakup threshold, our experiment can only state unequivocally that the actual BBU threshold in the injector recirculator is: a) above 215 μA for optical setting 1; and b) a substantial fraction of the threshold currents calculated by TDBBU for a broad variety of optical conditions.

Finally, it is noted that the data analysis of the HOM measurement is based on the assumption that the HOMs are isolated resonances in a single cavity. This is the case for the CEBAF/Cornell superconducting cavities which have HOM Q values on the order of 10^4. In principle the measurement should be performed on all important

HOMs in all the cavities for all (recirculation) optical conditions to be sure to identify the HOM that is limiting in the sense of multipass BBU. In this experiment the two highest Q modes at 1899 MHz and 2110 MHz were measured for a single cavity for various optical conditions and the measured Q used in the TDBBU simulations.

The TDBBU simulations do take into account the full dynamics of all the cavities and HOMs interacting with a recirculated beam. They indicate that coherent behavior of the same mode in two or more different cavities in the linac can occur because the modes have nearly identical growth rates (the imaginary part of the oscillation frequency that gives rise to exponential growth). The present experiment would be directly sensitive to these coherent effects. The single cavity HOM data analysis would simply yield a threshold current smaller than that for a single HOM acting alone because the multiple HOMs would act roughly as a single giant HOM with a shunt impedance approximated as the sum of the individual HOM shunt impedances resulting in lower threshold currents for given recirculation optics.

The single pass measurement using both kicker and pickup was performed to investigate the possiblility of using this technique to obtain the shunt impedance of the HOMs. The shunt impedance obtained was an order of magnitude estimate due the difficulty in determining the matrix element ratio. Part of the difficulty arises because the exact nature of the HOM resonance is unknown. It may be due to a single HOM in a single cavity or to more than one acting coherently. The theory developed in Chapter 2 needs to be extended to treat coherent effects between cavities.

Compared to traditional bench measurements, this technique gives direct indication of the most important and potentially destructive HOMs in any superconducting or room temperature structure under study. This is a result of the fact that the actual deflection of the beam due to the HOM (or some combination of modes acting coherently) is detected. In a bench measurement of a complicated cavity structure, the possibility exists that an important HOM is not noticed because of difficulties in coupling to the mode using traditional probes and antennas. The mode would therefore only show up as a beam deflection. As a final remark, if the cavity and pickup are some multiple of π phase advance apart (in other words, for this experiment $M_{34}^{(p)} = 0$) the HOM deflection could not be detected at the pickup because the pickup detects net off-axis displacement and not deflection angles. This is equivalent to the case of insensitivity due to inadequate probe coupling in a bench measurement. The situation could be corrected by careful beamline optical design and/or using more than one pickup to detect the HOM signals. The optics could be changed using quadrupoles to provide adjustment of the phase advance over some reasonable range.

Chapter 5

Discussion and Conclusion

The experimental study of multipass BBU in this thesis was undertaken to determine possible machine performance limitations for accelerators that use superconducting cavities. The specific accelerator used was the CEBAF superconducting linac. The primary experimental tool used was the RF measurement of the cavity HOMs which indicated no substantial evidence of multipass BBU in the CEBAF injector. The primary theoretical tools used for comparison were the DIMAD calculations of the optics and the TDBBU simulations of multipass BBU. The simulations also indicated that multipass BBU would not occur in the CEBAF injector where the beam conditions are most favorable for it to occur (low energy and large recirculation transfer matrix elements). In addition, an estimate of the shunt impedance R/Q of a HOM was made using a single pass RF measurement using a stripline pickup. This estimate was in order of magnitude agreement with HOM shunt impedances calculated in the bench tests of the cavities [Am84] for the most important TM_{110} like modes. The main conclusion for CEBAF is that multipass BBU should not limit machine performance. Practically this means that the full design CW current of 200 μA can be accelerated in the linac with no beam loss due to multipass BBU. At 4 GeV this represents a beam power of 800 kW!

The analysis of the recirculation RF measurements was done by fitting the results of the relatively simple recirculating linac model discussed in Chapter 2. The lower bound on the threshold currents resulting from these fits were below the values calculated in the simulations which used fits to measured recirculation transfer matrix elements. This was primarily due to the noise evident on the measured peak of the HOM resonance. Reduction of this RF noise would be the aim of improving the measurement technique in the future. Alternatively, one can improve sensitivity by extending the measurements to higher CW beam currents through improvements in recirculation optics (primarily in bends B1 and B2).

For the optics measurements the discrepancy between the measured matrix elements and the calculated values was primarily due to the non-ideal optical behavior of bends B1 and B2. As was pointed out, the dispersion was found to be a non-linear function of momentum and roughly a factor of two smaller than the linear computation so that by inference the sine and cosine like optics is also a non-linear function of momentum and different than the simple linear theory. A non-linear theory based on a sextupole contribution in the end field of the bend dipoles was developed to take into account this behavior. The basic problem with the bends was that the dipoles used were simply not designed to bend the beam by 45°. Coupled with the difficulties of matching the beam through the second pass which resulted in large β functions, it is reasonable that the computed and measured matrix elements were not in agreement. Improvement in the matrix element measurement would have resulted from better readback accuracy for the values of the corrector current. This was the largest error in determining the deflection angles. Another improvement would have been to use the linac RF beam position monitoring system to record beam centroid offsets instead of the camera/viewer stopgap diagnostic used. This system is designed to measure the beam centroid to .1 mm; unfortunately it was not functional at the time of the recirculation experiment.

Finally, an interesting application of the RF measurement techniques used in these experiments is in feedback stabilization of the beam. Instead of using the kickers and pickup to measure HOMs and beam resonances, a feedback circuit could be designed between kicker and pickup to actively damp resonance motion of the beam. Ideally such a system would detect a HOM kick using a pickup and apply an appropriate kick to the beam instantaneously proportional to the HOM kick using a kicker. The net result of such a system would be to actively damp any growing oscillation of the beam. Of course no electronics system has infinite bandwidth, so the HOM kick must be anticipated for bunches about to arrive at the kicker by using the transport matrix that describes the accelerator. Interesting questions arise concerning gain, bandwidth, kicker and pickup design and power requirements of such a system. Reference [Ga92] describes general aspects of this kind of feedback control of a charged particle beam.

Appendix A

Dipole Beam Orbit Geometry

The general beam orbit geometry in a parallel faced dipole magnet is described in this appendix. Figure A.1 shows the circular beam orbit in a dipole of effective length l_e, total bend angle θ, and entrance and exit edge angles α and β respectively. From the geometry,

$$\gamma = \frac{\pi - \theta}{2} \tag{A.1}$$

$$\gamma + \beta + \delta = \frac{\pi}{2} \tag{A.2}$$

$$\gamma - \delta + \alpha = \frac{\pi}{2}. \tag{A.3}$$

The angles δ and θ can be expressed in terms of the entrance and exit edge angles:

$$\delta = \frac{\alpha - \beta}{2} \tag{A.4}$$

$$\theta = \alpha + \beta. \tag{A.5}$$

Equation A.5 indicates that specification of at least two of the three angles α, β or θ is required to specify the orbit. Using the law of cosines on the isosceles triangle where two sides have length ρ and one side has length h results in

$$h^2 = 4\rho^2 \sin^2(\theta/2) \tag{A.6}$$

The hypotenuse h is given by

$$\cos(\delta) = \frac{l_e}{h} \tag{A.7}$$

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Figure A.1: Beam orbit geometry for a parallel faced dipole magnet.

and using equation A.6 the radius of curvature is

$$\rho = \frac{l_e}{2\ \sin(\theta/2)\cos(\delta)}. \tag{A.8}$$

The beam orbit in the dipole is thus determined completely by two of the three angle parameters and the magnet effective length through equations A.4, A.5 and A.8.

Two specific cases of the geometry are important for the dipoles used in the recirculation arc. The first, which corresponds to a normal entrance angle, is

$$\alpha = 0 \tag{A.9}$$

$$\beta = \theta \tag{A.10}$$

$$\rho = \frac{l_e}{\sin(\theta)} \tag{A.11}$$

$$s = \rho\{1 - \cos(\theta)\} \tag{A.12}$$

and is denoted as "case 1". The second, which corresponds to equal entrance and exit angles, is

$$\alpha = \beta = \frac{\theta}{2} \tag{A.13}$$

$$\rho = \frac{l_e}{2\ \sin(\theta/2)} \tag{A.14}$$

$$s = \rho\{1 - \cos(\theta/2)\} \tag{A.15}$$

and is denoted as "case 2". The quantity s is the sagitta defined to be the maximum lateral extent the beam travels as it passes from entry to exit of the dipole. The beam orbit is made to pass through the dipole such that the sagitta is centered. Mathematically this means that for a magnet pole of width W,

$$w = \frac{W - s}{2} \tag{A.16}$$

for both cases. The dipoles that make up the injection chicane, energy recovery chicane, and bends B1 and B2 are designed to transport the beam according to case 1 or case 2.

The sagitta of the orbit in the bend dipoles (~ 1.9 cm for a bend angle of $45°$) was an issue because of the small width of the pole piece (~ 8.1 cm). This sagitta was a factor of three larger than the orbit the bend dipoles were designed for ($\sim .6$ cm for a bend angle of $\sim 15°$) in the MUSL-2A microtron. The beam therefore travels a longer distance in the dipole end fringing fields and closer to the dipole edges. The measured dispersion asymmetry and low dispersion value can be traced to the large sagitta in the recirculator dipole application. In modeling the bends, a sextupole contribution was added to the dipole fringe fields in an attempt to explain the dispersion measurement and is described in Appendix B.

Appendix B

Dipole Sextupole Fringe Field Model

The bend model was based on a dipole model that resulted from an analysis of the B1 dispersion measurements (the details of this measurement are discussed in Chapter 3, section 3.4). The initial measurement analysis along with DIMAD calculations are described fully in reference [Do91]. The dispersion was measured with a first pass beam at 45 MeV and with quadrupoles Q1 and Q2 unpowered. The dipoles were cycled to eliminate hystesis effects. The dispersion at the exit of B1 (the viewscreen after D4) was found to be 2.21 m when increasing magnetic field of the bend dipoles to simulate a low momentum particle and 2.94 m when lowering the magnetic field of the dipoles to simulate a high momentum particle. The average linear dispersion derived from the complete set of data (both from raising and lowering the beam energy) was 2.57 m. The DIMAD calculation of the linear dispersion using a simple dipole model resulted in 5.52 m for the dispersion at the end of the bend, as shown in figure B.1. This value deviates significantly from the measured values implying that a more realistic model must be used for the bend dipoles. The new model must take into account not only the factor of two difference in the value but also the asymmetry in the value depending on particle momentum.

Measurements of the field integral of bend dipole D1 indicated a quadratic variation in the field integral of $\Delta Bl/Bl = -.5\%$ from the center of the pole to ± 1.5 cm on either side of center. This was taken into account in the dipole model by adding a sextupole field variation according to,

$$\frac{\Delta Bl}{B\rho} = k_2^d x^2 \tag{B.1}$$

$$\frac{\Delta Bl}{B\rho} = \frac{\Delta Bl}{Bl} \times \frac{l_e}{\rho} \times \left(\frac{x}{.015\ m}\right)^2 \tag{B.2}$$

where k_2^d is the sextupole multipole strength, ρ is the radius of curvature, l_e is the effective length of 19.1 cm, and x is the horizontal coordinate. The sextupole repre-

Figure B.1: First pass linear dispersion in B1 and B2 for the various stages of dipole model development.

$$k_0^d = k_2^d x_\circ^2 \tag{B.4}$$

which is simply the value of the slope (quadrupole contribution) and magnitude (dipole contribution) of the sextupole contribution at the location of the design orbit at each end of the dipole which in these dipoles is half the sagitta (since the sagitta is split symetrically about the centerline of the dipole) given by case 2 in Appendix A ($x_\circ = -.0095$ m). Using these parameter values yields a sextupole strength of $-17.0/\text{m}^2$, a quadrupole strength of .32/m, and a dipole strength of .0015. These strengths resulted in a calculated linear dispersion of 3.85 m from DIMAD, much closer to the measured value of 2.57 m, and is shown in figure B.1.

Using the "detailed chromatic analysis" feature of DIMAD, a simulation of the orbit displacement at the exit of B1 for both raising and lowering the beam energy by the same momentum bite used in the dispersion measurement ($\delta p/p = \pm.35\%$) was performed. The dispersion computed when lowering the energy was 3.39 m and when raising the energy was 4.40 m indicating qualitative agreement with the observed asymmetry. A modest increase in the sextupole strength by a factor of 1.83 above that based on the measurements in an attempt to fit the measured dispersion when lowering the beam energy yielded a linear dispersion value of 2.76 m shown in figure B.1 and a dispersion asymetry of 2.21 m when lowering the energy and 3.51 m when raising the energy.

Though not perfect, the sextupole fringe field model of the dipole does a reasonable job at accounting for the reduction of the original calculated linear dispersion of 5.5 m due to non-linear focussing of the dipole fringe fields. Furthermore it also explains the observed asymmetry in the dispersion when raising or lowering the energy. The dipoles in B1 and B2 (except the small corrector D5) were all used in the MUSL-2A microtron as approximately 15° bends of case 2. The orbit sagitta was therefore a factor of three smaller than in the present application in the recirculation arc. This boils down to the fact that the beam travels a larger distance in the dipole fringe field and most importantly passes closer to the pole edges at the ends of the dipoles in B1 and B2 in the recirculator than it did in the original use of the dipoles in MUSL-2A.

In terms of the sextupole fringe field model of the dipoles, the asymmetry in the calculated dispersion is qualitatively understood in terms of the focussing the beam experiences as it traverses the end field region of the dipoles where there is a significant magnetic field gradient. A low momentum particle is bent more and arrives at each dipole entrance (and exit) displaced farther from the centerline than the nominal particle; it therefore experiences a larger field gradient and hence stronger focussing. The opposite is true for a high momentum particle. The stronger focussing experienced by the low momentum particle manifests itself as a systematic reduction in the dispersion from the average linear value as compared to that for the high momentum particle.

The next step in the analysis is to calculate the effect of Q1 and Q2 focussing on the calculated B1 dispersion using the sextupole model of the dipole fringe fields. The sextupole strength initially used is obtained from the fit to the dispersion measurement (see figure B.1). The measurements described in Chapter 3 indicate that the dispersion was zero (both slope and magnitude) when the quadrupoles were set to a strength $k = 2.37$/m. Using this value in a linear dispersion computation using the sextupole strength fit to the dispersion measurements results in .77 m of dispersion at the end of the bend on the first pass as shown in figure B.1 indicating incomplete dispersion suppression. To model the dispersion-suppressed mode of the bend, the sextupole strength was used to fit the linear dispersion as well as slope to zero at the end of B1. This fit required that the sextupole strength be increased by a factor of 4.23 to model the measured dispersion-suppressed mode of the bends and is shown in figure 3.8 of Chapter 3. The bends were normally run in the dispersion suppressed mode except when B1 was used to measure the energy. The same dipole model was included in the description of bend B2 dipoles D7, D8, D9 and D10 so that the first order optics in the dispersion suppressed mode are identical to that of B1.

Appendix C

Kicker and Pickup Operation and Design

The kicker and pickup devices used in the RF measurements were designed to produce transverse beam deflections and detect transverse beam offsets over a wide range of frequencies. These broadband requirements are met by the stripline type of kicker or pickup shown respectively in figure C.1. These devices are based on two stripline electrodes that support TEM waves along their length as shown in the figures. The figures show the kicker and pickup connected to the circuit used to operate each in the transverse sense (both kicker and pickup have an equivalent longitudinal mode of operation). This appendix is not meant to be an exposition of the complete theory of kicker and pickup operation. The devices are discussed in the context of the application used for these experiments. Reference [Go92] gives an extensive discussion of the theory of the important kicker and pickup devices in use at accelerators presently.

The kicker setup in figure C.1 shows the kicker driven from a source signal V_s which is split into two signals V_+ and V_-. Each signal then travels along the transmission line of impedance Z_L formed by each stripline electrode. Each stripline is terminated in some standard impedance Z_c which for these experiments was 50 Ω. The signal V_+ is phase shifted by the hybrid coupler 180° relative to V_-. The beam enters from the

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Figure C.1: Kicker and pickup setup schematic.

left on axis and is assumed to follow a straight line trajectory at constant velocity through the kicker–a good approximation for electrons above 5 MeV. Assuming a straight line trajectory is justified because the transverse kick is small compared to the longitudinal momentum of the beam.

The paradox in understanding kicker operation lies in the fact that the striplines support TEM waves. Relativistic electrons should not be deflected at all by a TEM wave because the $\mathbf{v} \times \mathbf{B}$ force on the particle is exactly canceled by the force due to the electric field $\mathbf{E}$. The answer lies in a remarkable theorm known as the Panofsky-Wenzel theorm which relates the transverse momentum kick imparted to the beam to the transverse gradient of the longitudinal energy change of the beam due to the kicker. The theorem is written in phasor form as,

$$p_{\perp} = \frac{i\nabla_{\perp}(\Delta E)}{\omega} \tag{C.1}$$

$$\Delta E \equiv \int_{z_{\circ}}^{z_{\circ}+l} E_z(x, y, t)dz \tag{C.2}$$

where $_{\perp}$ denotes the dimensions transverse to z, l is the kicker length, and the complex factor i indicates that the transverse kick is 90° out of phase with the longitudinal electric field produced by the kicker. The theorem results from substitution of Faraday's law into the Lorentz force equation and a bit of rearranging. For the situation of the stripline kicker, the only component of the electric field longitudinal to the beam direction occurs at the ends of the electrodes. The electrodes are driven 180° out of phase so as to produce the maximum transverse gradient of the longitudinal electric field thereby maximizing the kick $p_{\perp}$. The origin of the kick is now clear, the transverse gradient of the time varying longitudinal electric field produces a time varying magnetic field at the electrode ends according to Faraday's law. The induced magnetic field produces a net kick at the electrode ends. Driving the two electrodes in phase would produce a longitudinal kicker. No transverse kick would occur in this case because the longitudinal electric field at the electrode ends would have no transverse gradient.

The final consideration for the kicker design is computation of the kick given by equation C.1. This is in principle accomplished by solving the boundary value problem for the time varying longitudinal electric field and inserting the result into the Panofsky-Wenzel theorem. This has been done [Go92] and the result is quoted here in terms of a figure of merit known as the transverse kicker shunt impedance defined as,

$$R_{\perp} \equiv \frac{|p_{\perp} c/e|^2}{2P_i} \tag{C.3}$$

where P_i is the total power input to the hybrid coupler. For the stripline case this quantity is given by

$$R_{\perp} = 8Z_L \left(\frac{g_{\perp} l}{h}\right)^2 \left(\frac{\sin(kl)}{kl}\right)^2 \tag{C.4}$$

$$g_{\perp} = \tanh\left(\frac{\pi w}{2h}\right) \tag{C.5}$$

where $g_{\perp}$ is a geometric factor arising from the specific electrode geometry and k is the wavenumber of the kicker electromagnetic field. The quantity of interest is the maximum transverse angular kick for a given longitudinal beam momentum and is defined by

$$\theta_{\circ} = \frac{|p_{\perp}|}{p_i}, \tag{C.6}$$

where $p_i = 5.6$ MeV is the beam momentum at the entrance of the kicker. Using equations C.3 and C.4 in C.6 yields,

$$\theta_{\circ} = \frac{4e\sqrt{P_i Z_L}}{p_i c} \left(\frac{g_{\perp} l}{h}\right) \left|\frac{\sin(kl)}{kl}\right| \tag{C.7}$$

for the angular kick in terms of the known parameters of the system.

Equation C.7 is now used to explain the choices made for the kicker parameters used for this experiment. The first parameter to choose is the kicker center frequency. Noting that the function $|\sin(kl)/kl|$ in equation C.7 is maximum at $k = 0$ (or equivalently at DC exitation $f_k = 0$) the center frequency of choice should be placed as near to this maximum as possible. This can also be seen from the Panofsky-Wenzel theorm which shows that the kick varies inversely as the frequency. Kickers therefore become more efficient at lower frequencies. Another reason for going to as low a frequency as possible is that broadband power is expensive and costs increase with center frequency.

A good choice for the center frequency is when the factor $\sin(k_\circ l) = 1$ or $k_\circ l = \pi/2$ resulting in $l = \lambda_\circ/4$. The length of each electrode is then set to a quarter of the wavelength at the center frequency of exitation of the kicker. For these experiments the HOMs of most interest lie in the neighborhood of 2.0 GHz so that a kicker of with a center frequency of 500 MHz can be used to excite these modes by virtue of aliasing due to the bunched beam at 1.5 GHz. For the kicker used in this application $l = 15.0$ cm.

The effective bandwidth can be found by determining the point at which the angular kick is a factor of $1/\sqrt{2}$ down from its value at the center frequency (3 dB point) or when

$$\left|\frac{\sin(kl)}{kl}\right| = \frac{1}{\sqrt{2}}\left|\frac{\sin(k_\circ l)}{k_\circ l}\right| \tag{C.8}$$

at a value for k above $k_\circ$. Equation C.8 reduces to the trancendental equation

$$\left|\frac{\sin(kl)}{kl}\right| = \frac{\sqrt{2}}{\pi} \tag{C.9}$$

which results in $kl = 2.01$ corresponding to a maximum frequency of $f_k = 639.8$ MHz. The kicker can therefore be considered to have an effective 3 dB bandwidth from DC up to 639.8 MHz for a design center frequency of 500 MHz. As a practical matter the power amplifier was rated from 100 MHz to 1.0 GHz so not all of the bandwidth available from the kicker was used.

The flatness of the kick response for the two HOMs that were measured (using a kicker frequency of 402.5 MHz for the 1899 MHz mode and 613.5 MHz for the 2110 MHz mode-see figure 4.2) is estimated by making use of equation C.7. The flatness over the range of frequencies that the HOM response is appreciable is defined as the ratio of the maximum kick at the low frequency end of each range to the minimum kick at the high frequency end of each range where the ratio is expressed in dB. Performing these calculations by using equation C.7 results in a flatness of .001 dB for the 1899 MHz mode and .01 dB for the 2110 MHz mode. This is indeed very flat and indicates that for the worst case (the 2110 MHz mode) the variation in the angular kick over the range of frequencies of a typical CEBAF HOM is on the order of .1 %.

The geometric factors w = 2.5 cm and h = 1.7 cm were set to maximize $g_\perp$ consistent with the beam pipe diameter and other mechanical constraints. Using these parameters and a typical stripline impedance $Z_L \sim 75\ \Omega$ and amplifier power P_i of 25 W in equation C.7, the angular kick at the center frequency of 500 MHz and a beam momentum of 5.6 MeV is

$$\theta_\circ = .17 \text{ mrad} \tag{C.10}$$

or somewhat smaller than typical phase space angles of the particles in the beam.

As a final remark, the kicker is a directional device in the sense that no net kick is imparted to the beam if the beam direction is reversed from that shown in figure C.1. This directional effect can be understood by considering the kicker excited at its center frequency of 500 MHz and the beam is assumed to travel at the speed of light (certainly the case for these experiments). The beam is kicked as it enters the upstream gap and subsequently travels downstream which takes a time equivalent to a $\pi/2$ phase shift at 500 MHz. By the time the beam arrives at the downstream gap the traveling wave of the kicker has also undergone a $\pi/2$ phase shift because it is traveling upstream. There is an additional phase shift of π due to the opposite polarity of the field between downstream and upstream electrode ends for a grand total of 2π phase shift between the kick recieved at the upstream end from that at the downstream end. The kicks for this case are therefore in phase and there is net deflection. For the case where the beam and traveling wave of the kicker travel in the same direction (beam direction opposite to that in figure C.1) there is only a net π phase shift between downstream and upstream electrode ends due to the polarity difference. This results in no (or very little) net deflection due to cancelation of the upstream and downstream kicks. The kicker when operated as a pickup also exhibits this type of directionality.

Figure C.1 shows the setup for operation of the stripline device as a pickup. The beam enters displaced by an amount x off axis thereby generating a pulse on the upstream end of each electrode. For each electrode, the pulse divides and half travels out to the hybrid combiner and the other half travels to the downstream end of the electrode. At the downstream end the beam induces a pulse of opposite polarity to the pulse generated at the upstream end. Half the negative pulse cancels the positive pulse that traveled with the beam downstream and the other half travels upstream and out to the hybrid combiner. The net signal out of each electrode is bipolar with each peak separated in time by $2l/c$. The impedance Z_c is not necessary in principle because of the signal cancelation between the pulse generated at the upstream gap and the that at the downstream gap. It is included in practice to absorb spurious reflections generated in the circuit from impedance mismatches. The pulse cancellation at the downstream end explains why the pickup is directional. If the beam direction were reversed from that shown in figure C.1 no signal would emerge from the striplines because of the downstream cancellation effect.

The relative magnitude of the pulses generated on each stripline electrode depends upon the fraction of the beam image current intercepted by each electrode. The image current fraction depends upon the transverse dipole moment (or current moment) of the beam Ix. Subtracting each electrode signal using the hybrid combiner results in a difference signal V_p that is proportional to the current moment. The proportionality constant is known as the transverse transfer impedance and is defined by

$$Z_\perp \equiv \frac{V_p}{Ix}. \tag{C.11}$$

The transfer impedance has been calculated for the pickup geometry as with the kicker geometry [Go92] and is given by

$$Z_{\perp} = \sqrt{\frac{Z_c Z_L}{2}} \left(\frac{2g_{\perp}}{h}\right) e^{i(\pi/2-kl)} \sin kl. \tag{C.12}$$

The power out of the hybrid for a given current moment Ix is given by

$$P_o = \frac{|V_p|^2}{Z_c} \tag{C.13}$$

and is expressed in terms of the pickup parameters by inserting equations C.11 and C.12 into equation C.13. The result is,

$$P_o = \frac{Z_L}{2} \left(\frac{2g_{\perp}}{h}\right)^2 (Ix)^2 \sin^2(kl). \tag{C.14}$$

Equation C.14 indicates that by selecting a quarter wave stripline electrode the response is maximized. Constructing the pickup identically to the kicker with a center frequency of 500 MHz will therefore be optimal for detecting HOM signals aliased down from 2.0 GHz. Note however that the precise frequency response of the kicker and pickup differ significantly. The pickup has a simple $\sin^2(kl)$ response, whereas the kicker has a $(\sin(kl)/kl)^2$ response.

Using equation C.14 for the pickup with center frequency at 500 MHz, the 3 dB points occur at frequencies of 250 and 750 MHz for a total bandwidth of 500 MHz. The same analysis as was done with the kicker is now used to determine the flatness of the pickup response. Performing the calculations using equation C.14 results in a flatness of .0009 dB for the 1899 MHz mode and .005 dB for the 2110 MHz mode. This is also very flat and indicates that for the worst case (the 2110 MHz mode) the variation in the pickup response over the range of frequencies of a typical CEBAF HOM is on the order of .06 %. This is somewhat better than that for the kicker because of the difference in frequency response when the kicker is used as a pickup.

An important parameter to estimate is the amount of noise power generated by the pickup when a beam is present. Equation C.14 can be used to determine this power by assuming a minimum detectable curent moment Ix. To estimate this current moment this analysis borrows from experience gained with the CEBAF arc beam position monitors which are essentially the same device as the stripline pickup except that they use wire electrodes and are a quarter wavelength long at 1.5 GHz. The main result for the arc monitors is that they are sensitive to current moments down to $Ix = .1\ \mathrm{mm} \cdot \mu\mathrm{A}$. Using the same parameters for the kicker in equation C.14 (both kicker and pickup are mechanically identical) and assuming the pickup is excited by a $.1\ \mathrm{mm} \cdot \mu\mathrm{A}$ current moment modulation at the center frequency of 500 MHz, the power output from the pickup is

$$P_o = 5 \times 10^{-15}\ \mathrm{W} = -113\ \mathrm{dBm}. \tag{C.15}$$

This is indeed a very small power and corresponds to a pickup output voltage $V_p = 3.5\ \mu\mathrm{V}$ into $Z_c = 50\ \Omega$. To increase the output power right at the pickup, a pre-amplifier was mounted in the accelerator tunnel approximately half a meter away from the pickup at the output of the hybrid combiner. The pre-amplifier consisted of three broadband (.01 to 1.0 GHz) low noise preamps with a gain of 22 dB each. The preamps combined in series had a total gain of 66 dB. The preamps effectively raised the noise floor from -113 dBm to a "comfortable" -47 dBm which corresponds to ~ 20 nW or equivalently 7 mV into 50 Ω. The preamps were necessary because the noise floor of the network and spectrum analyzers was -100 dBm which is an order of magnitude above that for the pickup itself.

Appendix D

RF Measurement Raw Data

Setting 1		
HOM frequency (MHz)	CW Beam Current (μA)	$\|S_{21}\|$ (dB)
1899	10.0	-61.3 $\pm$ .8
1899	17.0	-57.0 $\pm$ .6
1899	27.3	-52.9 $\pm$ .5
1899	37.9	-49.9 $\pm$ .5
1899	47.2	-48.1 $\pm$ .4
1899	57.0	-46.3 $\pm$ .4
1899	67.4	-45.1 $\pm$ .4
2110	10.0	-80.9 $\pm$ 1.0
2110	17.1	-76.6 $\pm$ .9
2110	27.3	-72.9 $\pm$ .9
2110	37.9	-70.1 $\pm$ .8
2110	46.1	-67.1 $\pm$ .9
2110	56.8	-64.6 $\pm$ .9

Table D.1: RF measurement raw data for setting 1.

Setting 2		
HOM frequency (MHz)	CW Beam Current (μA)	$\|S_{21}\|$ (dB)
1899	10.0	-62.2 $\pm$ .2
1899	20.0	-56.6 $\pm$ .2
1899	30.6	-53.1 $\pm$ .2
1899	40.1	-51.2 $\pm$ .2
1899	51.7	-49.2 $\pm$ .2
1899	61.7	-47.4 $\pm$ .2
1899	70.3	-46.8 $\pm$ .2
2110	9.9	-76.3 $\pm$ .5
2110	20.1	-70.9 $\pm$ .3
2110	29.9	-67.9 $\pm$ .3
2110	40.1	-66.0 $\pm$ .2
2110	50.8	-63.4 $\pm$ .4
Setting 3		
1899	10.0	-72.9 $\pm$ .3
1899	20.0	-66.9 $\pm$ .3
1899	30.2	-63.6 $\pm$ .3
1899	40.1	-61.2 $\pm$ .3
1899	50.3	-59.5 $\pm$ .3
1899	60.0	-58.2 $\pm$ .4
2110	5.0	-88.8 $\pm$ .6
2110	15.0	-80.0 $\pm$ .6
2110	24.8	-76.1 $\pm$ .5
2110	35.2	-73.5 $\pm$ .4
2110	45.6	-71.2 $\pm$ .3
2110	55.0	-69.8 $\pm$ .3

Table D.2: RF measurement raw data for settings 2 and 3.

Setting 4		
HOM frequency (MHz)	CW Beam Current (μA)	$\|S_{21}\|$ (dB)
1901	5.0	-91.6 ± .5
1901	15.0	-84.3 ± .6
1901	25.0	-79.2 ± .7
1901	35.0	-77.3 ± .6
1901	44.9	-74.6 ± .8
1901	50.2	-73.4 ± .7
2110	5.0	-94.9 ± .6
2110	9.9	-90.0 ± .8
2110	20.3	-85.8 ± .6
2110	30.0	-81.9 ± .8
2110	40.0	-79.0 ± .7
Setting 5		
1899	12.1	-59.2 ± .6
1899	22.4	-54.0 ± .4
1899	32.5	-50.7 ± .3
1899	43.7	-46.4 ± .2
1899	52.8	-44.8 ± .2
2110	12.2	-79.7 ± .7
2110	22.3	-74.9 ± .4
2110	32.1	-71.7 ± .5
2110	43.7	-68.4 ± .5
2110	53.1	-66.9 ± .5
2110	64.8	-64.7 ± .5
2110	74.3	-63.2 ± .6
2110	83.2	-62.0 ± .4

Table D.3: RF measurement raw data for settings 4 and 5.

Setting 6		
HOM frequency (MHz)	CW Beam Current (μA)	$\lvert S_{21}\rvert$ (dB)
1899	12.9	-52.7 ± .3
1899	23.4	-47.8 ± .3
1899	33.9	-44.5 ± .2
1899	43.5	-42.3 ± .2
1899	53.6	-40.2 ± .2
1899	63.9	-38.8 ± .2
2110	12.9	-71.5 ± .6
2110	23.5	-67.2 ± .3
2110	33.8	-63.8 ± .3
2110	43.4	-61.7 ± .4
2110	53.7	-60.0 ± .3
Energy Recovery		
1899	5.0	-64.6 ± .7
1899	9.9	-62.6 ± .6
1899	15.0	-62.8 ± .6
1899	20.5	-57.9 ± .5
1899	23.0	-55.2 ± .4
1899	24.9	-55.5 ± .5
1899	31.4	-55.2 ± .5

Table D.4: RF measurement raw data for setting 6 and energy recovery.

Single Pass (Using the Cavity as a Pickup)		
HOM frequency (MHz)	CW Beam Current (μA)	$\|S_{21}\|$ (dB)
1899	9.9	-68.7 ± .5
1899	19.8	-63.3 ± .2
1899	29.8	-59.9 ± .2
1899	39.5	-57.7 ± .1
1899	50.3	-55.6 ± .1
1899	57.8	-54.3 ± .1
1899	69.8	-52.8 ± .1
1899	79.6	-51.8 ± .1
1899	100.0	-49.7 ± .1
1899	119.0	-48.2 ± .1
1899	135.0	-47.1 ± .1
1899	161.0	-45.5 ± .1
1899	179.0	-44.7 ± .1
1899	199.0	-44.0 ± .1
1899	218.0	-43.1 ± .1
1899	239.0	-42.5 ± .1
1899	260.0	-41.8 ± .1
1899	275.0	-41.5 ± .1

Table D.5: Single pass RF measurement raw data using a kicker and a superconducting cavity.

Single Pass (Using the Cavity as a Pickup)		
HOM frequency (MHz)	CW Beam Current (μA)	$\lvert S_{21}\rvert$ (dB)
2110	9.9	-86.2 $\pm$ 1.0
2110	19.8	-81.7 $\pm$ .6
2110	29.8	-78.2 $\pm$ .5
2110	39.5	-76.4 $\pm$ .5
2110	50.3	-74.5 $\pm$ .5
2110	57.8	-73.5 $\pm$ .5
2110	69.8	-71.7 $\pm$ .5
2110	79.6	-70.7 $\pm$ .4
2110	100.0	-68.4 $\pm$ .4
2110	119.0	-67.4 $\pm$ .4
2110	135.0	-65.9 $\pm$ .4
2110	161.0	-64.5 $\pm$ .4
2110	179.0	-63.7 $\pm$ .2
2110	199.0	-62.9 $\pm$ .2
2110	218.0	-62.1 $\pm$ .2
2110	239.0	-61.4 $\pm$ .2
2110	260.0	-60.9 $\pm$ .2
2110	275.0	-60.5 $\pm$ .1
Single Pass (Using an RF stripline pickup)		
1899	9.9	0.0 $\pm$ 1.3
1899	49.6	14.1 $\pm$ .6
1899	105	21.1 $\pm$ .1
1899	150.9	24.5 $\pm$ .3
1899	195.2	27.0 $\pm$ .1

Table D.6: Single pass RF measurement raw data using a kicker and a superconducting cavity and a kicker and pickup.

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Vita

Nicholas S. R. Sereno was born on September 1, 1965 in Libertyville, Illinois. He attended the University of Illinois at Urbana-Champaign from the fall of 1983 to the spring of 1987, when he received a Bachelor of Science degree in physics. He entered graduate school at the University of Illinois at Urbana-Champaign in the fall of 1987, and received the Master of Science degree in physics in the spring of 1989. He is a member of the American Physical Society.

EXPERIMENTAL STUDIES OF MULTIPASS BEAM BREAKUP AND ENERGY RECOVERY USING THE CEBAF INJECTOR LINAC

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Beam breakup (BBU) instabilities in superconducting linacs are a significant issue due to the potentially high Q of the cavity higher-order modes (HOMs). The CEBAF accelerator, which employs five pass recirculation through two superconducting linacs to accelerate high CW current (up to 200 μA), poses unique instability problems. An experimental investigation of multipass BBU at CEBAF has been completed using a single recirculation through the CEBAF injector linac. This recirculator is calculated to be more sensitive to the instability than the full CEBAF accelerator. Successful recirculation through the injector of a beam with greater than 200 μA average current from an injection energy of 5.6 MeV to a final energy of 80.1 MeV indicates that BBU should not be a problem in the full CEBAF accelerator. In addition, we were able recover all the energy from a 30 μA 42.8 MeV first pass beam by recirculating it out of phase and decelerating it to a final energy equal to the injection energy (5.6 MeV).

The recirculator constructed in the CEBAF injector was designed so that the recirculation leg optics could be changed in an attempt to induce multipass BBU. The various optical settings were modeled using the computer code DIMAD and the resulting recirculation transfer matrices were used in the multipass BBU code TDBBU. Threshold currents for the onset of multipass BBU were calculated using TDBBU for each optical setting. The computed threshold currents were found to be at least an order of magnitude larger than the maximum average current available from the CEBAF injector (~ 200 μA).

The primary experimental investigation of multipass BBU consisted of a series of RF measurements using a stripline kicker to deflect the beam to excite cavity HOMs and a superconducting cavity as a pickup device to detect HOM signals. The

(normalized) amplitude of the cavity HOM signal was measured as a function of the average CW beam current for each optical setting. These data were analyzed in terms of a simplified model of a recirculating linac. Based on an analysis of the simple model, a lower bound on the threshold current was determined from the data for each optical setting. The lower bound on the threshold current inferred from the data for the majority of recirculator optical settings was beyond the currents available from the CEBAF injector. The RF experiment combined with the results of the TDBBU calculations indicate that the full CEBAF superconducting linac should not be limited by multipass BBU for CW beam currents well above the maximum design current of 200 μA.

In addition, a single pass measurement was performed using a stripline pickup (identical to the kicker device) to detect the HOM deflection of the beam. In this experiment, the actual deflection of the electron beam due to a HOM was observed using the pickup. The data obtained in this experiment were used to estimate the value of the shunt impedance R/Q of the HOM detected.

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Contents

List of Tables

List of Figures