Study of wiggler CSR effect on electron ring of MEIC

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Motivation

• In high-luminosity collider machines, to lower down the lepton beam emittance, people usually use damping wigglers to achieve the goal by extracting a large fraction of the synchrotron radiation and thus increasing the radiation damping rate (or, shortening the damping time).

• Fanglei’s previous presentation (Feb. 5, MEIC R&D Meeting) about potential damping wiggler design for MEIC reminds me about J. Wu et al. work:

• The following slides demonstrate some results reproduced from the above two papers, and apply the developed code to MEIC e-Ring case.
Outline

• Basic theory and assumption
• CSR impedance from a wiggler
• Underlying physics
• Application: MEIC e-Ring with damping wiggler
• Summary
Theoretical background

- step 1: Vlasov equation, with longitudinal equations of motion,
  \[
  \frac{\partial \rho}{\partial s} - \eta \delta \frac{\partial \rho}{\partial z} - \frac{r_0}{\gamma} \frac{\partial \rho}{\partial \delta} \int_{-\infty}^{\infty} dz' \, d\delta' \, W(z - z') \rho(\delta', z', s) = 0
  \]
- step 2: adding a (harmonic) perturbation
  \[\rho = \rho_0(\delta) + \rho_1(\delta, z, s) \quad \rho_1 = \hat{\rho}_1 e^{-i\omega_s/c + i k z}.\]
- step 3: linearization of Vlasov equation
  \[(\omega + c k \eta \delta) \hat{\rho}_1 = i \frac{r_0 c}{\gamma} \frac{\partial \rho_0}{\partial \delta} Z(k) \int d\delta \, \hat{\rho}_1(\delta).\]
- step 4: dispersion equation
  \[1 = \frac{i r_0 c Z(k)}{\gamma} \int \frac{d\delta \,(d\rho_0/d\delta)}{\omega + c k \eta \delta} \]

\[1 = -\frac{i Z(k) \Lambda}{\sqrt{2\pi} k} \int_{-\infty}^{\infty} df \frac{p e^{-p^2/2}}{\Omega \pm p} \]

\[Z(k) = \frac{Z_D(k)}{C} + \frac{Z_W(k)}{L_W C} \]

\[Z_D(k) = -iA \frac{k^{1/3}}{R^{2/3}}, \quad Z(k) = -i2k_w \frac{k}{k_0} \left[ \gamma_E + \log \left( \frac{4k}{k_0} \right) + i \frac{\pi}{2} \right] \]

\[Z(k) = -i \frac{6\Gamma[11/6]}{5\sqrt{\pi} \Gamma[4/3]} A \left( \frac{Kk_w}{\gamma} \right)^{2/3} k^{1/3} \]

(low-frequency approximation)

(high-frequency approximation)
CSR impedance from a dipole

- Assume steady-state and ultrarelativistic beam, the CSR impedance in a dipole can be expressed as

\[ Z_D(k) = -iA \frac{k^{1/3}}{R^{2/3}} \text{ with } A = 3^{-1/3} \Gamma(\frac{2}{3})(\sqrt{3}i - 1). \]

- Physical picture:

\[ \frac{dE}{cdt} = W_\parallel(s,z) = eE_\perp(s,z)\sin\theta \]

\[ E_\perp(s,z) \approx \frac{2Ne\lambda(s,z)}{r(s,z)} \]
CSR impedance from a wiggler

- Physical picture:
  separated by m $\lambda_w$, $W_\parallel$ always 0
  separated by [(m+1)/2] $\lambda_w$, $W_\parallel$ max.

\[
Z(k) = -ik \frac{K^2}{\gamma^2} \int_0^\infty d\xi G(\xi) e^{-4i(k/k_0)\xi}
\]

\[
G(\xi) = \frac{2}{\pi} \int_0^\pi d\hat{\xi} \frac{\sin \Delta \cos \hat{\xi} + (1 - \cos \Delta) \sin \hat{\xi}}{B(\Delta, \hat{\xi})}
\]
The spectrum are shown in Figs. 4 and 5, respectively. Eqs. (14) and (22).

where

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Simplest case: cold beam

- When Landau damping is negligible, i.e. cold beam case, the dispersion relation

\[ 1 = \frac{i r_0 c Z(k)}{\gamma} \int \frac{d\delta \, (d\rho_0/d\delta)}{\omega + c k \eta \delta} \quad \text{with} \quad \rho_0(\delta = \Delta p/p) = \delta(\delta_0) \]

can be greatly simplified and the growth rate can be estimated to be

\[ \omega = \sqrt{ic^2 n_0 r_0 \eta k Z(k)} / \gamma. \]

and the instability growth rate \[ \tau^{-1} \propto \text{Im}(\omega) \propto \sqrt{k Z(k)}. \]

- Usually, \( Z(k) \propto k^\varepsilon \), with \( \varepsilon > 0 \). For example, for dipole CSR, \( \varepsilon = 1/3 \)
- Thus, \( k \uparrow (\lambda \vee) \), \( \tau^{-1} \uparrow \)
- However, there are damping mechanisms, e.g. Landau damping due to finite energy spread and beam emittance, and synchrotron radiation damping (of course, quantum excitation would be involved.).
The parameters driving the design of the damping rings are the damping rate. We therefore need a pre-damping ring not compatible with the required extracted emittances and emittances, up to 30,000 mm-mrad. The necessary damping ring, required to reduce the emittance of the beam must be damped to normalized emittances of 3 mm-mrad. Designed to accept a beam with normalized emittances of 150 mm-mrad vertically, the damping times are 4.85, 5.09, and 2.61 ms.

**Table 1: MDR Damping Wiggler Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>1.98 GeV</td>
</tr>
<tr>
<td>Wiggler peak field</td>
<td>2.15 T</td>
</tr>
<tr>
<td>Wiggler period</td>
<td>0.27 m</td>
</tr>
<tr>
<td>Total wiggler length</td>
<td>46.25 m</td>
</tr>
<tr>
<td>Energy loss/turn from dipoles</td>
<td>247 keV</td>
</tr>
<tr>
<td>Energy loss/pass from wiggler</td>
<td>530 keV</td>
</tr>
<tr>
<td>Damping times $\tau_{x,y,z}$</td>
<td>4.85, 5.09, 2.61 ms</td>
</tr>
</tbody>
</table>

**Table 2: Principal Parameters of the Main Damping Rings**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>$E$</td>
</tr>
<tr>
<td>Circumference</td>
<td>$C$</td>
</tr>
<tr>
<td>Number of stored trains</td>
<td></td>
</tr>
<tr>
<td>Natural emittance</td>
<td>$\gamma e_0$</td>
</tr>
<tr>
<td>Tunes</td>
<td>$\nu_x, \nu_y$</td>
</tr>
<tr>
<td>Natural chromaticity</td>
<td>$\xi_x, \xi_y$</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>RF voltage</td>
<td>$V_{RF}$</td>
</tr>
<tr>
<td>RF acceptance</td>
<td>$\varepsilon_{RF}$</td>
</tr>
<tr>
<td>Energy spread (rms)</td>
<td>$\sigma_\delta$</td>
</tr>
<tr>
<td>Bunch length (rms)</td>
<td>$\sigma_z$</td>
</tr>
<tr>
<td>Integrated wiggler field</td>
<td>$\int B_w^2 ds$</td>
</tr>
<tr>
<td>Energy loss/turn</td>
<td>$U_0 + U_w$</td>
</tr>
<tr>
<td>Damping times $\tau_{x,y,z}$</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4-7.** Layout of main damping ring.
NLC main damping ring

- Dipole only
- Dipole + wiggler
- Wiggler only (unstable)
  - Wiggler only (stable)
To obtain the same results as in the paper, somehow we need to slightly adjust the rms energy spread from $9.09 \times 10^{-4}$ to $8.6 \times 10^{-4}$.

**NLC main damping ring**
Underlying physics

- Region (I): long wavelength, negligible Landau damping
  \[ \tau^{-1} \propto \text{Im}(\omega) \propto \sqrt{kZ(k)}. \]
- Region (II): shorter wavelength, more Landau damping, more phase mixing

![Graph showing growth rate vs. CSR instability wavelength (mm)]

- Region (I) dominated by CSR impedance
- Region (II) dominated by Landau damping (phase mixing)
Application: MEIC e-Ring

Note: beam and lattice parameters are provided by Fanglei.
## Application: MEIC e-Ring

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>2.154</td>
<td>km</td>
</tr>
<tr>
<td>Dipole radius</td>
<td>64 (average)</td>
<td>m</td>
</tr>
<tr>
<td>Total bending angle</td>
<td>540</td>
<td>deg</td>
</tr>
<tr>
<td>Momentum compaction factor, $\alpha_c$</td>
<td>0.00215</td>
<td></td>
</tr>
<tr>
<td>Wiggler peak field</td>
<td>1.6</td>
<td>Tesla</td>
</tr>
<tr>
<td>Wiggler period</td>
<td>0.2</td>
<td>m</td>
</tr>
<tr>
<td>Wiggler total length</td>
<td>24</td>
<td>m</td>
</tr>
<tr>
<td>Beam energy</td>
<td>10</td>
<td>GeV</td>
</tr>
<tr>
<td>Particles in a bunch</td>
<td>$5 \times 10^{10}$ (vary)</td>
<td></td>
</tr>
<tr>
<td>RMS fractional energy spread</td>
<td>$1.14 \times 10^{-3}$</td>
<td></td>
</tr>
</tbody>
</table>
Application: MEIC e-Ring

• Numerical simulation shows that, based on the given parameters, there is no such instability found in the e-Ring system.

• Comparing MEIC e-Ring with NLC damping ring, we found:
  – dipole radius is much larger, causing smaller CSR effect
  – rms fractional energy spread is a bit larger, resulting in more Landau damping
  – smaller fraction of wiggler total length to the whole ring (or, much larger ring circumference)

• What if the energy spread becomes half of the given number? \( N_b = 5 \times 10^{10}, \sigma_\delta = 0.57 \times 10^{-3} \)
Application: MEIC e-Ring

- Would the instability be suppressed by shielding of vacuum chamber?
  \[ \lambda < \lambda_c \sim 4\sqrt{2} b \sqrt{\frac{b}{R}} \]  
  [R. Warnock and P. Morton, Part. Accel. 25, 113 (1990)]
- Given the dipole radius \( R = 64 \) m and \( N_b = 5 \times 10^{10} \), to effectively suppress the instability requires the pipe radius \( b \leq 14 \) cm, which can be easily achieved.
- Assume pipe radius \( b = 3 \) cm (which is a usual case?), what about the intensity threshold, given \( \sigma_\delta = 0.57 \times 10^{-3} \)?

![CSR instability wavelength (mm) vs Particle number per bunch](image)
### Application: MEIC e-Ring

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutoff wavelength (assume $b = 3$ cm)</td>
<td>3.67</td>
<td>mm</td>
</tr>
<tr>
<td>Intensity threshold at cutoff (wiggler on)</td>
<td>$25 \times 10^{10}$</td>
<td></td>
</tr>
<tr>
<td>Growth time at cutoff (wiggler on)</td>
<td>22.7</td>
<td>μsec</td>
</tr>
<tr>
<td>assume $N_b = 27 \times 10^{10}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Synchrotron oscillation frequency</td>
<td>1~10 (assume)</td>
<td>kHz</td>
</tr>
<tr>
<td>Longitudinal radiation damping time</td>
<td>4.12</td>
<td>msec</td>
</tr>
</tbody>
</table>
Summary

• By solving the dispersion relation derived from (linearized) Vlasov equation, we can estimate the instability growth rate, given a set of beam parameters.
• Given further the information of vacuum chamber, i.e. pipe radius $b$, we can estimate the threshold intensity $N_{b,th}$.
• Since the calculation is fast, it can be used to optimize the e-Ring design with insertion of damping wiggler.
Possible improvement of the presented work

- e-Ring optimization with damping wiggler [see Section IV of J. Wu et al., PRST-AB 6, 104404 (2003)]
- 1-D linearized Vlasov formulation → 2-D
  - to account for Landau damping effect from finite transverse emittance
- coasting beam approximation → bunched-beam effect should be taken into account
- only \{steady-state CSR + wiggler\} impedances are included → \{entrance transient + exit propagation effects\} of CSR can be considered, as well as \{wall shielding effect\} should be incorporated
- Vlasov equation → Vlasov-Fokker-Planck equation (VFP)
  - VFP takes into account the effect of synchrotron radiation induced quantum excitation
- lumped model → distributed model
an instability while the results for the ATF are very similar to those of the NLC damping ring shown in Fig. 1. One interesting effect can be seen at the wiggler radiation fundamental frequency and the odd harmonics where the ring is actually stabilized by the wiggler CSR impedance. This is opposite to the single-pass behavior through a wiggler where there is an instability at the wiggler fundamental frequency usually referred to as the free-electron laser (FEL) instability. The effect will be discussed further in Appendix A but is a direct result of the positive sign of the momentum compaction in the ring. To illustrate this, Fig. 2 is a plot for the NLC damping ring with identical wiggler and arc parameters but the momentum compaction is assumed to be opposite in sign. Here, one can see that at low frequencies the growth is slightly lower. In contrast, the wiggler CSR impedance makes the system less stable at the wiggler fundamental frequency unlike the case with positive momentum compaction. Furthermore, if the magnitude of the momentum compaction is reduced, the system will become unstable similar to that illustrated in Appendix B. This is similar to the FEL instability; however, it is also noted in Appendix A that our theory does not fully treat the FEL instability.

As seen in Figs. 1 and 2, the instability is most important at relatively low frequency. The longest instability

<table>
<thead>
<tr>
<th>TABLE I. Parameters and results for the NLC main damping ring [16], the TESLA damping ring [17], and the KEK ATF prototype damping ring [18]. The parameter $F_w$, defined in Eq. (19), is the ratio of the ISR power emitted in the wiggler to that emitted in the arc bending magnets.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Circumference $C$/km</td>
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<tr>
<td>Dipole radius $R$/m</td>
</tr>
<tr>
<td>Total bending angle $\Theta/2\pi$</td>
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<tr>
<td>Momentum compaction $\alpha/10^{-4}$</td>
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<tr>
<td>Synchrotron frequency $Q_s$/kHz</td>
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<tr>
<td>Extracted $X$ emittance $\gamma\epsilon_x/10^{-6}$ m</td>
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<tr>
<td>Extracted $Y$ emittance $\gamma\epsilon_y/10^{-8}$ m</td>
</tr>
<tr>
<td>Energy $E$/Gev</td>
</tr>
<tr>
<td>Energy rms spread $\nu_0/10^{-4}$</td>
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<tr>
<td>Bunch rms length $\sigma_z$/mm</td>
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<tr>
<td>Particles in a bunch $N_e/10^{10}$</td>
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<tr>
<td>Wiggler peak field $B_w$/T</td>
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<tr>
<td>Wiggler period $\lambda_w$/m</td>
</tr>
<tr>
<td>Wiggler total length $L_w$/m</td>
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<tr>
<td>Wiggler $\beta$ function $\beta_{x,w}$/m</td>
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<tr>
<td>Pipe radius $b$/cm</td>
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<tr>
<td>$F_w$</td>
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<tr>
<td>Cutoff wavelength $\lambda_c$/mm</td>
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<tr>
<td>Threshold at cutoff (wiggler off) $N_t/10^{10}$</td>
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<tr>
<td>Threshold at cutoff (wiggler on) $N_t/10^{10}$</td>
</tr>
<tr>
<td>Growth time at cutoff (wiggler off) $\tau/\mu$s</td>
</tr>
<tr>
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