Coherent Electron Cooling

Use of an Electron Beam for Stochastic Cooling

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Outline

Principal idea
Polarization of electron plasma (beam) by a fast ion
Potentially possible amplification
Region of amplification
Limitation of cooling rate due to ion shielding
Arrangements for microwave instabilities in e-beam
Shottky-noise limitation and suppression of noise
Limitation due to non-linear saturation
Phasing
CEC on FEL (preliminary estimates)
Conclusions and outlook
Coherent electron cooling (CEC) was proposed 27 years ago

- General idea: amplify response of e-beam to an ion by a micro-wave instability of the beam
- A few instabilities have been shown
- CEC advantages/disadvantages compared to:
  - EC: Gain in cooling rate
  - Complicate BT
  - SC: Very large FB (30 GHz – optics)
  - Precise phasing required
  - OSC: Effective in a wide energy range
  - Small signal delay
  - Intense e-beam required
  - Signal gain is limited

What changed in last 10 years?
- Relativistic DC EC realized (FNAL)
- ERL realized (JLab)
- SASE FEL realized (UCLA - DESY)
- ERL-based HEEC on the way (BNL)
  And more…
Fast ion wake in electron gas

Fast ion produces polarization of e-gas:

\[ n = n_0 + \tilde{n}; \quad \tilde{n} + \omega_e^2 \tilde{n} = -\omega_e^2 Ze \delta(r - \tilde{v}t) \]

\[ \langle \text{In magnetized beam:} \quad T_e \Rightarrow T_{ez} \approx e^2 n_0^{1/3} \approx 10^{-4} eV \rangle \]

Ion interacts with charge image spread over distances \( \rho \):

\[ \frac{\nu_e}{\omega_e} \equiv \rho_D < \rho \leq \rho_{sh} \equiv \frac{\nu}{\omega_e} \quad \text{-area of collective response} \]

In stable plasma, ion receives drag force

\[ F_0 = \frac{(Ze)^2}{\rho_{sh}^2} \int \frac{d\rho}{\rho} \]

Polarization is small:

\[ \frac{\tilde{n}}{n_0} \approx \frac{\delta\rho}{\rho_{sh}} \approx \frac{1}{n_0 \rho_{sh}^3} \]

Thus, there is an enhancement potential

\[ G \leq n_0 \rho_{sh}^3 \quad (\approx 10^6)! \]

• A micro-wave instability is called…
• E-beam should be cool enough…
• Phasing is required…
Maximum CEC rate

• Maximum field initiated by a single ion
  (after amplification at absence of friends…)

\[ E \Rightarrow 4\pi n_0 e \rho_{sh} = 4\pi n_0 e \nu / \omega_e \]

• The momentum transfer is then

\[ \Delta p_\perp = F \cdot \tau \cong Z e^2 4\pi n_0 / \omega_e^2 = Z m_e \nu = \frac{Z}{A} \frac{m_e}{m_p} p_\perp \]

• And, the cooling time then would be

\[ \tau_c \cong \frac{A}{Z} \frac{m_p}{m_e} \frac{1}{f_0} \]
Microwave electron instabilities -1

Introduce **parametric plasma resonance** \( \ddot{n} + \omega^2_e(t)\dot{n} = 0 \)

- Change electron density up and down every 90 degree of plasma phase advance

**Possible way:**

- E-beam in modulated solenoid: \( \omega^2_e(s) = \frac{4J/A}{\gamma vr^2(s)} \), \( r(s) = r_1 \leftrightarrow r_2; \)

- Start at \( \dot{n} = 0 \), then after each single period:

- After \( q \) periods: \( \frac{\dot{n}_{q+1}}{\dot{n}_0} = \left(\frac{r_2}{r_1}\right)^q \)

**Correction:** magnetized plasma is non-isotropic: \( \omega_e \Rightarrow \omega_e \times \cos \theta \)
Microwave electron instabilities-2

- Negative longitudinal mass instability

An example:

- Beam angle
  \[ \theta_0 := \frac{B_u / B_s}{\kappa_u \lambda_c - 1} \quad \hat{\lambda}_c = \frac{p}{eB_s} \]

- The angle grows with energy at \( \kappa_u \hat{\lambda}_c < 1 \)

- Translation velocity:
  \[ \nu_z^2 = 1 - \frac{1}{\gamma^2} - \theta_0^2 ; \]

  decreases with energy at
  \[ \frac{1}{\mu} = \frac{d\nu_z}{d\gamma} = \frac{1}{\gamma^3} - \frac{1}{2} \frac{d\theta_0^2}{d\gamma} < 0 \]

Coulomb instability:

\[ \Lambda = \sqrt{\frac{4\pi ne^2}{-\mu c^2}} \]
Microwave electron instabilities -3

FEL instability

• Undulator period
  \[ 2\pi \hat{\kappa}_u \]

• Radiation wave length
  \[ \lambda = \frac{1}{2} \lambda_u \left( \frac{1}{\gamma^2} + \theta_u^2 \right) \]

• Plasma beat wave length
  \[ l_e = \frac{\gamma_r}{2} \sqrt{\frac{\gamma J_A}{J(1 + \gamma^2 \theta_u^2)}} \]

• SASE gain length
  \[ l_G \approx l_e \left( \frac{\hat{\kappa}_u}{l_e \gamma^2 \theta_u^2} \right)^{1/3} \]
  at \[ \sqrt{\hat{\kappa}_u l_G} \leq \gamma_r \]

• Optimal arrangement:
  \[ \hat{\kappa}_u \approx \gamma_r \min(1, \frac{l_e}{l_c}) \]
Limitations of CEC due to ion interactions

• Conventional stochastic cooling limit:

\[
(\tau_c)_{\text{min}} \geq \frac{N_{\Delta \varphi} \Delta \varphi}{2 \pi \Delta f_0} = \left( \frac{J_{\text{peak}}}{e} \right) \frac{f_0}{(\Delta \omega)^2} \frac{\Delta f_0}{\Delta f_0}
\]

\[
\Delta \omega = \frac{c}{l_\perp}
\]

• CEC:

\[
\Delta \omega = \frac{\gamma \beta c}{\min(\rho_{sh}, \sigma_\perp)}
\]

Reduction of mixing limit by a factor of

\[
\left[ \frac{\gamma l_\perp}{\min(\rho_{sh}, \sigma_\perp)} \right]^2
\]
Schottky noise limitation of CEC

- Normal Schottky impact:

\[ \dot{T}_{\text{scat}} \approx -\frac{m_e}{m_i} \dot{T}_{\text{cool}} \]

Rates gain in CEC:

\[ \dot{T}_{\text{cool}} \rightarrow \times G \quad \dot{T}_{\text{scat}} \rightarrow \times G^2 \]

It yields

\[ G_{\text{max}} \approx \frac{m_i}{m_e} \]

Suppressed Schottky noise:

\[ \dot{T}_{\text{scatt}} \rightarrow \times \Gamma^{-2} \]

Then

\[ G_{\text{max}} \Rightarrow \frac{m_i}{m_e} \Gamma^2 \]
Suppression of Schottky noise

• Frequency range: \[ \omega = k / \beta c \quad k \geq 1 / \sigma \]

Possible ways:
• 3/2 e-gun regime: \[ \Gamma^2 \approx (eU / T_{cath})^{1/2} \]
• Adiabatic acceleration (more suppression)
• Thermal relaxation along a low energy drift (most effective, in principle)
• Fast acceleration case: implement plasma gymnastics to compensate for instability mode (not effective in case of FEL amplification)

An “absolute” suppression limit:

\[ \Gamma_{max}^2 \approx \frac{e^2 n_e \rho_{sh}^2}{T_{ez}} = \frac{m}{M} \frac{T_i}{T_{ez}} ; \quad G_{max} \approx \frac{T_i}{T_{ez}} \quad (\Rightarrow 10^7 - 10^9) \]
Gain limitation due to non-linear saturation

Maximum gain for an absolute cooled e-beam:

\[ G_1 \approx (n_e \sigma^3 / \gamma) \approx N_e \frac{\sigma \parallel}{\gamma \sigma_z} \]

Maximum gain for a real e-beam

\[ G_2 \approx \Gamma (n_e \sigma^3 / \gamma)^{1/2} \quad (\Gamma < \sqrt{n_e \sigma^3 / \gamma}) \]

Maximum gain for real i-beam:

\[ G_3 \approx (n_e \sigma^3 / \gamma)/(n_i \sigma^3 / \gamma)^{1/2} \]

Max. gain, at all:

\[ G \leq \min(G_2, G_3, \Gamma^2 \frac{m_p}{m_e}) \quad (\approx 10^3 - 10^4) \]
Transverse vs longitudinal CEC

- Due to beam transport conditions and nature of microwave instabilities, transverse e-polarization is low compared to the longitudinal one. Therefore, the transverse drag force is small compared to the longitudinal one.
- Solution in general: arrange for dispersive cooling
- Method: ion dispersion + tilt of electron ellipsoids

Non-achromatic chicane installed at the exit of the FEL before the kicker section turns the fronts of the charged planes.

An alternative option: create transverse gradient of electron energy, by introducing the gradient SRF (no bend required...).
CEC phasing

• The “good mixing” in CEC is extraordinary good (super-large frequency bandwidth!)

\[ \Delta s = \frac{\Delta \gamma}{\gamma^3} l \approx \frac{\lambda}{4} \frac{4l}{\lambda_u} \frac{\Delta \gamma}{\gamma} < \frac{\lambda}{4} \]

• The “bad mixing” for non-bent ion beam is not bad…

• However, the e-beam response experiences some delay (necessary for amplification)

• The most advantageous way to compensate for delay time seems to be:
  
  Increase (or modulate) electron energy

• In case of CEC on FEL, the “light” overtakes the ion…

Optimal phasing needs more study
FELs and high-energy electron cooling

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29th International FEL Conference
August 26-31, 2007, BINP, Novosibirsk
And so, my fellow Americans, ask not what your country can do for you; ask what you can do for your country.

And so, my fellow FELers, ask not what storage ring can do for FELs: ask what FELs can do for your storage rings!

Vladimir Litvinenko

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ultra-relativistic case (\(\gamma >> 1\)), longitudinal cooling

Most versatile phasing option

- Hadrons
- Electrons

Modulator: region 1 about a quarter of plasma oscillation
Longitudinal dispersion for hadrons
Amplifier of the e-beam modulation via SASE FEL

Most economical option

Kicker: region 2
CEC on FEL

- **Modulator: Interaction region 1**
  Length: about a quarter of plasma oscillation

\[
\omega_{pe} = \sqrt{\frac{4\pi n_e e^2}{m_e}} = c \sqrt{4\pi n_e r_e}
\]

Each hadron generates modulation in the electron density with total charge of about minus charge of the hadron, \( Z \)
CEC on FEL

Longitudinal dispersion for hadrons, time of flight depends on its energy:

\[(T-T_0) \nu_o = -D \frac{(E-E_0)}{E_o}\]

Amplifier of the e-beam modulation- SASE FEL

Electron density modulation is amplified in SASE FEL and made into a train with duration of N alternating hills (high density) and valleys (low density) with period of FEL wavelength

\[\lambda_o = \lambda_w \left(1 + a_w^2 \right) / 2 \gamma^2 ; N_c \sim L_{gain} / \lambda_w\]

Maximum gain of the electron density of SASE FEL is \(\sim 10^3\). 
A hadron with central energy (E₀) phased with the hill where longitudinal electric field is zero, a hadron with higher energy (E>E₀) arrives earlier and is decelerated, while hadron with lower energy (E<E₀) arrives later and is accelerated by the collective field of electrons.
# Cooling of hadron beams

<table>
<thead>
<tr>
<th>Machine</th>
<th>Species</th>
<th>Energy GeV/n</th>
<th>Synchrotron radiation, hrs</th>
<th>Electron cooling, hrs</th>
<th>CEC, hrs</th>
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<tbody>
<tr>
<td>RHIC</td>
<td>Au</td>
<td>100</td>
<td>20,961 ∞</td>
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<td>7,000</td>
<td>13/26</td>
<td>∞ ∞</td>
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</table>
Conclusions

- Coherent electron cooling seems to be a promising method to enhance the capabilities of electron/stochastic cooling. It might find important applications in a wide energy range of hadron beams in accelerators.
- At high energies, it might take full advantage of high gain FELs based on high brightness ERLs.
- Proof of principle experiment of cooling Au ions in RHIC at ~ 40 GeV/n is feasible with existing R&D ERL (oper. starts in 2009).
- Cooling 100 GeV/n ions and 250 GeV protons in RHIC seems to be straightforward.
- Cooling protons in LCH at 7 TeV seems to be possible, but may require slightly more elaborate scheme (buncher, etc.).
- Question of possible short-noise suppression in electron beam is very interesting and should be further studies.
Afterwards

It seems we see the prairie, but was the horse laying there?

A comprehensive study to follow…