Transparent Spin Resonance Crossing in Accelerators

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Spin Motion at Circular Accelerator

$$\frac{d\vec{S}}{d\theta} = [\vec{W} \times \vec{S}] \,,$$

Thomas-BMT equation

$\theta -$ **particle azimuth**

[Ya.S. Derbenev, A.M. Kondratenko, A.N. Skrinsky (1970)] :

The spin equilibrium closed orbit

$\vec{n}(\theta + 2\pi) = \vec{n}(\theta) -$ **periodical axis of precession**

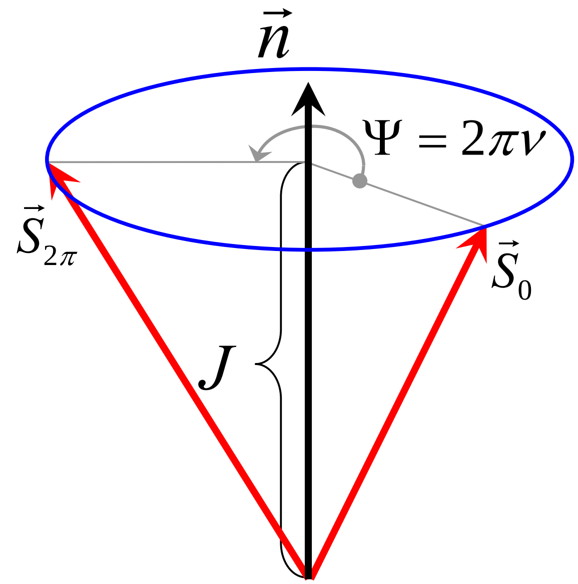
$$\vec{S} = J \cdot \vec{n} + \vec{S}_\perp \,, \qquad J = \vec{S} \cdot \vec{n}, \qquad \vec{S}_\perp \perp \vec{n}$$

Spin vector rotate around n-axis:

If $\vec{S}_0 \parallel \vec{n} \;\Rightarrow\; \vec{S}_{2\pi} = \vec{S}_0$

If $\vec{S}_0 \perp \vec{n} \;\Rightarrow\; \vec{S}_{2\pi} \perp \vec{n}, \;\; \angle\left(\vec{S}_0, \vec{S}_{2\pi}\right) = \Psi = 2\pi\nu$

$\nu -$ **spin precession tune**



Not equilibrium orbit

$$\vec{n}(\theta + 2\pi, I_i, \Psi_i + 2\pi) = \vec{n}(\theta, I_i, \Psi_i)$$

$I_i, \Psi_i -$ **act-phase variables of betatron motion**

$J = \vec{S} \cdot \vec{n} -$ **Spin Adiabatic Invariant**

$\vec{w} = \Delta\vec{W} -$ **spin deviation**

$$\Rightarrow \begin{cases} \Delta\vec{n} - \text{spread of n-axises} \\ \Delta\nu - \text{spread of spin tune} \end{cases}$$

In ideal accelerator $\quad \vec{n} = \vec{e}_z \,, \quad \nu = \gamma G$

$G = (g-2)/2 -$ **gyromagnetic anomaly**

$\vec{\Pi} = \langle \vec{S} \rangle -$ **vector of polarization,**

$D = 1 - |\vec{\Pi}| -$ **degree of depolarization**

$$\vec{\Pi} = \langle J\,\vec{n} \rangle + \langle \vec{S}_\perp \rangle = \langle J \rangle \langle \vec{n} \rangle$$

Spin Motion Near Single Spin Resonance

Near coherent (imperfection) resonance

$$\vec{W} = \nu_0 \vec{e}_z + w_k \left(\vec{e}_x \cos k\theta + \vec{e}_y \sin k\theta \right)$$

$$\vec{n} = \frac{\varepsilon_k}{\sqrt{\varepsilon_k^2 + w_k^2}} \vec{e}_z + \frac{w_k}{\sqrt{\varepsilon_k^2 + w_k^2}} \left(\vec{e}_x \cos k\theta + \vec{e}_y \sin k\theta \right)$$

$\nu = k + \sqrt{\varepsilon_k^2 + w_k^2}$ **is spin tune**

$\varepsilon_k = \nu_0 - \nu_k$ **is resonance detune**

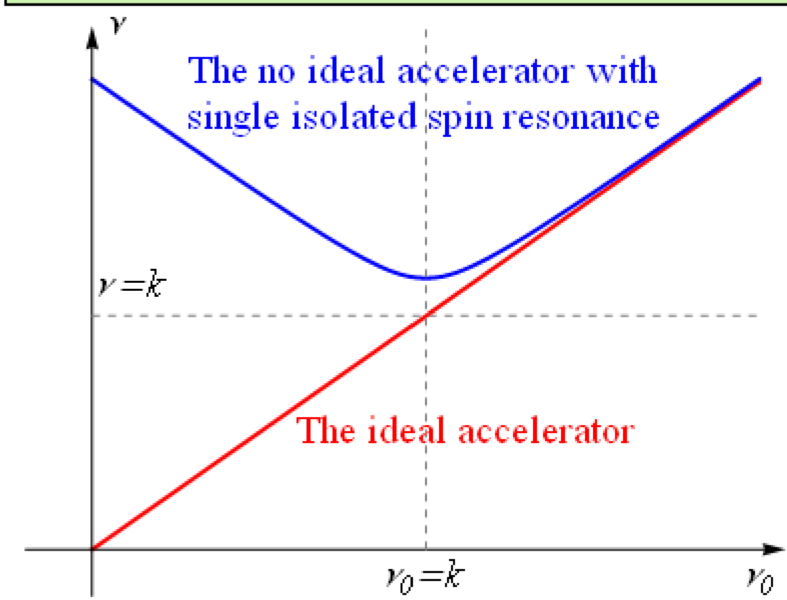
w_k **is resonance strength**

Field in **Resonance coordinate system**, which rotates around vertical axis with resonance frequency, is $\boxed{\vec{h} = \varepsilon_k \vec{e}_z + \vec{w}_k}$

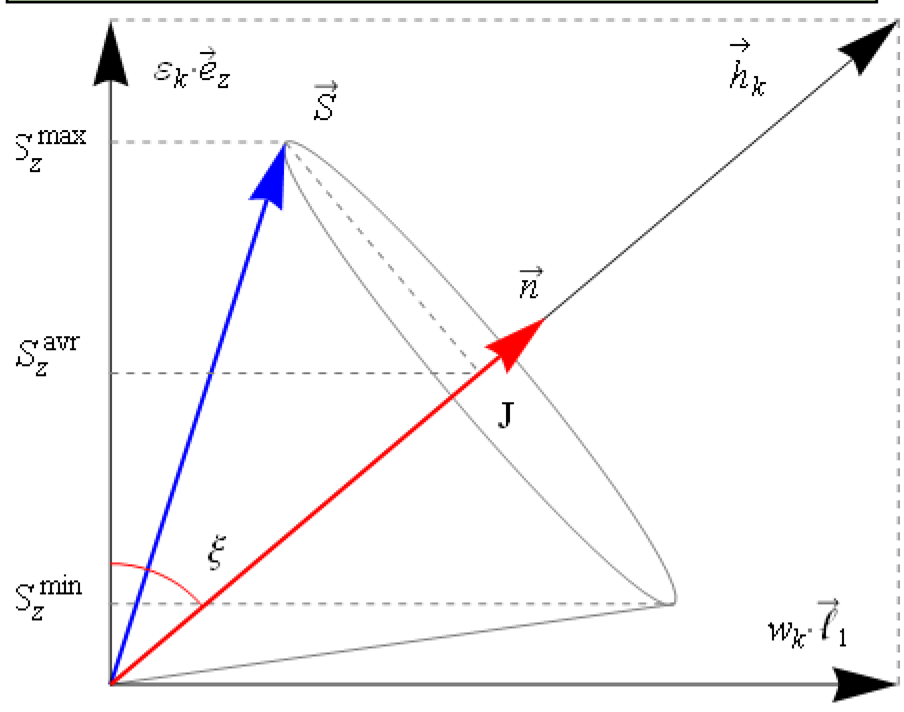
Hence, spin precession axis

$$\vec{n} = \frac{\vec{h}}{h} = \frac{\varepsilon_k}{\sqrt{\varepsilon_k^2 + w_k^2}} \vec{e}_z + \frac{w_k}{\sqrt{\varepsilon_k^2 + w_k^2}} \vec{\ell}_1$$

In this system, spin tune $\nu = \sqrt{\varepsilon_k^2 + w_k^2}$



Dependence of spin tune ν on Gγ in accelerator with and without resonance.



Spin precession near the spin resonance

$\vec{S}$ – **blue arrows**

$\vec{n}$ – **red arrows**

$\varepsilon_k^{\text{eff}}$ – effective resonance region

Spin adiabatic invariant (**SAI**) is $J = \vec{S}\cdot\vec{n}$

$J = const$ when

$$\left|\frac{d\vec{h}_k}{d\theta}\right| << h_k^2 \quad \text{or} \quad \left|\frac{d\varepsilon_k}{d\theta}\right| << w_k^2 + \varepsilon_k^2$$

Adiabatic crossing

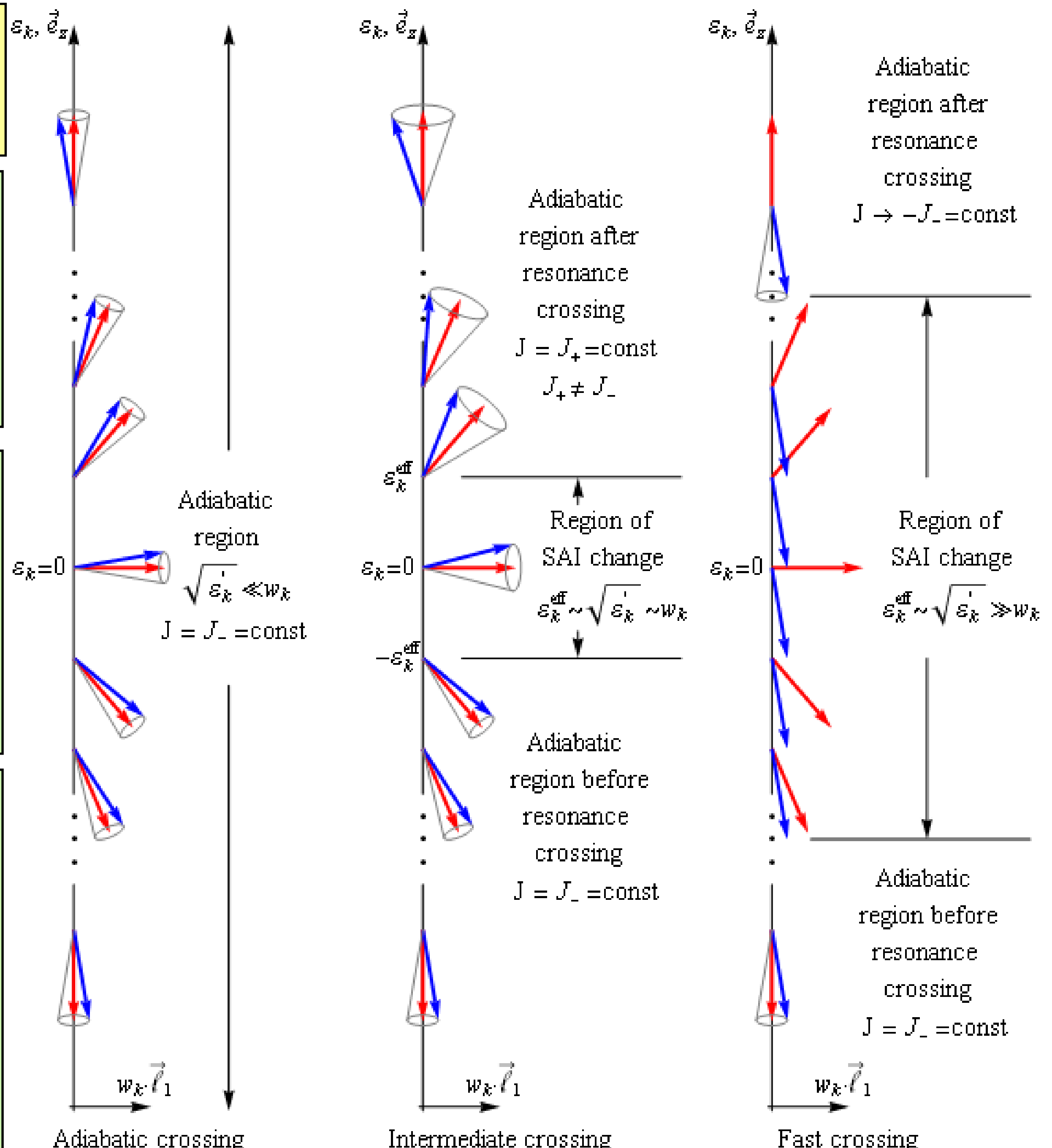
$$\varepsilon_k' << w_k^2 \quad \left(J_{after} = J_{before}\right)$$

Intermediate crossing

$$\varepsilon_k' \sim w_k^2 \quad \left(J_{after} \neq J_{before}\right)$$

Fast crossing

$$\varepsilon_k' >> w_k^2 \quad \left(J_{after} \to -J_{before}\right)$$



Generalized F.S. formula

$S_z^i = S_z(-\infty)$ – initial vertical spin component

$S_z^f = S_z(+\infty)$ – final vertical spin component

$æ = \dfrac{w_k}{\sqrt{\varepsilon'}}$ – normalized resonance strength

M.Froissart, R.Stora, 1960

$$S_z^f = \left[2\exp\left(-\frac{\pi æ^2}{2}\right) - 1\right] S_z^i$$

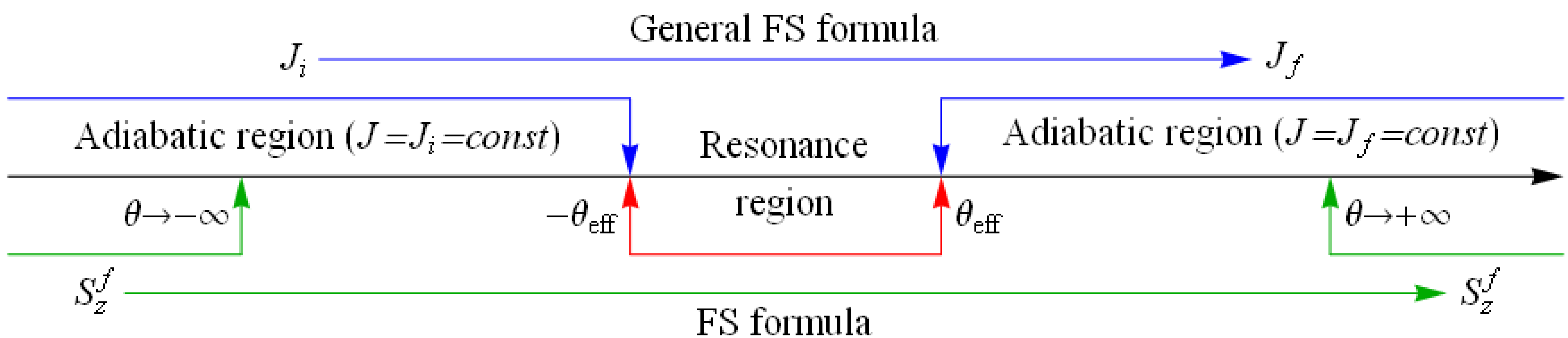
SAI magnitude becomes equal to vertical spin component magnitude only in the limit $|\theta| \Rightarrow \infty$:

$$J_i = \vec{S} \cdot \vec{n}(\theta_i) \Rightarrow -S_z^i$$

$$J_f = \vec{S} \cdot \vec{n}(\theta_f) \Rightarrow S_z^f$$

Generalized F.S. formula

$$J_f = \left[1 - 2\exp\left(-\frac{\pi æ^2}{2}\right)\right] J_i$$



In adiabatic region vertical spin component is $\langle S_z \rangle = J\cos\xi = J \dfrac{\varepsilon_k}{\sqrt{\varepsilon_k^2 + w_k^2}}$

Single crossing of the spin resonance

Spinor representation

$$\vec{S} = \chi^+ \vec{\sigma}\, \chi, \qquad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \qquad \frac{d\chi}{d\theta} = -\frac{i}{2}\left(\vec{\sigma}\cdot\vec{W}\right)\chi$$

$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ − Pauli matrices

The field coordinate system

$$\vec{e}_1 = \left[\vec{\ell}_2 \times \vec{n}\right]; \quad \vec{e}_2 = \vec{\ell}_2; \quad \vec{e}_3 = \vec{n} = \vec{h}_k / h_k$$

$$\vec{W} = \left\{0;\ \frac{w_k\, \varepsilon_k'}{h_k^2};\ h_k\right\}$$

The case of $\varepsilon_k' = 0$

$$\chi(\theta) = M_z(\Psi)\chi(0),$$

$$J_f = \left(1 - 2\left|M_{12}\right|^2\right)J_i \quad \Rightarrow \quad J = const$$

The case of $\varepsilon_k' \neq 0$

$$\chi(\theta_f) = M(\theta_f, \theta_i)\chi(\theta_i),$$

$$M(\theta_f, \theta_i) = M_z(\Psi_f)M_y(2\alpha)M_z(\Psi_i)$$

$$J_f = \left(1 - 2\left|M_{12}\right|^2\right)J_i \quad \Rightarrow$$

Generalized FS formula

$$J_f = \cos 2\alpha\ J_i = \left[1 - 2\exp\left(-\frac{\pi æ^2}{2}\right)\right]J_i$$

$$\Psi = \int_0^\theta h_k d\theta \qquad M_z(\Psi) = \exp\left(-\frac{i\Psi}{2}\sigma_z\right)$$

$$M_y(\alpha) = \exp\left(-i\frac{\alpha}{2}\sigma_y\right),$$

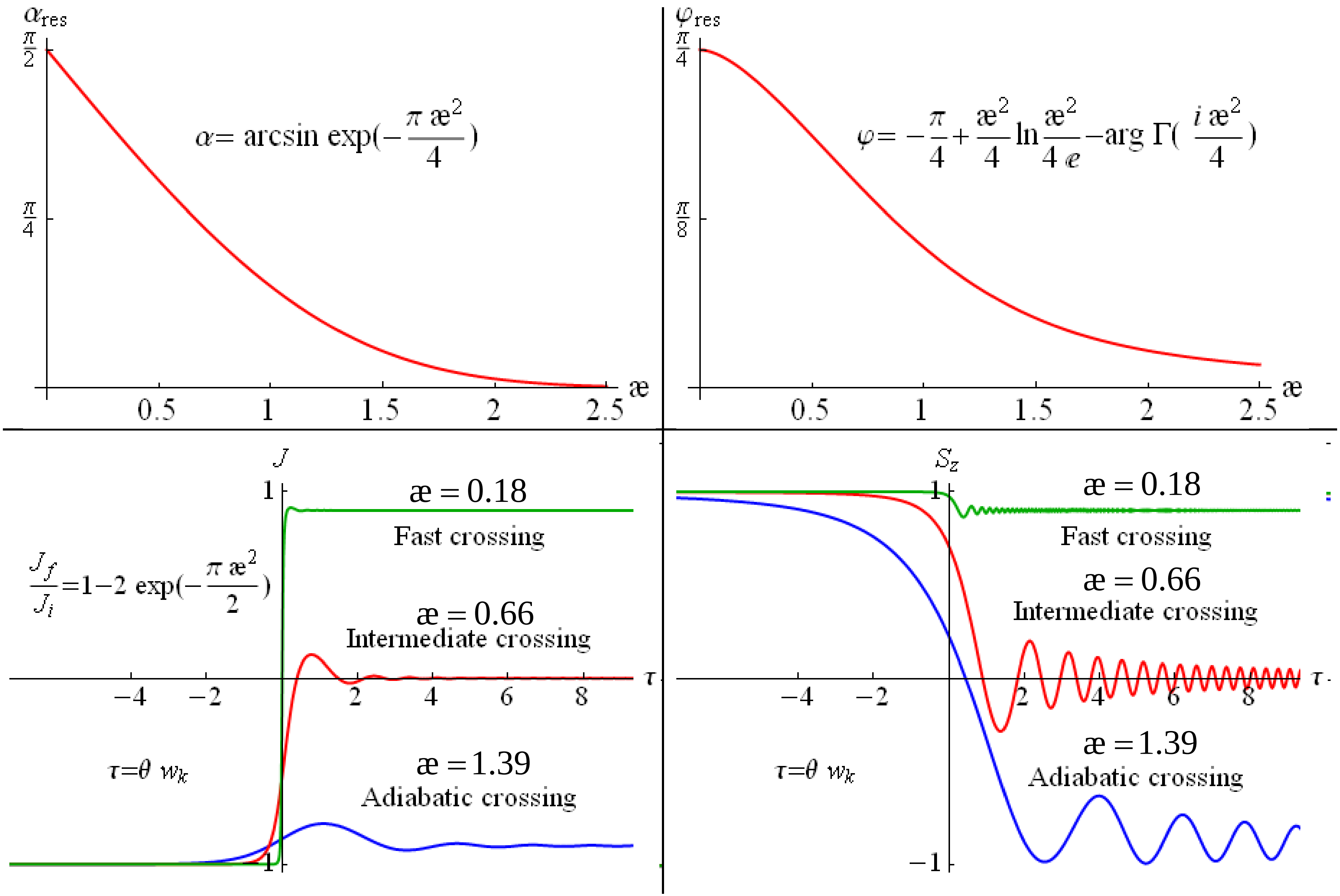
$$\Psi_i = \int_{\theta_i}^0 h_k d\theta + \varphi, \quad \Psi_f = \int_0^{\theta_f} h_k d\theta + \varphi,$$

$$\sin(\alpha) = \exp\left(-\frac{\pi æ^2}{4}\right), \qquad æ = \frac{w_k}{\sqrt{\left|\varepsilon'\right|}}$$

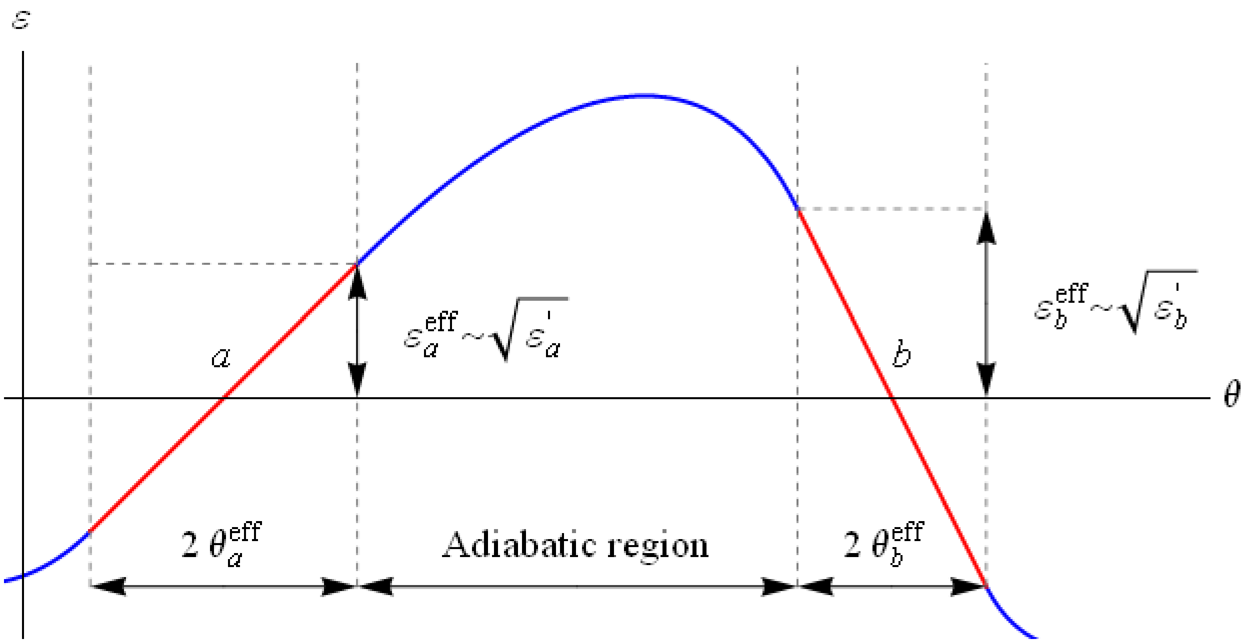
$$\varphi = -\frac{\pi}{4} + \frac{æ^2}{4}\ln\frac{æ^2}{4e} - \arg\Gamma\left(\frac{i\, æ^2}{4}\right)$$

$\Gamma(x)$ − Gamma function

Single crossing of the spin resonance



Two times crossing of the spin resonance



The spinor transformation matrix is

$$M(\theta_f,\theta_i)=M_z(\Psi_f)\,M_y(-2\beta)\,M_z(\Psi_{ab})\,M_y(2\alpha)\,M_z(\Psi_i)$$

$$\sin(\alpha)=\exp\left(-\frac{\pi\,æ_a^2}{4}\right),\qquad \sin(\beta)=\exp\left(-\frac{\pi\,æ_b^2}{4}\right),$$

$$\Psi_i=\int_{\theta_i}^{a}h_k\,d\theta+\varphi_a \qquad \Psi_f=\int_{b}^{\theta_f}h_k\,d\theta+\varphi_b \qquad \Psi_{ab}=\int_{a}^{b}h_k\,d\theta+\varphi_a+\varphi_b$$

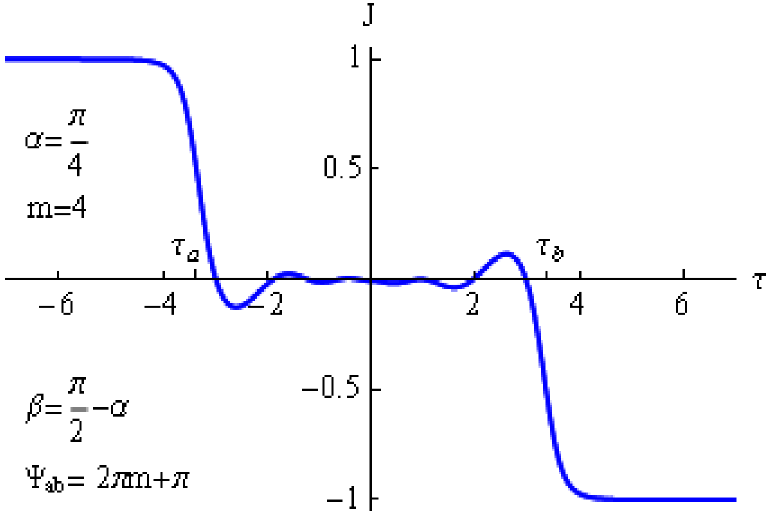
The condition of depolarization compensation (2 times crossing)

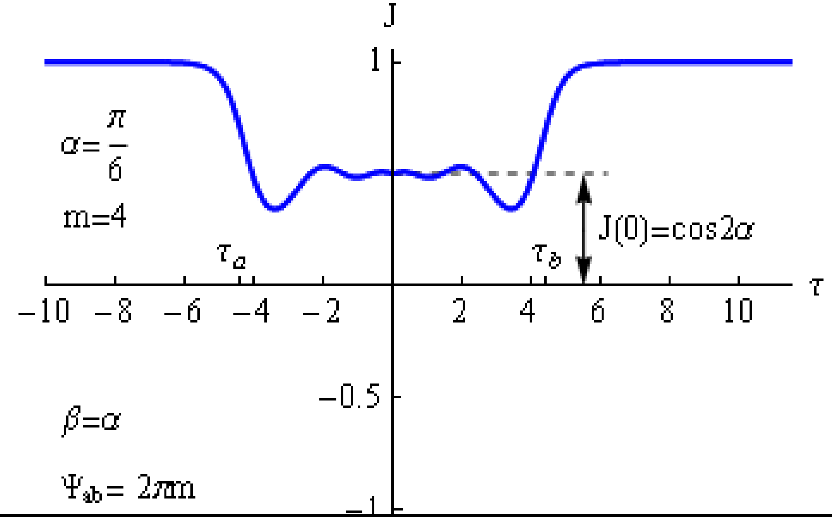
The case of SAI-flip $\left(J_f = -J_i\right)$

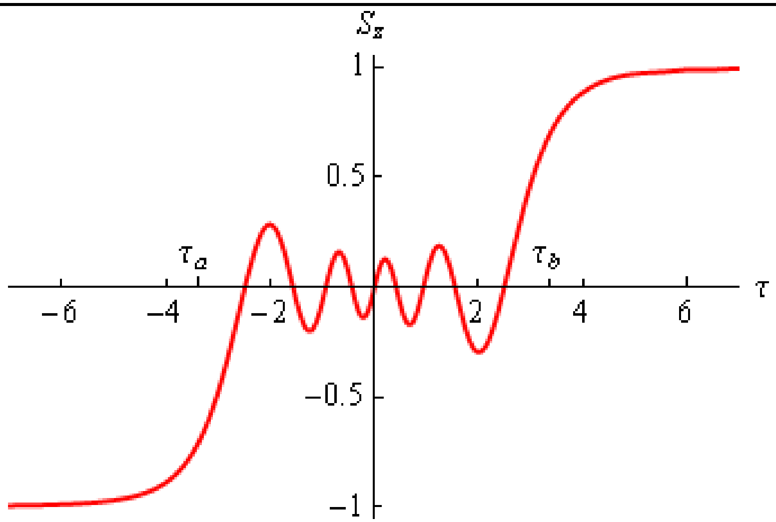
$$M_{11} = 0 \;\Rightarrow\; \alpha = \frac{\pi}{2} - \beta,\;\; \Psi_{ab} = 2\pi m + \pi$$

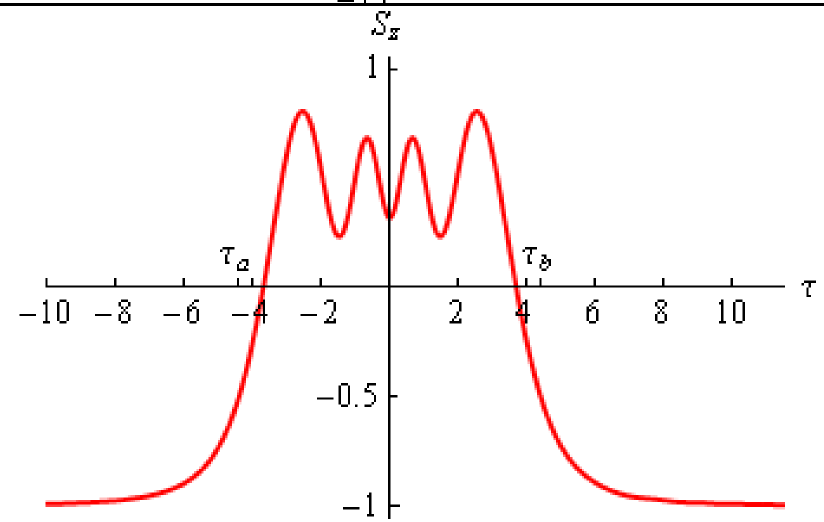
The SAI keeps sign $\left(J_f = J_i\right)$

$$M_{12} = 0 \;\Rightarrow\; \alpha = \beta,\;\; \Psi_{ab} = 2\pi m$$









Comparison with Results of A.W. Chao

"Spin Echo in Synchrotrons", 2005

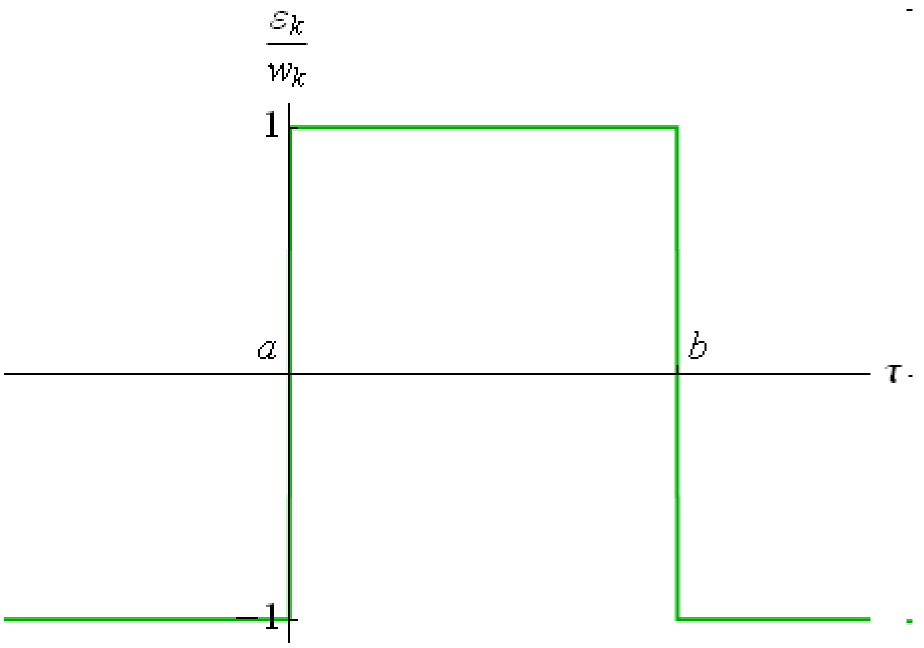
Condition when spin echo is not present after two spin detuning jumps:

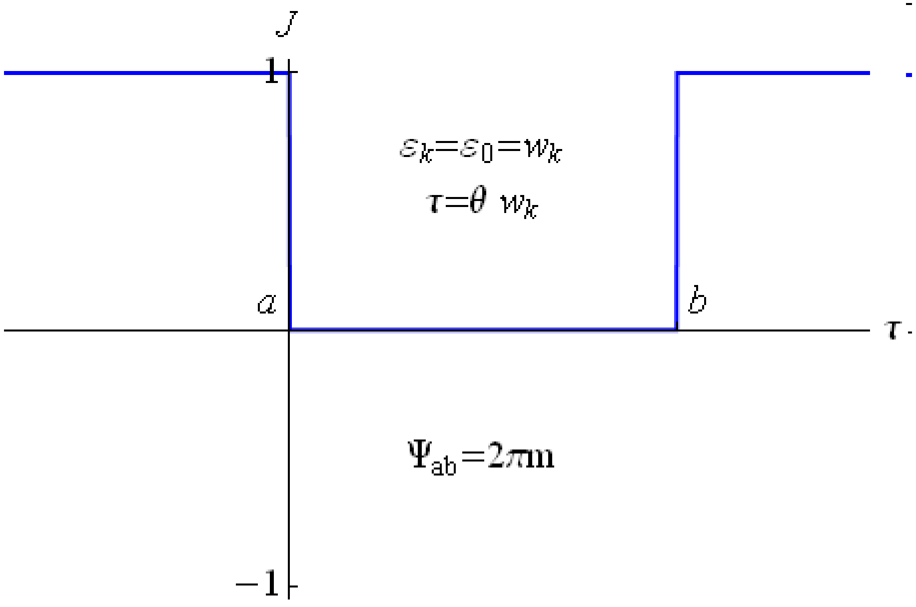
$$h_k(b-a) = 2\pi m$$

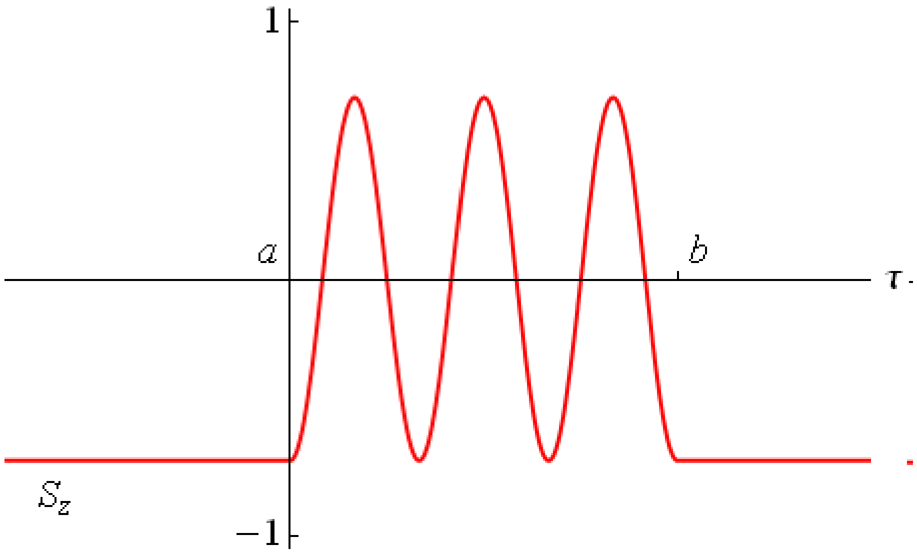
This result of A..Chao is similar to our result for the case

$$M_{12} = 0 \quad \text{and if} \quad \varphi_a + \varphi_b = 0$$

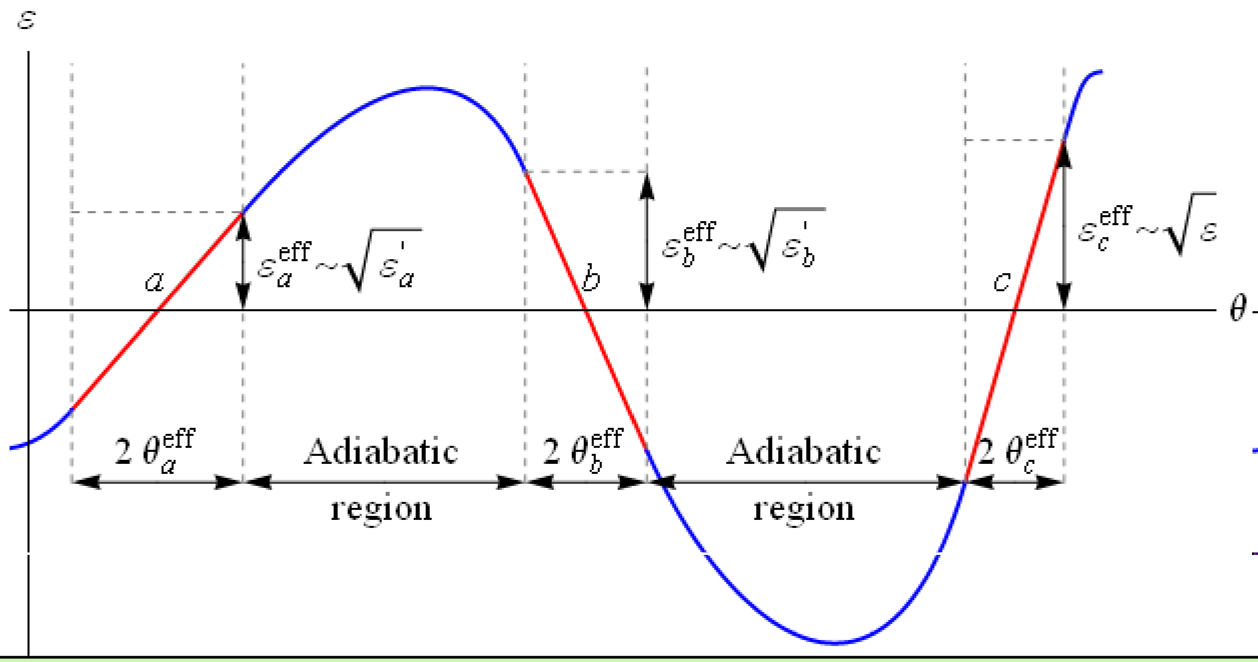
$$\Rightarrow \Psi_{ab} = h_k(b-a) = 2\pi m$$







Three times crossing of the spin resonance



The spinor transformation matrix is

$$M(\theta_f,\theta_i) = M_z(\Psi_f)\, M_y(2\gamma)\, M_z(\Psi_{bc})\, M_y(-2\beta)\, M_z(\Psi_{ab})\, M_y(2\alpha)\, M_z(\Psi_i)$$

$$\sin(\alpha) = \exp\left(-\frac{\pi\, æ_a^2}{4}\right), \qquad \sin(\beta) = \exp\left(-\frac{\pi\, æ_b^2}{4}\right), \qquad \sin(\gamma) = \exp\left(-\frac{\pi\, æ_c^2}{4}\right),$$

$$\Psi_i = \int_{\theta_i}^{a} h_k d\theta + \varphi_a, \quad \Psi_f = \int_{c}^{\theta_f} h_k d\theta + \varphi_c, \quad \Psi_{ab} = \int_{a}^{b} h_k d\theta + \varphi_a + \varphi_b, \quad \Psi_{bc} = \int_{b}^{c} h_k d\theta + \varphi_b + \varphi_c$$

The condition of depolarization compensation (3 times crossing)

The case of SAI-flip $\left(M_{11}=0,\ J_f=-J_i\right)$

$$\begin{cases}\cos u\ \cos\beta\cos(\alpha+\gamma)+\cos v\sin\beta\sin(\alpha+\gamma)=0\\ \sin u\ \cos\beta\cos(\alpha-\gamma)-\sin v\sin\beta\sin(\alpha-\gamma)=0\end{cases}$$

The SAI keep sign $\left(M_{12}=0,\ J_f=J_i\right)$

$$\begin{cases}\cos u\ \cos\beta\sin(\alpha+\gamma)-\cos v\sin\beta\cos(\alpha+\gamma)=0\\ \sin u\ \cos\beta\sin(\alpha-\gamma)+\sin v\sin\beta\cos(\alpha-\gamma)=0\end{cases}$$

$$u=\frac{1}{2}\left(\Psi_{ab}+\Psi_{bc}\right),\qquad v=\frac{1}{2}\left(\Psi_{ab}-\Psi_{bc}\right)$$

Depolarization compensation conditions stable in spin detuning deviation

$$\frac{\partial M_{11}}{\partial\varepsilon_k}=0\quad\text{or}\quad\frac{\partial M_{12}}{\partial\varepsilon_k}=0\qquad\frac{\partial\Psi_{ab}}{\partial\varepsilon_k}=\frac{\partial}{\partial\varepsilon_k}\left(\int_a^b h_k d\theta\right)=b-a>0\qquad\frac{\partial\Psi_{bc}}{\partial\varepsilon_k}=b-c<0$$

Obtain condition on the time of second crossing $$b=\frac{a\sin2\alpha+c\sin2\gamma}{\sin2\alpha+\sin2\gamma}$$

SAI-flip $\left(J_f=-J_i\right)$

$$\begin{cases}\beta=\alpha+\gamma-\dfrac{\pi}{2}\\ \Psi_{ab}=2\pi m_1\\ \Psi_{bc}=2\pi m_2\end{cases}\quad\textbf{or}\quad\begin{cases}\beta=\dfrac{\pi}{2}-\alpha-\gamma\\ \Psi_{ab}=2\pi m_1+\pi\\ \Psi_{bc}=2\pi m_2+\pi\end{cases}$$

The SAI keeps sign $\left(J_f=J_i\right)$

$$\begin{cases}\beta=\gamma-\alpha\\ \Psi_{ab}=2\pi m_1+\pi\\ \Psi_{bc}=2\pi m_2\end{cases}\quad\textbf{or}\quad\begin{cases}\beta=\alpha-\gamma\\ \Psi_{ab}=2\pi m_1\\ \Psi_{bc}=2\pi m_2+\pi\end{cases}$$

Depolarization compensation condition stable in resonance strength

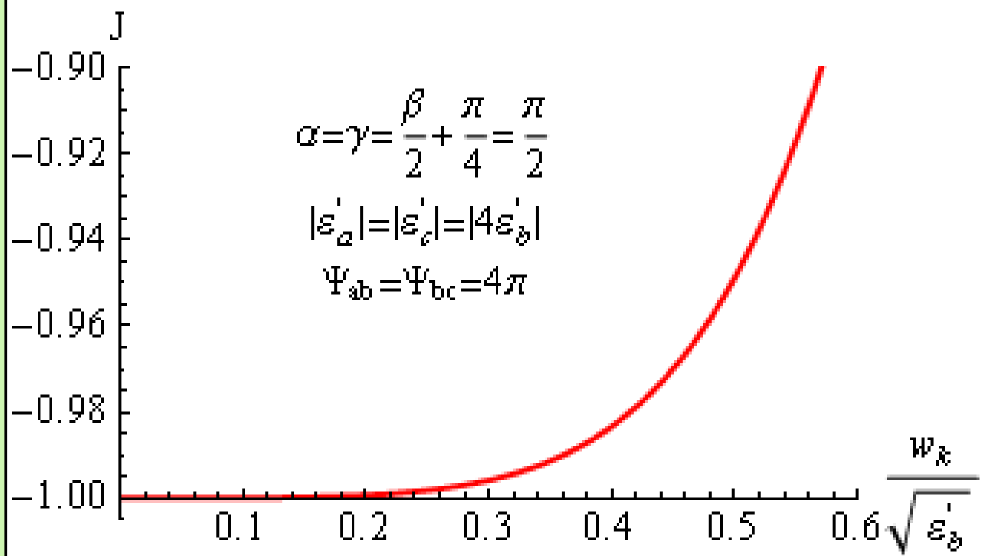
SAI-flip $\left(J_f = -J_i\right)$

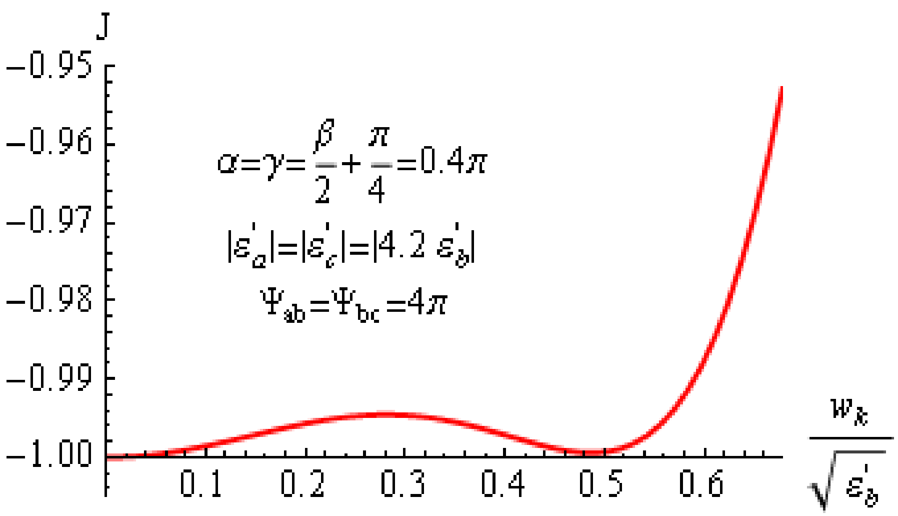
$$\beta = \alpha + \gamma - \frac{\pi}{2};\ \ \Psi_{ab} = 2\pi\, m_1;\ \ \Psi_{bc} = 2\pi\, m_2$$

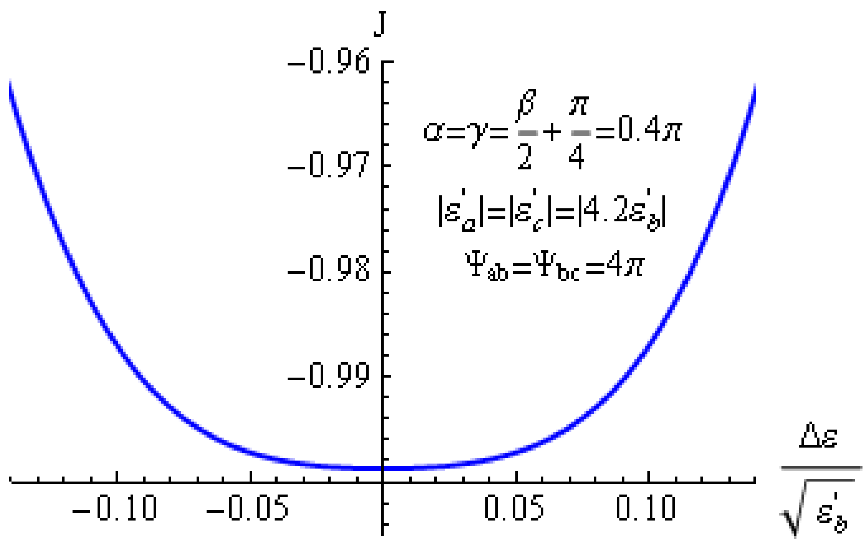
$$\alpha \to \frac{\pi}{2};\ \beta \to \frac{\pi}{2};\ \gamma \to \frac{\pi}{2}$$

$$b = \sqrt{|\varepsilon'_b|}\left(\frac{a}{\sqrt{|\varepsilon'_a|}} + \frac{c}{\sqrt{|\varepsilon'_c|}}\right),$$

$$\frac{1}{\sqrt{|\varepsilon'_b|}} = \frac{1}{\sqrt{|\varepsilon'_a|}} + \frac{1}{\sqrt{|\varepsilon'_c|}}$$







Summary

•We found simple depolarization compensation conditions for cases of multiple resonance crossing in analytical form.

•This method allows one to significantly reduce necessary resonance crossing rate in comparison with known methods.

•This method can also be applied to intrinsic resonances.

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A JUMP IN SPIN PRECESSION FREQUENCY AS A METHOD TO PASS SPIN RESONANCES

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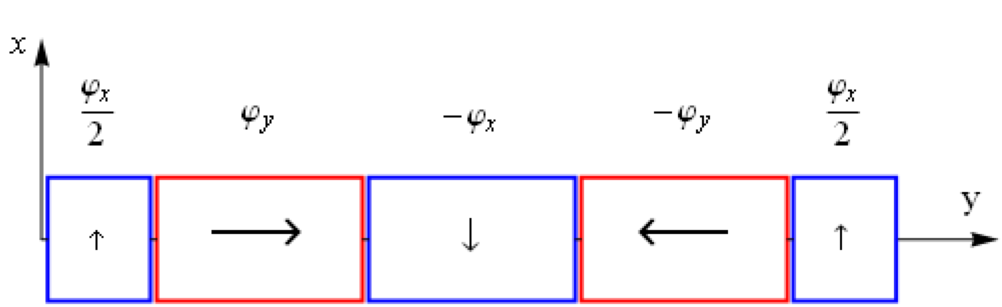
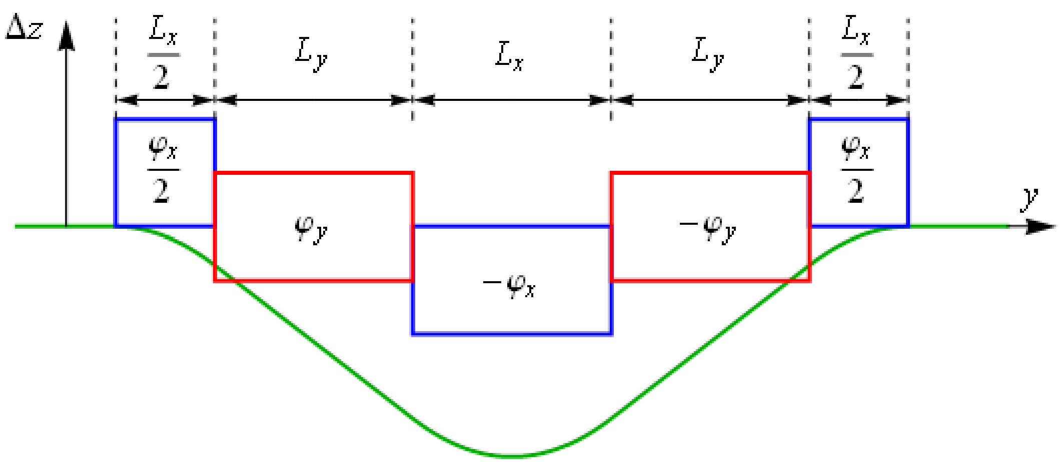
Scheme for shifting spin tune



φ_x – spin rotation angle in radial dipole

φ_y – spin rotation angle in solenoid

Spin tune shift: $\Delta\nu = \dfrac{\varphi_x \varphi_y}{2\pi}$



L_x – length of central dipole

L_y – length of one solenoid

Total length of insertion:

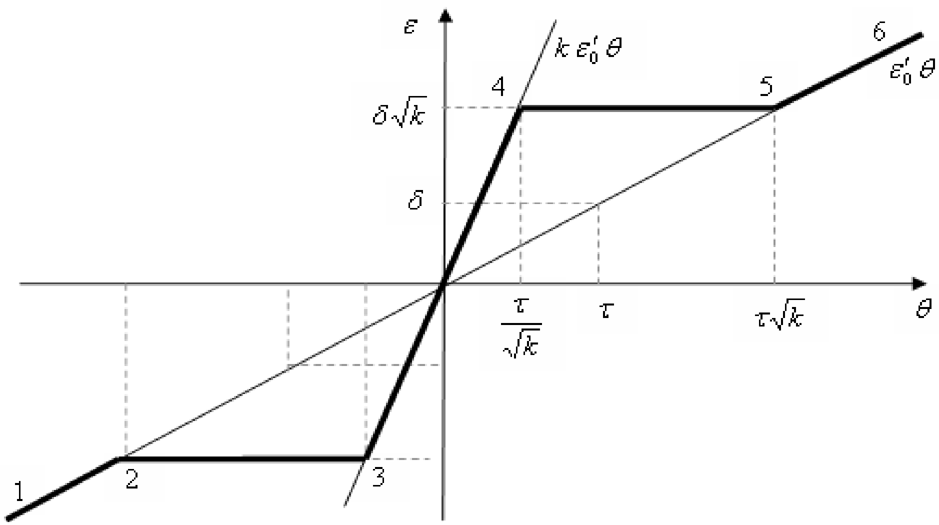
$$L_{tot} = 2(L_x + L_y)$$

Orbit's maximum vertical deviation:

$$|\Delta z_{\max}| = \frac{|\varphi_x|}{4\nu}\left(2L_y + L_x\right)$$

Betatron tune shift: $\Delta\nu_b = \dfrac{\pi}{2(1+G)^2}\dfrac{\beta}{L_y}\varphi_y^2$

Example of crossing spin resonance by spin tune jump for Nuclotron, JINR (Dubna)



$\varepsilon = \nu - \nu_k$ spin detuning

$$k = \varepsilon' / \varepsilon_0' = 100$$

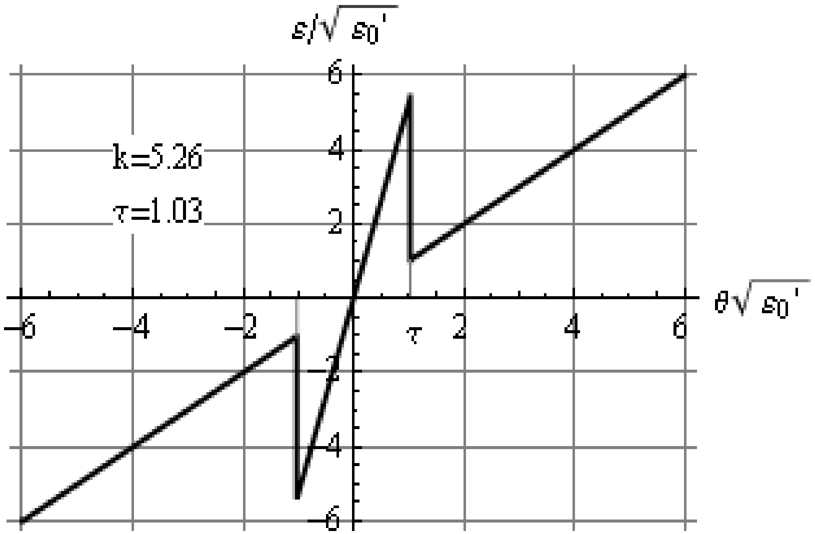
$$w_k = \sqrt{\varepsilon_0' / \pi} = 1.5 \times 10^{-3}$$

$$D_0 = 100\,\% \qquad D_k = D_k / k = 1\%$$

$$\Delta t = \frac{\tau}{\sqrt{k}\,\omega_0}$$

$L_x = 0.7$ m, $L_y = 1.0$ m, $L_{tot} = 3.4$ m, $\Delta\nu = 3 \cdot 10^{-2}$ $\left(\varepsilon_0' = 7 \cdot 10^{-6}\right)$, $\Delta t \sim 5$ μs						
k	γ	φ_x, $[rad]$	φ_y, $[rad]$	H_x, $[T]$	H_y, $[T]$	Δz_{max}, $[cm]$
1	14	0.6	0.3	1.6	4.7	1.7
	7	0.3	0.6	0.8	4.7	1.7
	2	0.2	1.0	0.4	1.9	3.8
3	14	0.2	0.3	0.50	4.7	0.5
	7	0.1	0.6	0.26	4.7	0.5
	2	0.06	1.0	0.13	1.9	1.2

Example of crossing spin resonance using depolarization compensation technique for Nuclotron, JINR (Dubna)



Spin detuning change pattern used in this example

$$D_k \to 0$$

$L_x = 0.7\ m,\quad L_y = 1.0\ m,\quad L_{tot} = 3.4\ m,\quad \Delta\nu = 10^{-2}\ \left(\varepsilon_0' = 7\cdot 10^{-6}\right),\quad \Delta t \sim 50\ \mu s$						
k	γ	φ_x, $[rad]$	φ_y, $[rad]$	H_x, $[T]$	H_y, $[T]$	Δz_{max}, $[cm]$
1	14	0.2	0.3	0.52	4.7	0.6
	7	0.1	0.6	0.26	4.7	0.6
	2	0.06	1.0	0.13	1.9	1.2
3	14	0.07	0.3	0.17	4.7	0.2
	7	0.035	0.6	0.086	4.7	0.2
	2	0.02	1.0	0.045	1.9	0.4

Conclusions

•Technique for shifting spin tune would allow one to preserve polarization at resonance crossing without substantially perturbing beam's betatron motion.

•Depolarization compensation technique allows one to substantially reduce required magnetic field integrals in insertion, which controls spin tune shift.