SUPERRADIANT EMISSION SCHEME, FREE ELECTRON SPIN-FLIP EMISSION OF RADIATION (FESFER)

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OUTLINE

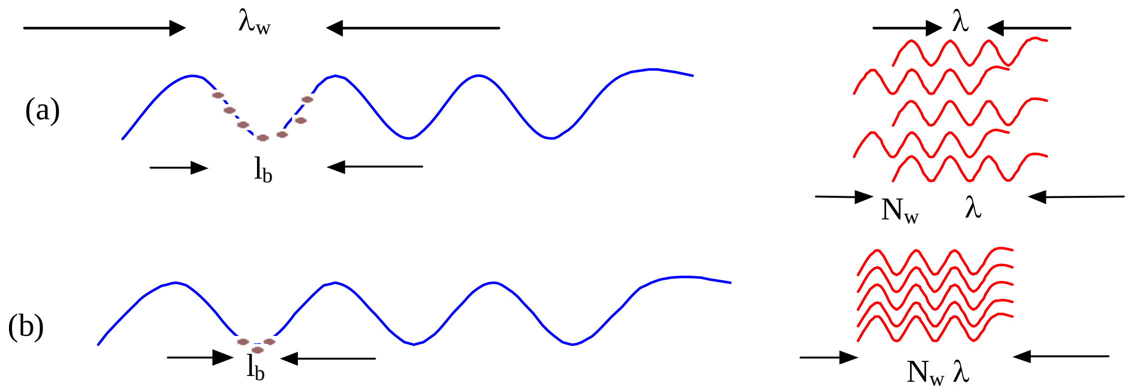
1. SUPERRADIANT* EMISSION – RADIATION SCHEMES AND GENERAL FORMULATION.

2. STIMULATED–SUPERRADIANCE FEL OSCILLATOR.

3. FREE ELECTRON SUPERRADIANT SPIN–FLIP EMISSION OF RADIATION.

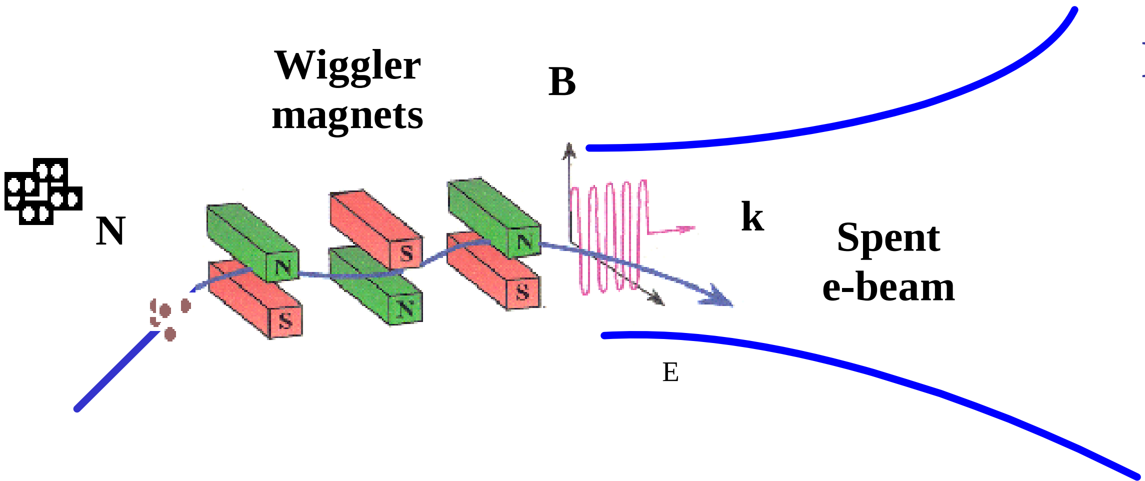
*R. H. Dicke, Phys. Rev. **93**, 99(1954)

(a) Spontaneous emission
(b) Superradiant emission



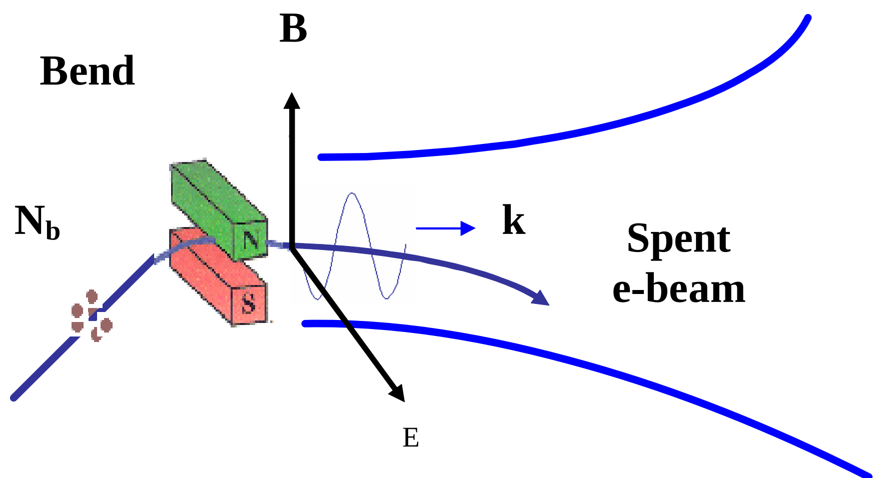
PB-FEL

I. Schnitzer, A. Gover, "The Prebunched FEL…", NIMPR A237, 124 (1985)



CSR

G.L. Carr et al, "High power THz radiation…", Nature 420, 153 (2002)



Other superradiant emission schemes

- Coherent Smith-Purcell
- Cerenkov Radiation
- Transition Radiation
- Cyclotron Resonant Emission (CRE)

Excitation of modes (Waveguide or Free Space)

$$\mathbf{E}(\mathbf{r},\omega)=\sum_{\pm q} C_q(z,\omega)\tilde{\mathbf{E}}_q(\mathbf{r})$$

$$\mathbf{H}(\mathbf{r},\omega)=\sum_{\pm q} C_q(z,\omega)\tilde{\mathbf{H}}_q(\mathbf{r})$$

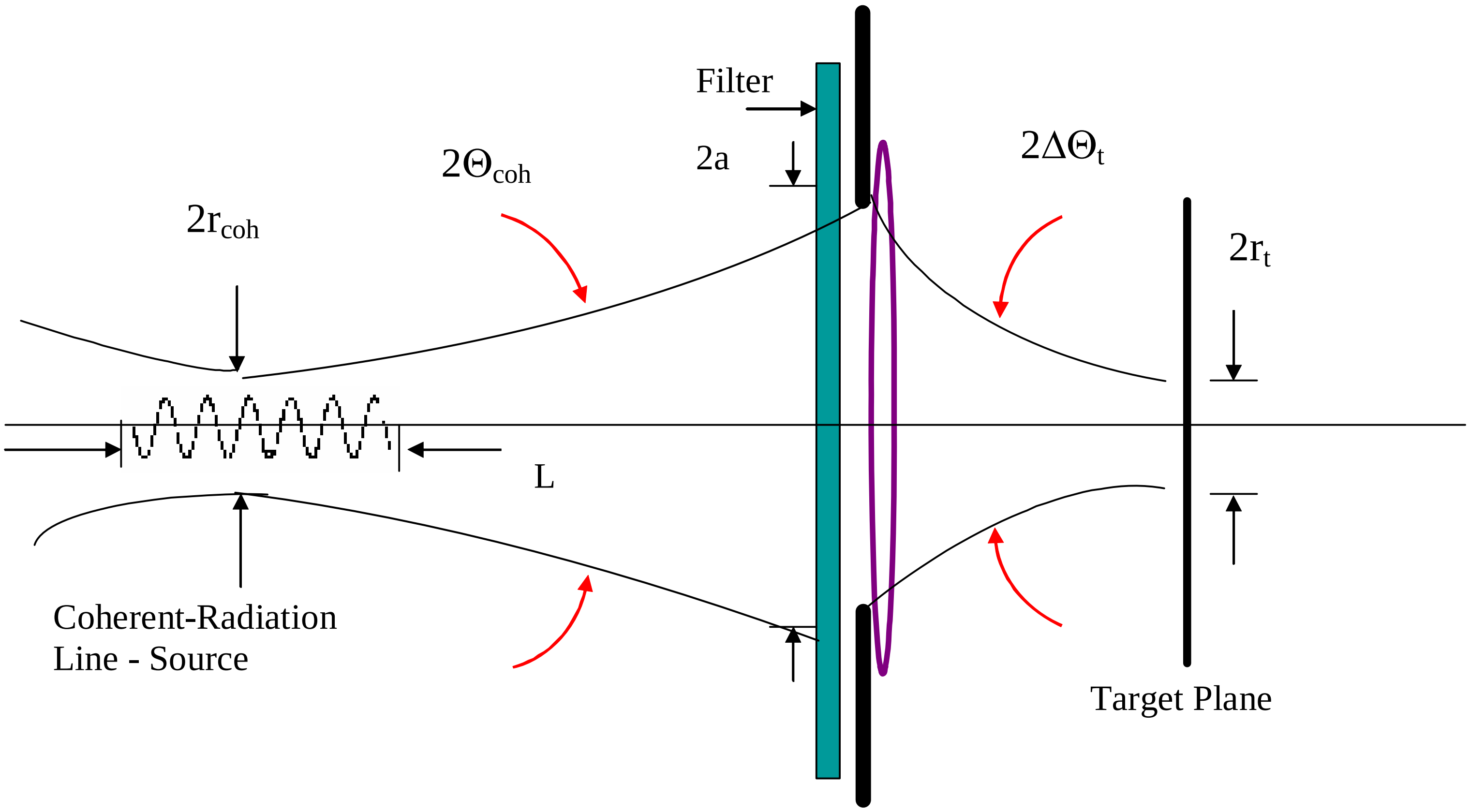
$$C_q^{out}(\omega)-C_q^{in}(\omega)=\sum_{j=1}^{N}\Delta C_{qj}=-\frac{1}{4\mathrm{P}_q}\sum_{j=1}^{N}\Delta \mathrm{W}_{qj}$$

$$\Delta \mathrm{W}_{qj}=-\mathrm{e}\int_{-\infty}^{\infty}\mathbf{v}_j(t)\cdot\tilde{\mathbf{E}}_q^{\,*}(\mathbf{r}_j(t))e^{i\omega t}dt$$

$$\frac{d\mathrm{W}_q}{d\omega}=\frac{2}{\pi}\mathrm{P}_q\left|C_q^{out}(\omega)\right|^2$$

$$\tilde{\mathbf{E}}_q=\tilde{\mathbf{E}}_{00}=\frac{w_0}{w(z)}\exp\left[-\frac{x^2+y^2}{w^2(z)}-ik\frac{x^2+y^2}{2R(z)}+i\varphi+ikz\right]$$

Useful Coherent Power from a spatially coherent source



Spatially Coherent Spectral Power

$$C_q^{out}(\omega) = C_q^{in}(\omega) + \Delta C_{qe}^{(0)}(\omega) \sum_{j=1}^{N} e^{i\omega t_{oj}} + \sum_{j=1}^{N} \Delta C_{qj}^{st}$$

$$\frac{dW_q}{d\omega} = \frac{2}{\pi} P_q \left\{ \left| C_q^{in}(\omega) \right|^2 + \right.$$

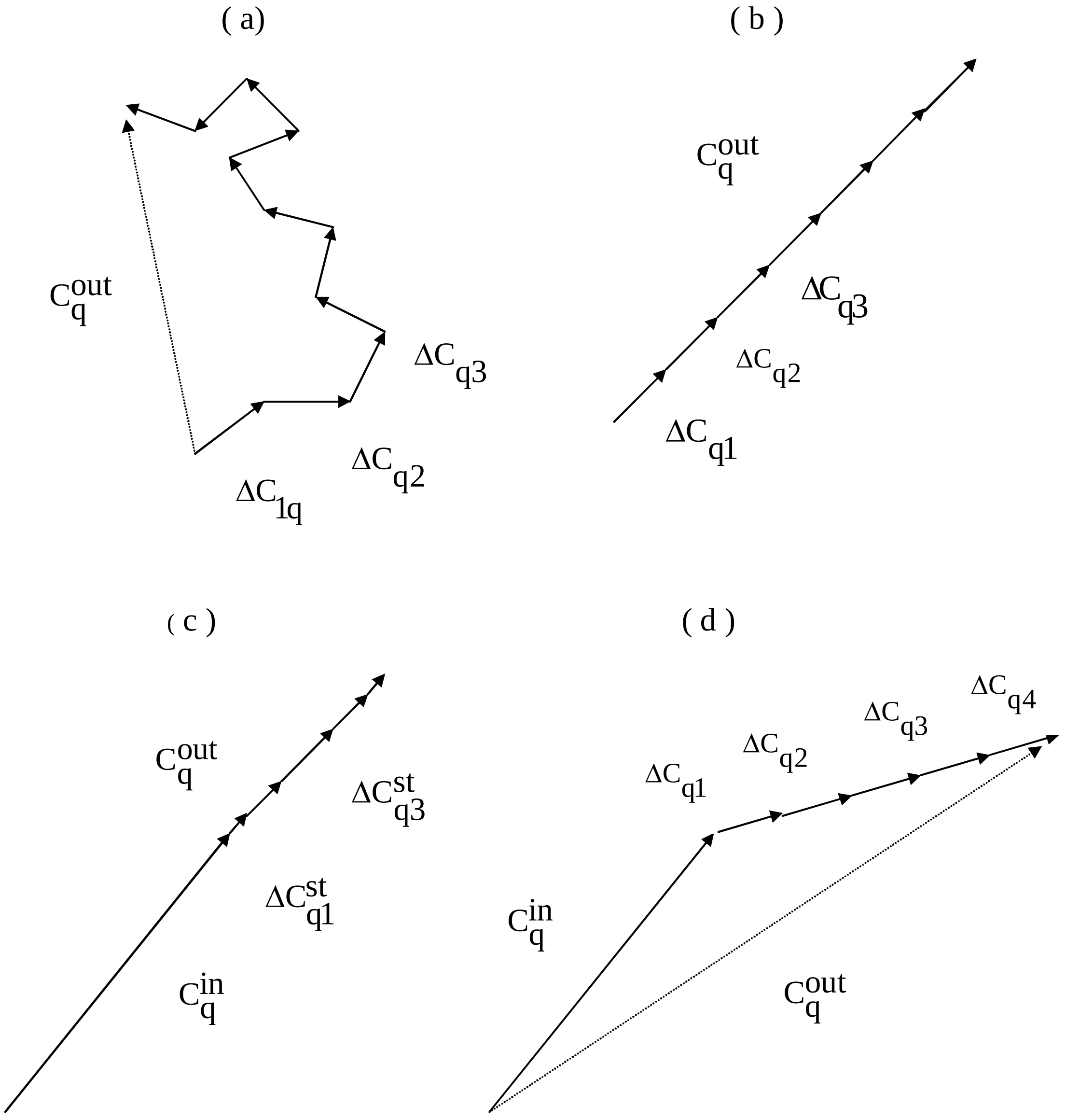
$$+ \left| \Delta C_{qe}^{(0)}(\omega) \cdot \sum_{j=1}^{N} e^{i\omega t_{oj}} \right|^2 +$$

$$+ \left[C_q^{in*}(\omega) \cdot \Delta C_{qe}^{(0)}(\omega) \cdot \sum_{j=1}^{N} e^{i\omega t_{oj}} + \text{c.c.} \right] +$$

$$\left. + \left[C_q^{in*}(\omega) \cdot \sum_{j=1}^{N} \Delta C_{qj}^{st}(\omega) + \text{c.c.} \right] \right\} \equiv$$

$$\equiv \left(\frac{dW_q}{d\omega} \right)_{in} + \left(\frac{dW_q}{d\omega} \right)_{sp/SR} + \left(\frac{dW_q}{d\omega} \right)_{ST-SR} + \left(\frac{dW_q}{d\omega} \right)_{st}$$

Superposition of radiation wavepackets:
a) Spontaneous emission b) Superradiance
c) Stimulated emission d) Stimulated superradiance

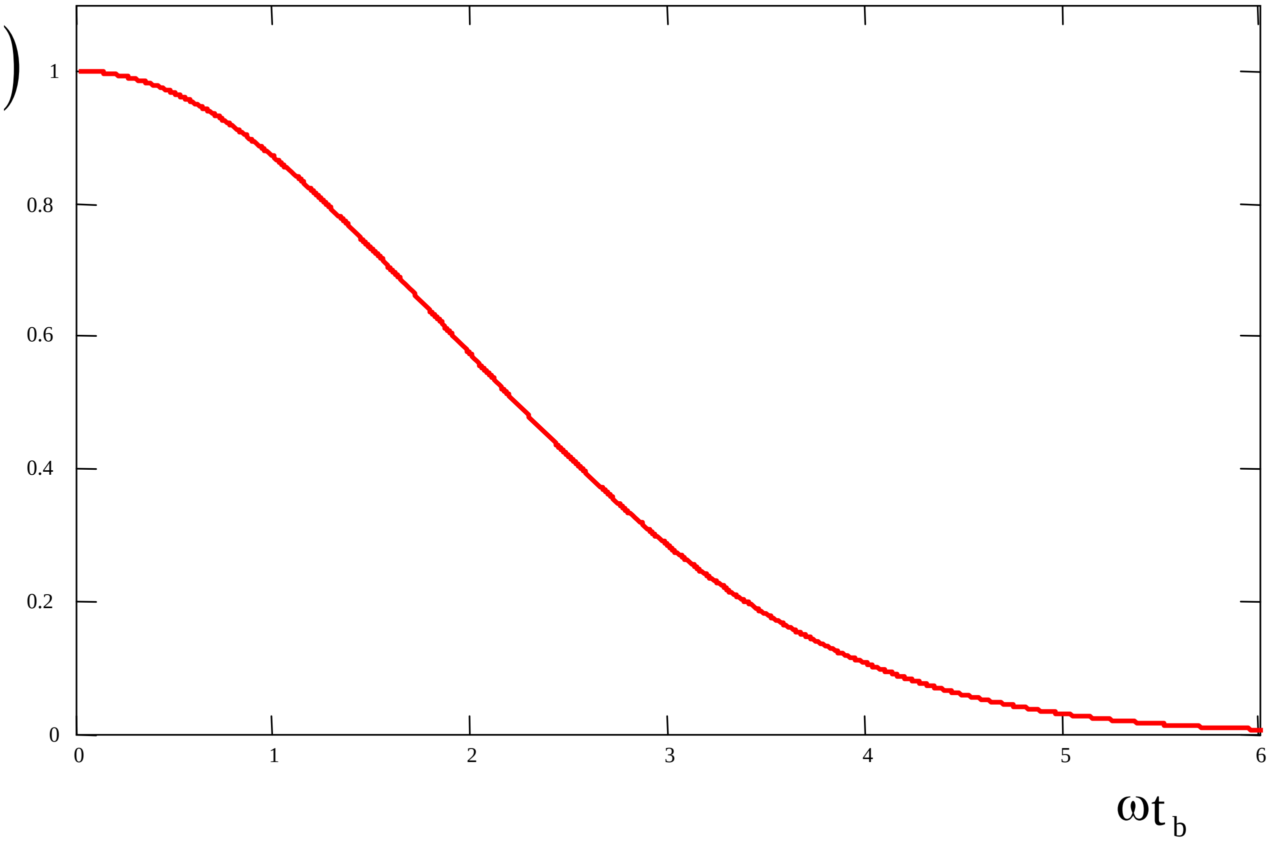


The Bunch Form-Factor

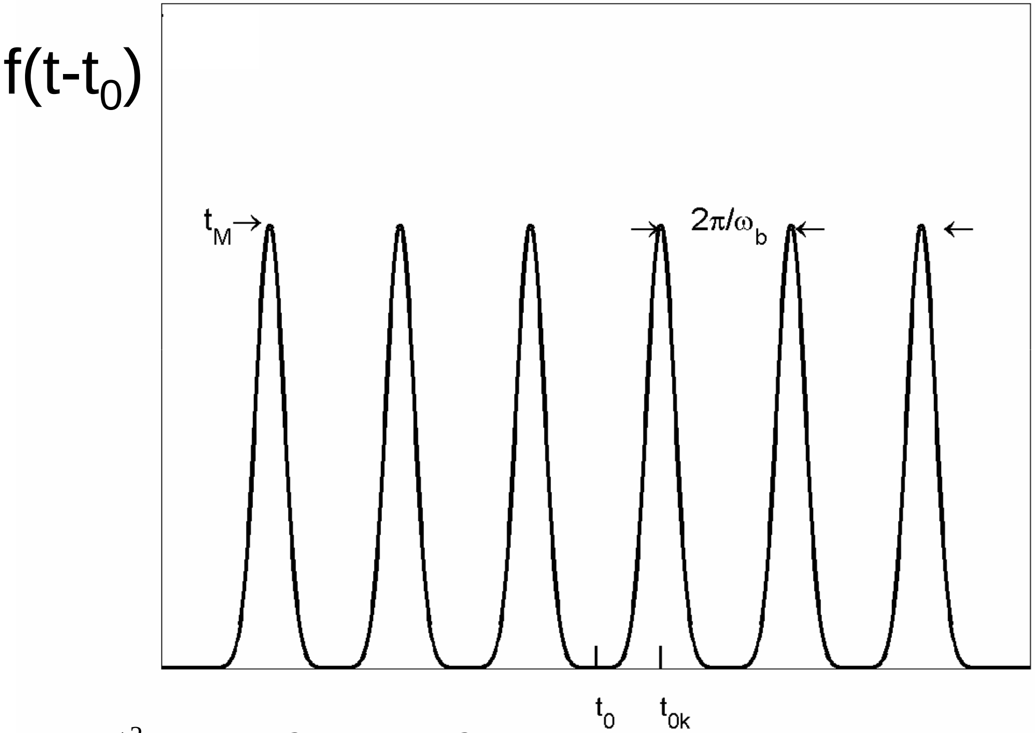
$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{1}{8\pi} \cdot \frac{\left|\Delta W_{qe}^{(0)}\right|^2}{P_q} \left|M_b(\omega)\right|^2 N^2 \qquad M_b(\omega) = \int_{-\infty}^{\infty} f\left(t_0'\right) e^{i\omega t_0'} dt_0'$$

for a Gaussian e-beam bunch distribution $f\left(t_0\right) = \exp\left(-t_0^2 / t_b^2\right) / \left(\sqrt{\pi} t_b\right)$:

$$\left|M_b(\omega)\right|^2 = \exp\left(-\omega^2 t_b^2 / 2\right)$$



The Macropulse Form-Factor for a Pulse Composed of a Periodic Train of N_M Micro-Bunches

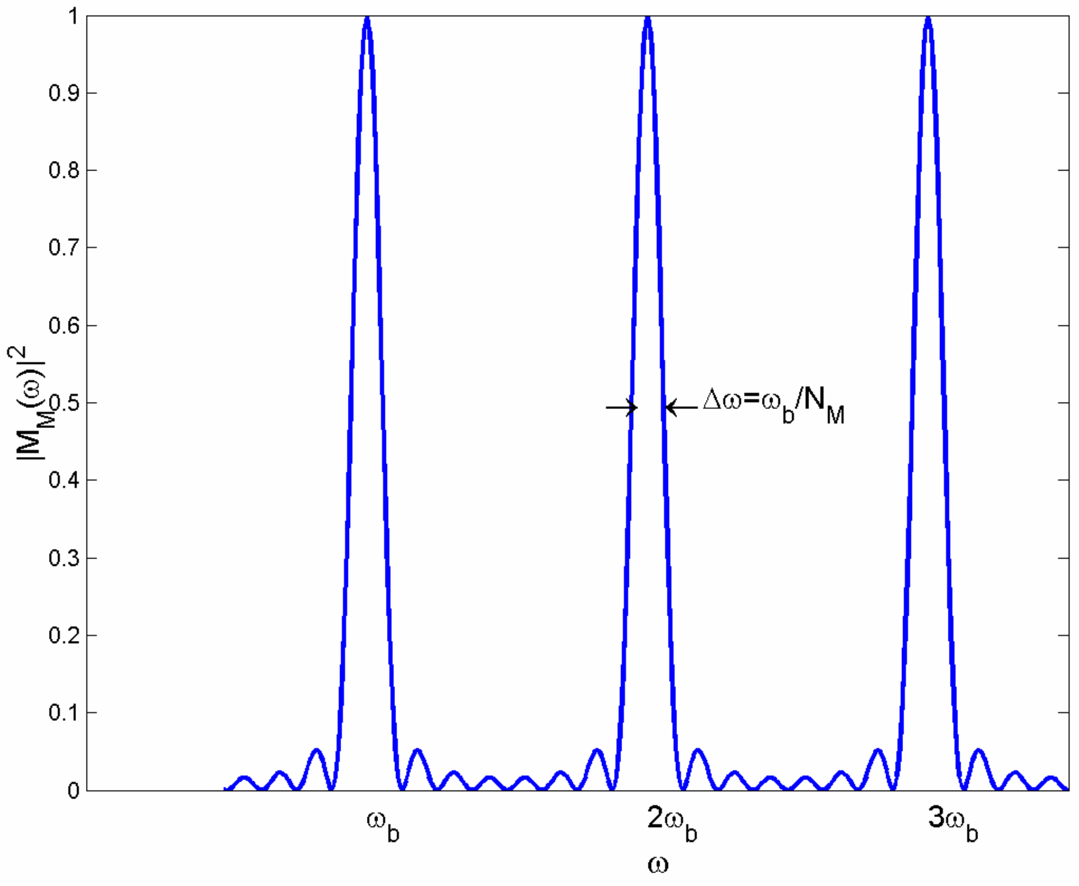


$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{N^2}{8\pi P_q}\left|\Delta W_{qe}^{(0)}(\omega)\right|^2 \left|M_b(\omega)\right|^2 \left|M_M(\omega)\right|^2$$

$$M_M(\omega) = \frac{\sin\left(N_M \pi\omega / \omega_b\right)}{N_M \sin\left(\pi\omega / \omega_b\right)}$$

$$\frac{\Delta\omega}{n\omega_b} = \frac{1}{nN_M}$$

The Macropulse Form-Factor Function Drawn for $N_M = 8$



Spatially Coherent Spectral Power

For PB-FEL:

$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{N_b^2 e^2 Z_q}{16\pi}\left(\frac{eB_w}{\beta_z \gamma mc}\right)^2 \frac{L^2}{k_w^2 A_{em}} \left|M_b(\omega)\right|^2 sinc^2(\theta L/2)$$

$$\theta = \omega / \mathrm{v_z} - \mathrm{k_z} - \mathrm{k_w}$$

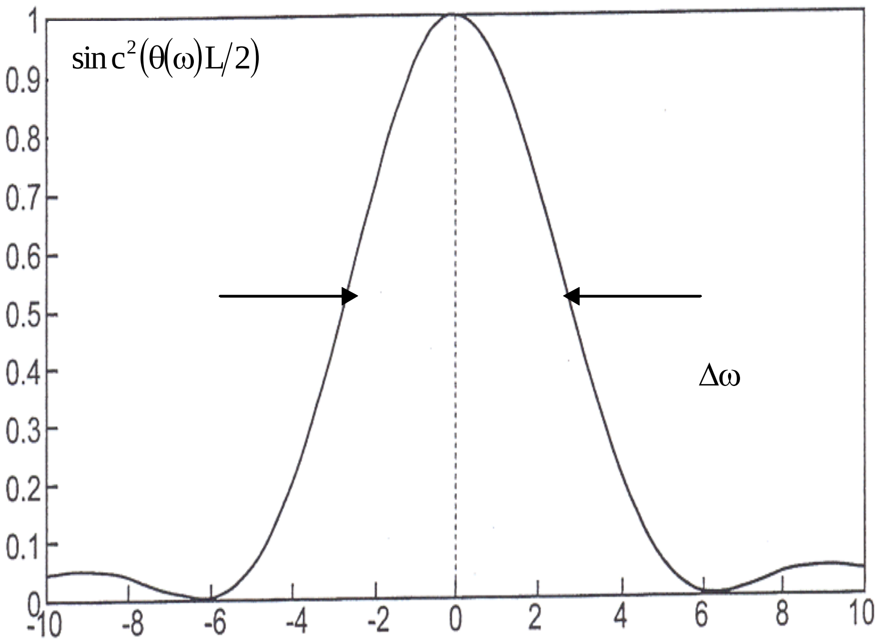
For CSR:

$$\left(\frac{dW_q}{d\omega}\right)_{SR} = \frac{N_b^2 e^2 Z_q}{4\pi}\left(\frac{eB_b}{\beta \gamma mc}\right)^2 \frac{L^4}{A_{em}} M_b(\omega)\left[\frac{d}{d\theta L} sinc(\theta L/2)\right]^2$$

$$\theta = \omega / \mathrm{v_z} - \mathrm{k_z}$$

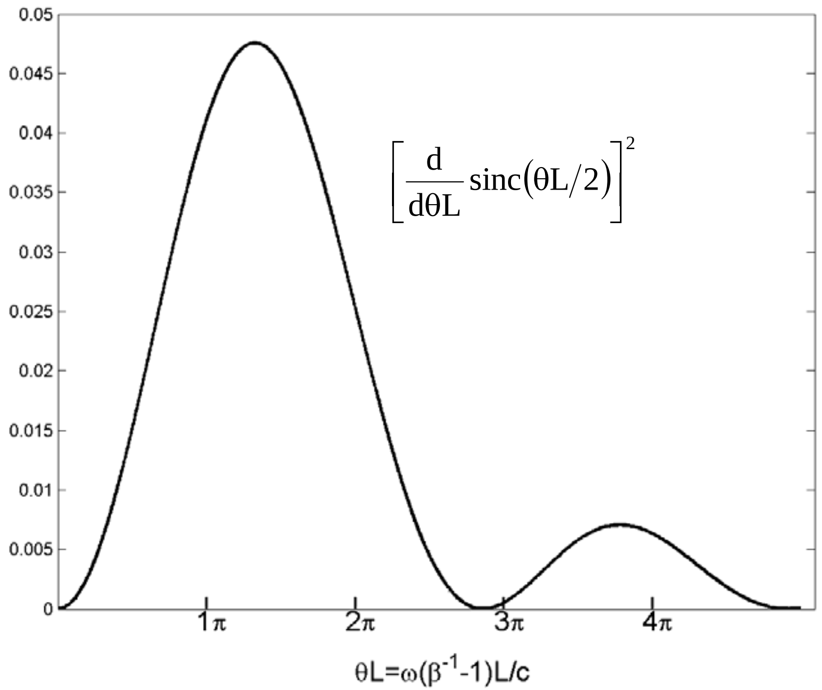
Line-Shape Function

PB-FEL

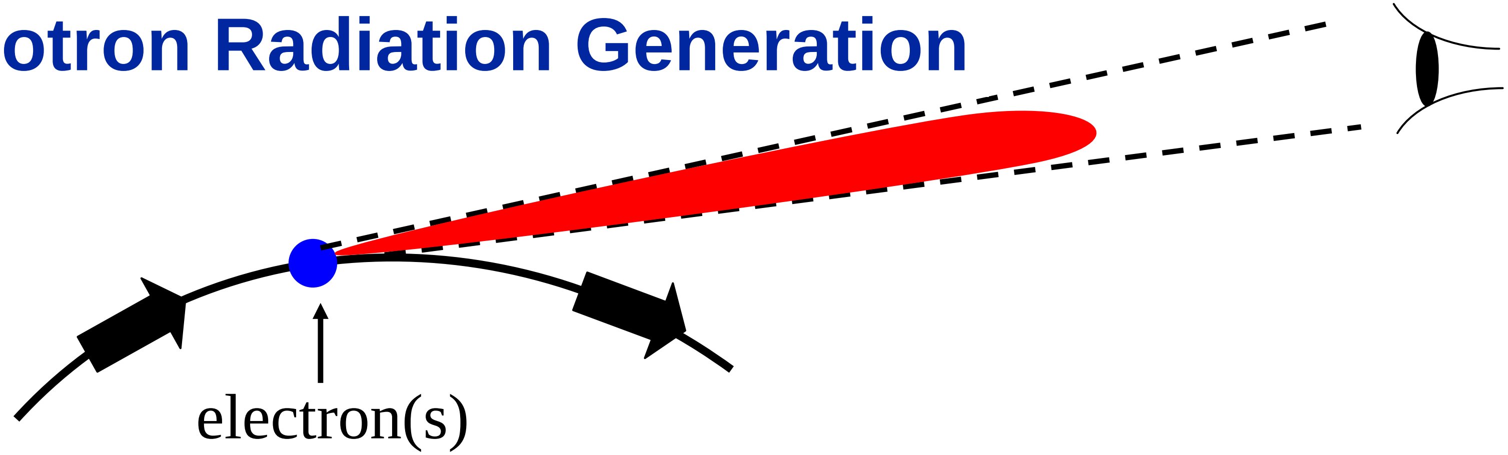


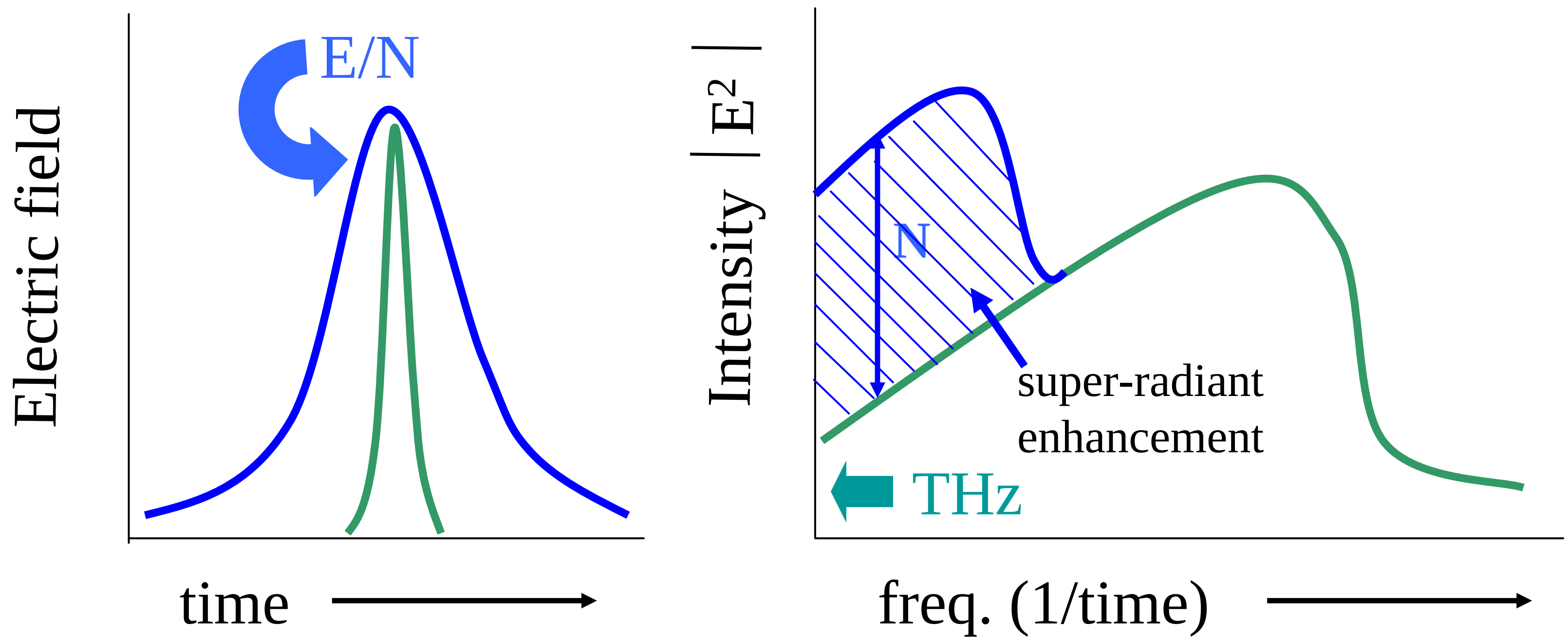
$\theta L = 2\pi(\omega - \omega_0)/\Delta\omega$

CSR



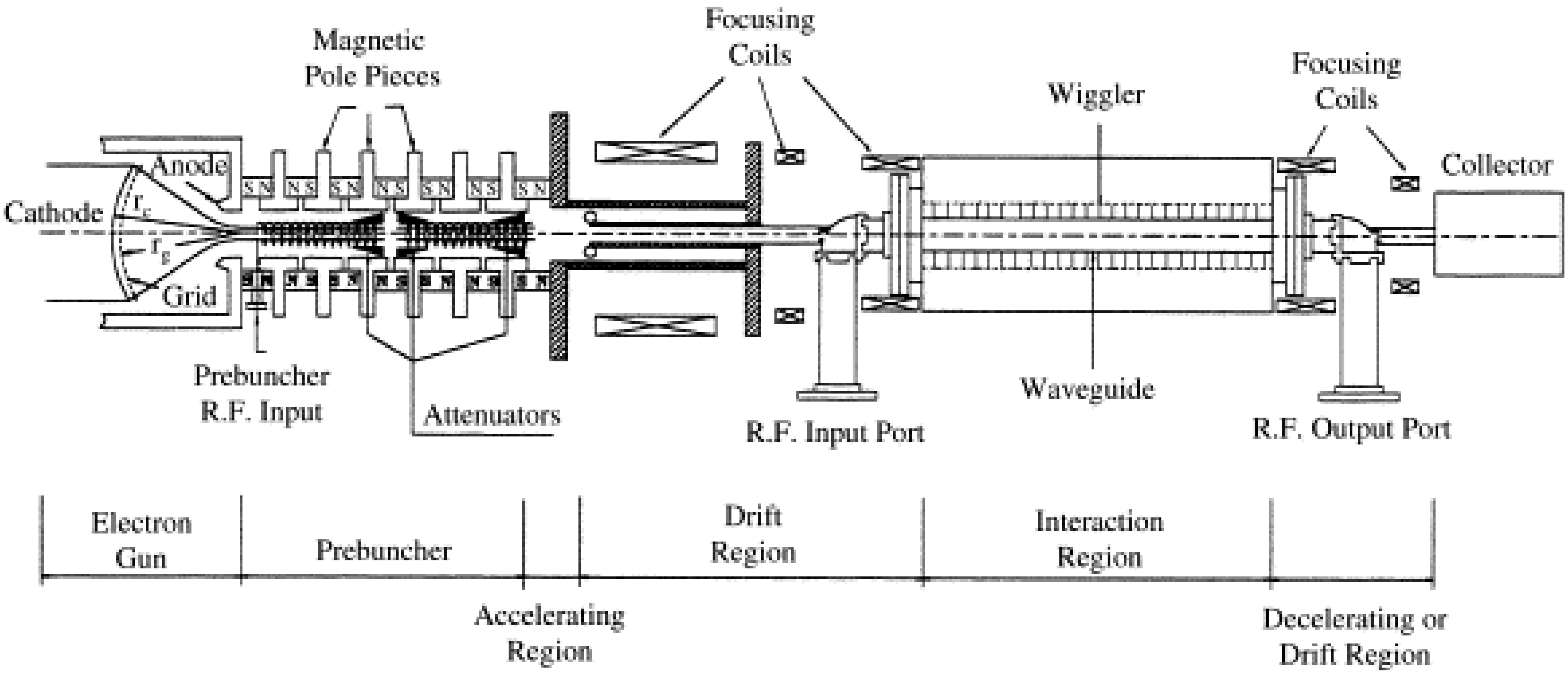
Synchrotron Radiation Generation



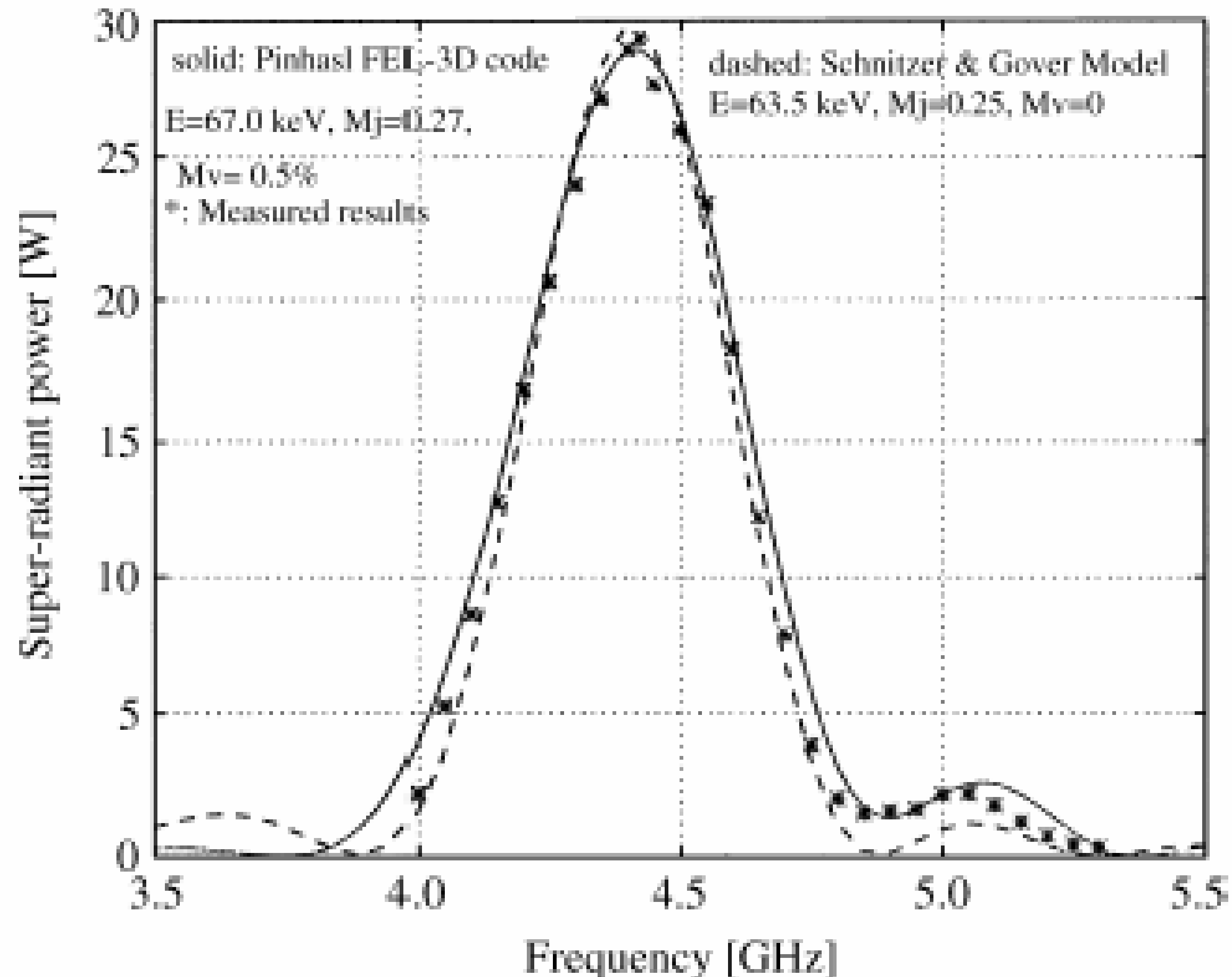


W.D. Duncan and G.P. Williams,"Infra-red Synchrotron Radiation From Electron Storage Rings", Applied Optics 22, 29l4 (1983).

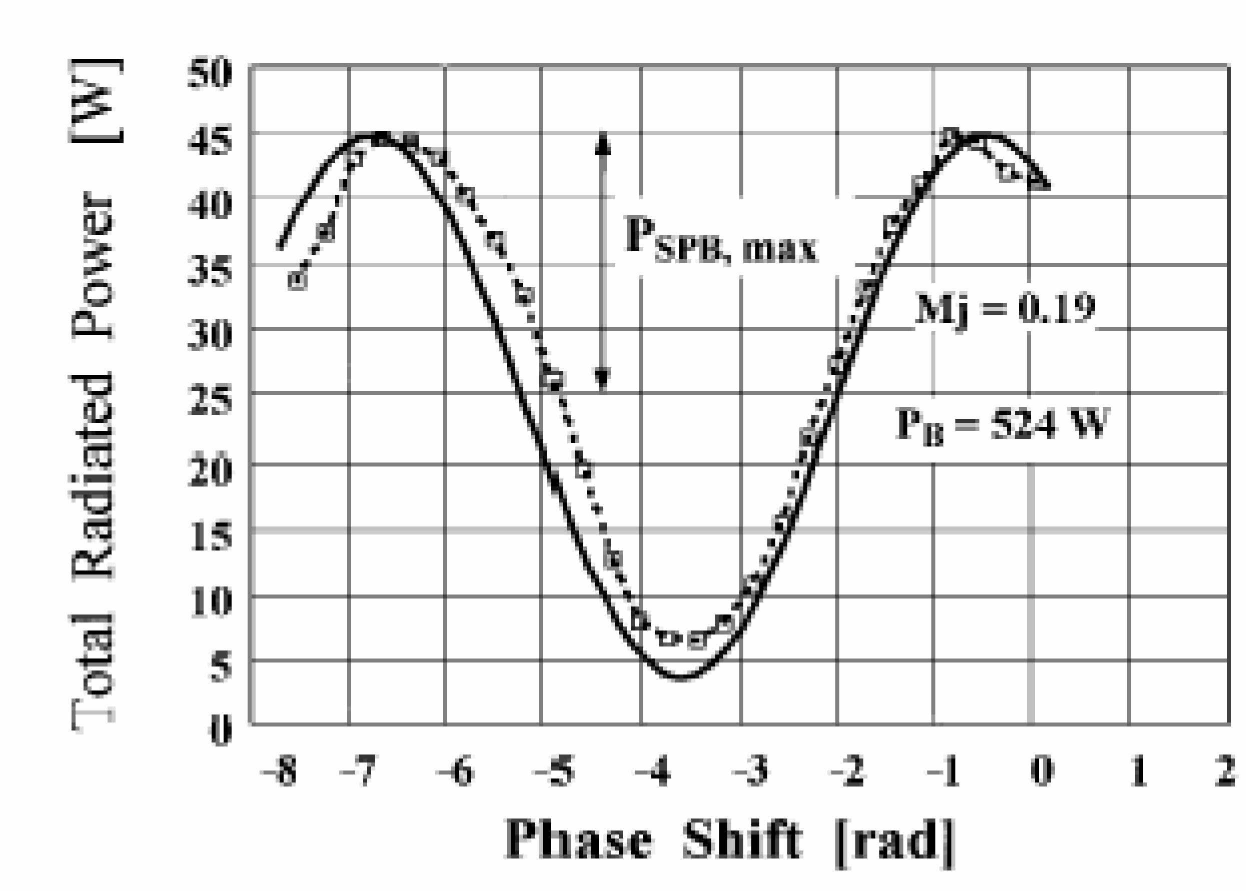
Schematic of the TAU table-top Prebunched-Beam Free Electron Maser.



PB – FEM SUPERRADIANCE MEASUREMENT

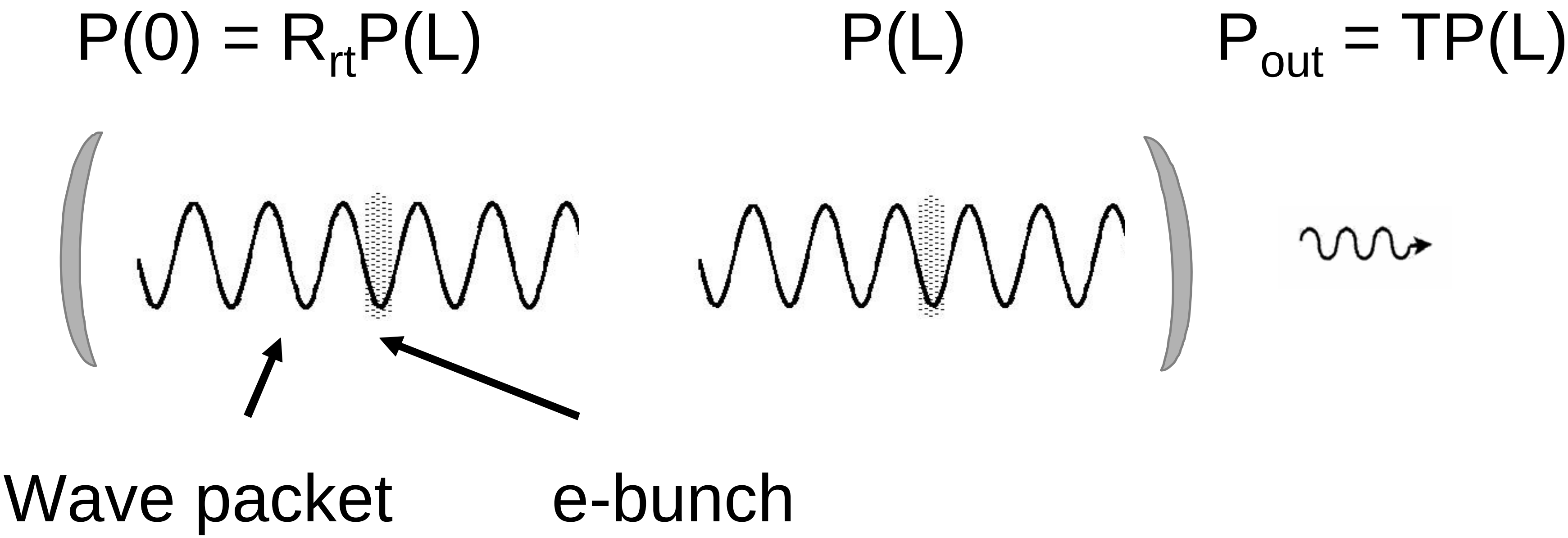


PB-FEL STIMULATED SUPERRADIANCE MEASUREMENT



STIMULATED– SUPERRADIANCE FEL OSCILLATOR

Stimulated-Superradiance PB-FEL Oscillator



The Pendulum Equation model – Saturated PB-FEL

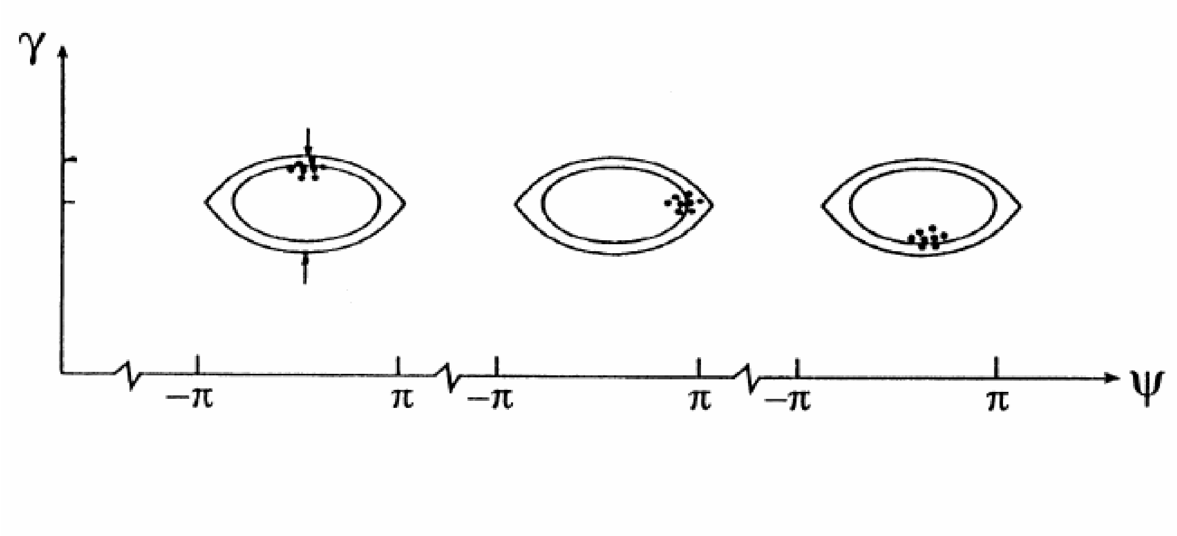
$$\frac{d^2\Psi}{dz^2} = -K_s^{\ 2} \sin\Psi$$

$$\theta = \frac{d\Psi}{dz}$$

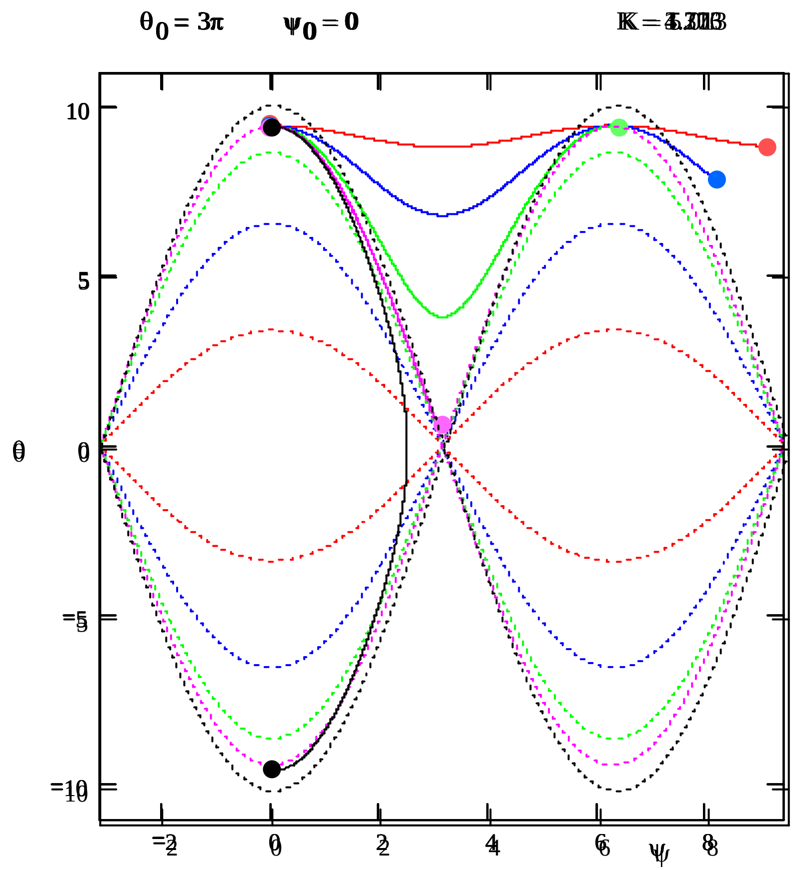
$$K_s = \frac{k\sqrt{a_w a_s / 2}}{\gamma\gamma_z \beta_z^2}$$

$$a_s = \frac{e\left|\tilde{\mathbf{E}}\right|\omega}{mc} = \frac{e\omega}{mc}\left(\frac{2P}{\sqrt{\mu/\varepsilon}A_{em}}\right)^{1/2}$$

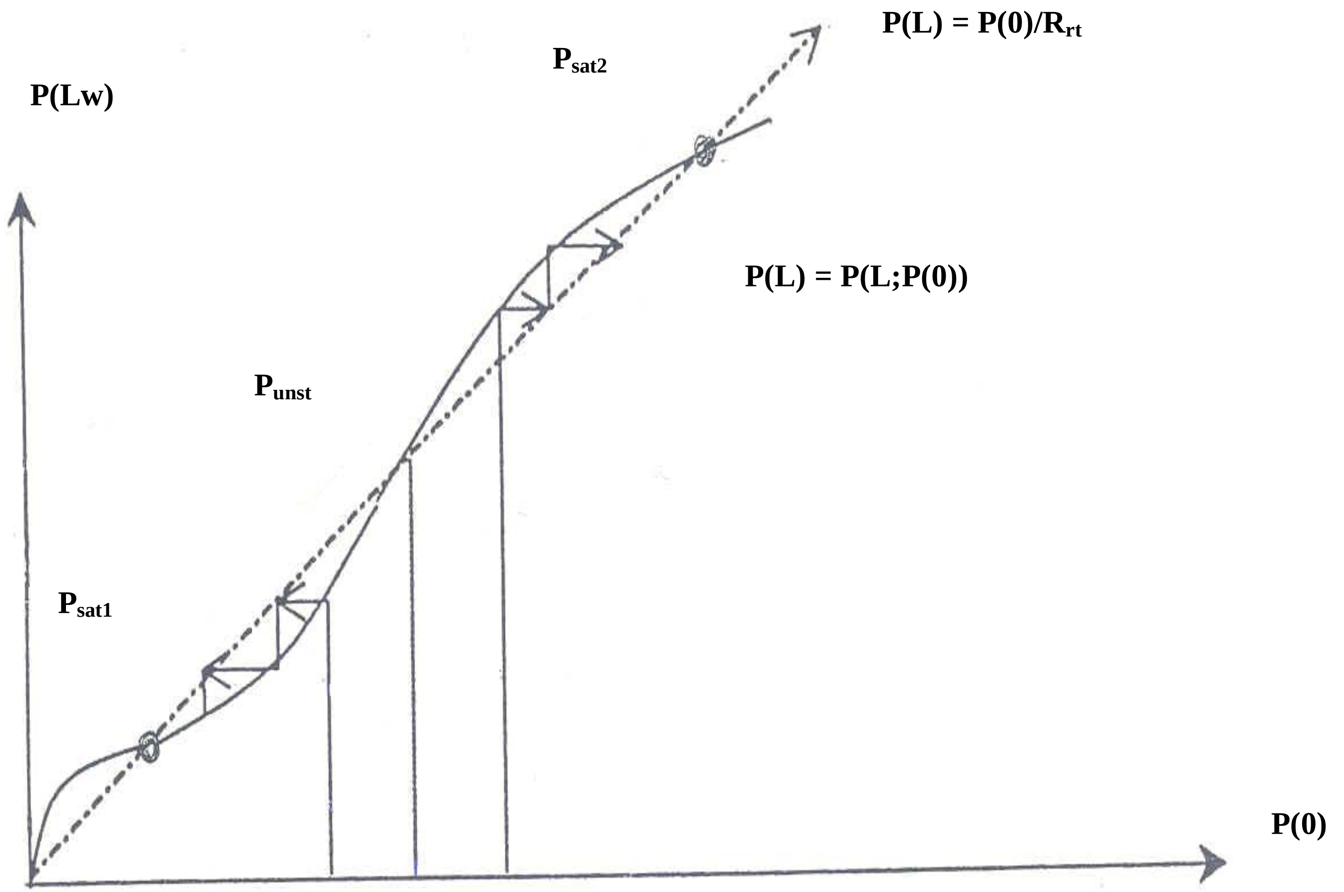
Ultimate Energy Extraction Efficiency Scheme in PB-FEL



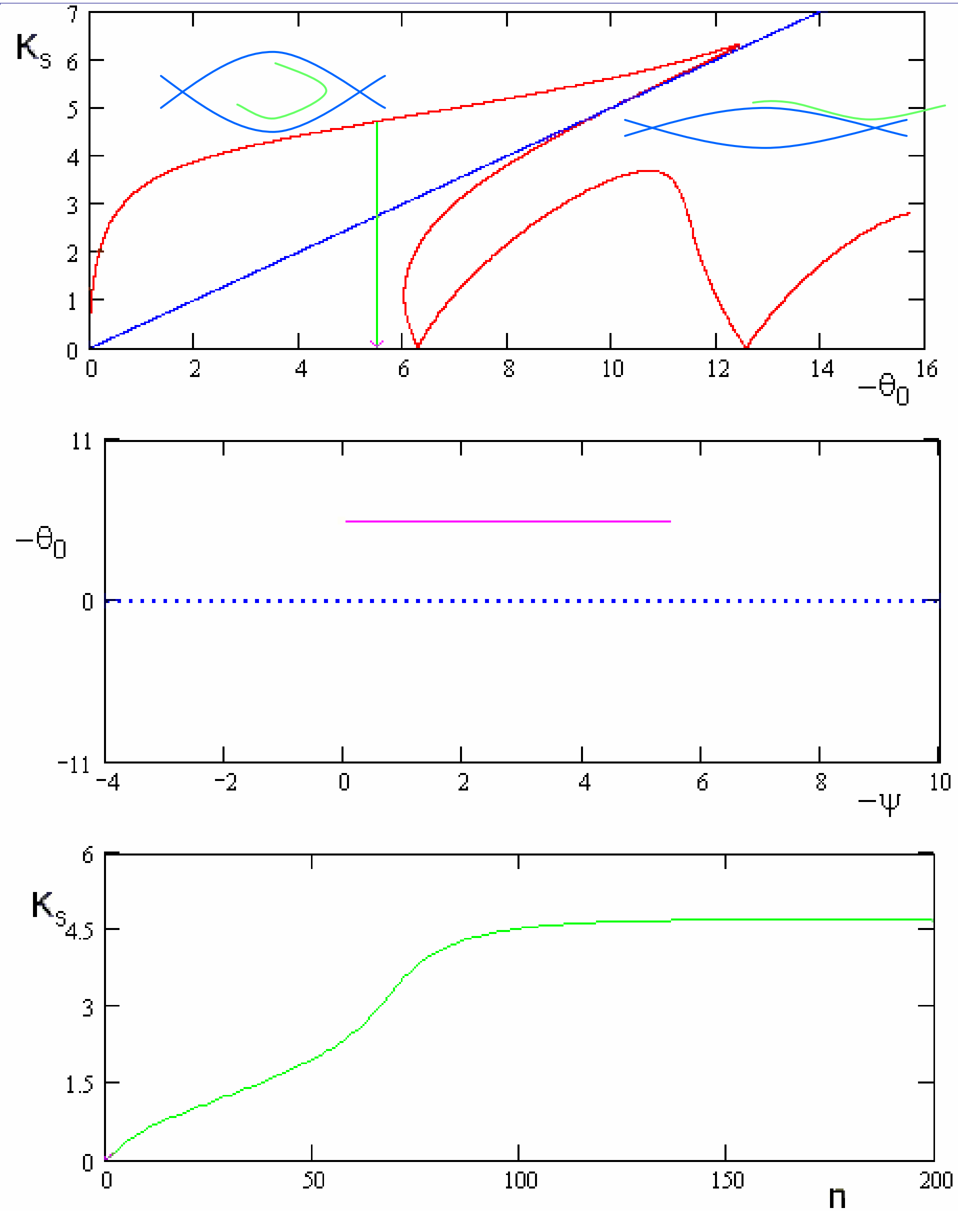
Saturation Dynamics of a Single e-Bunch in a fixed wiggler length for different K_s (stored power ~ K_s^4)



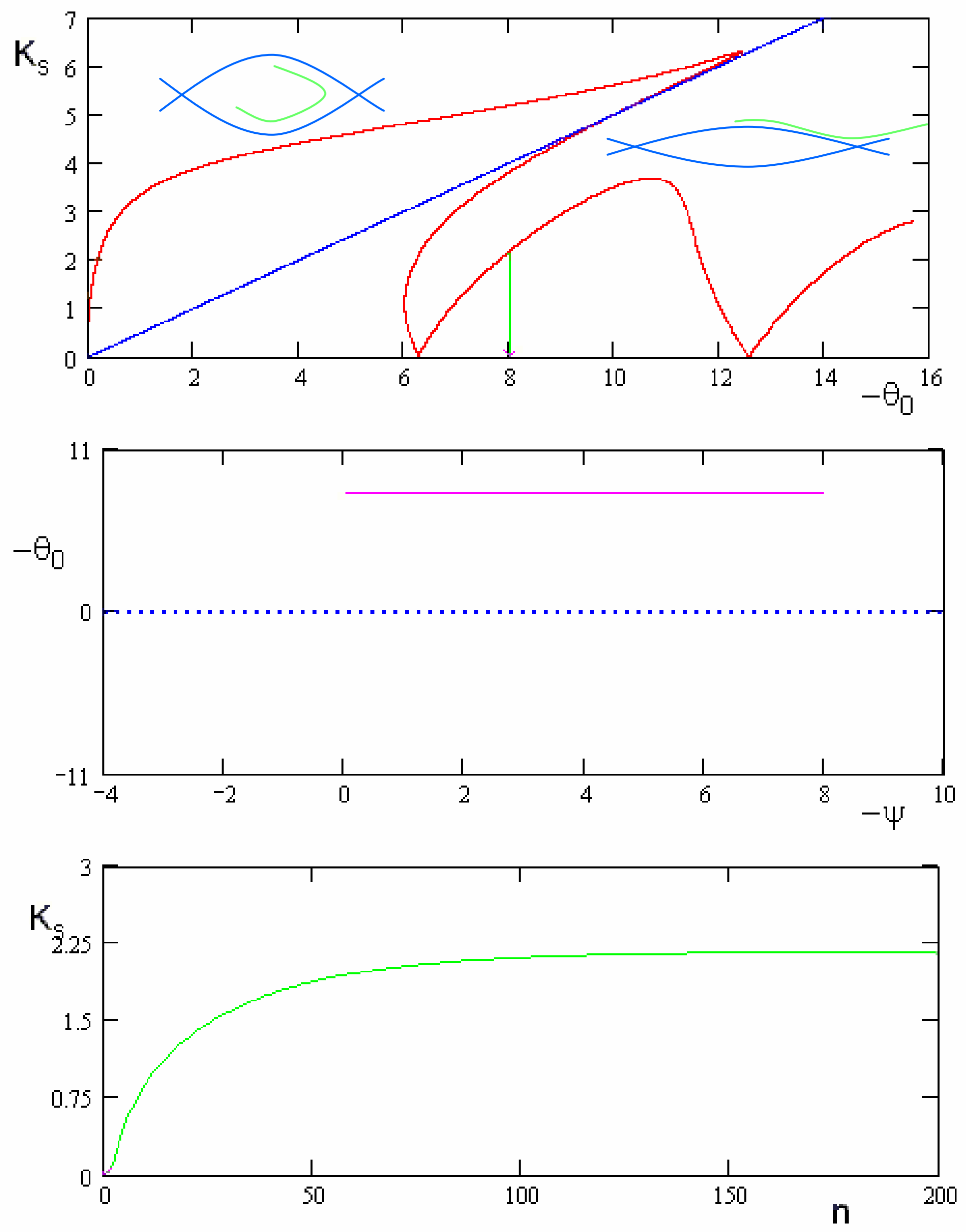
Bistability of PB-FEL Oscillator



CLOSED-TRAJECTORIES SATURATION STABLE POINT

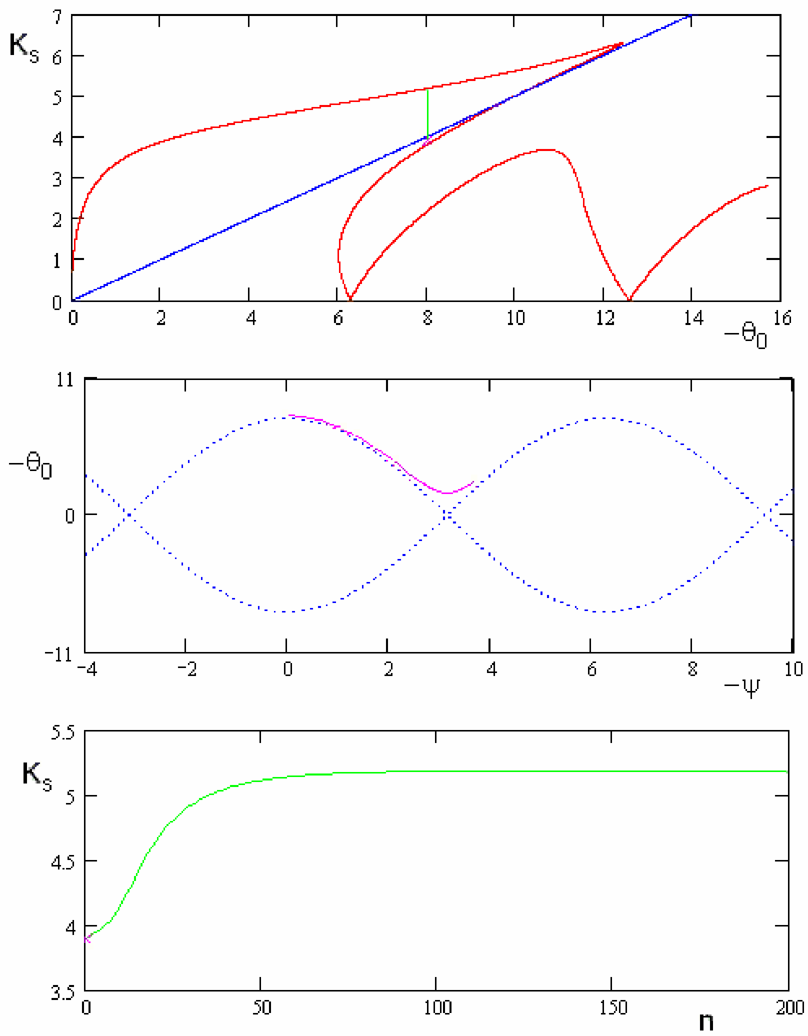


OPEN-TRAJECTORIES SATURATION STABLE POINT



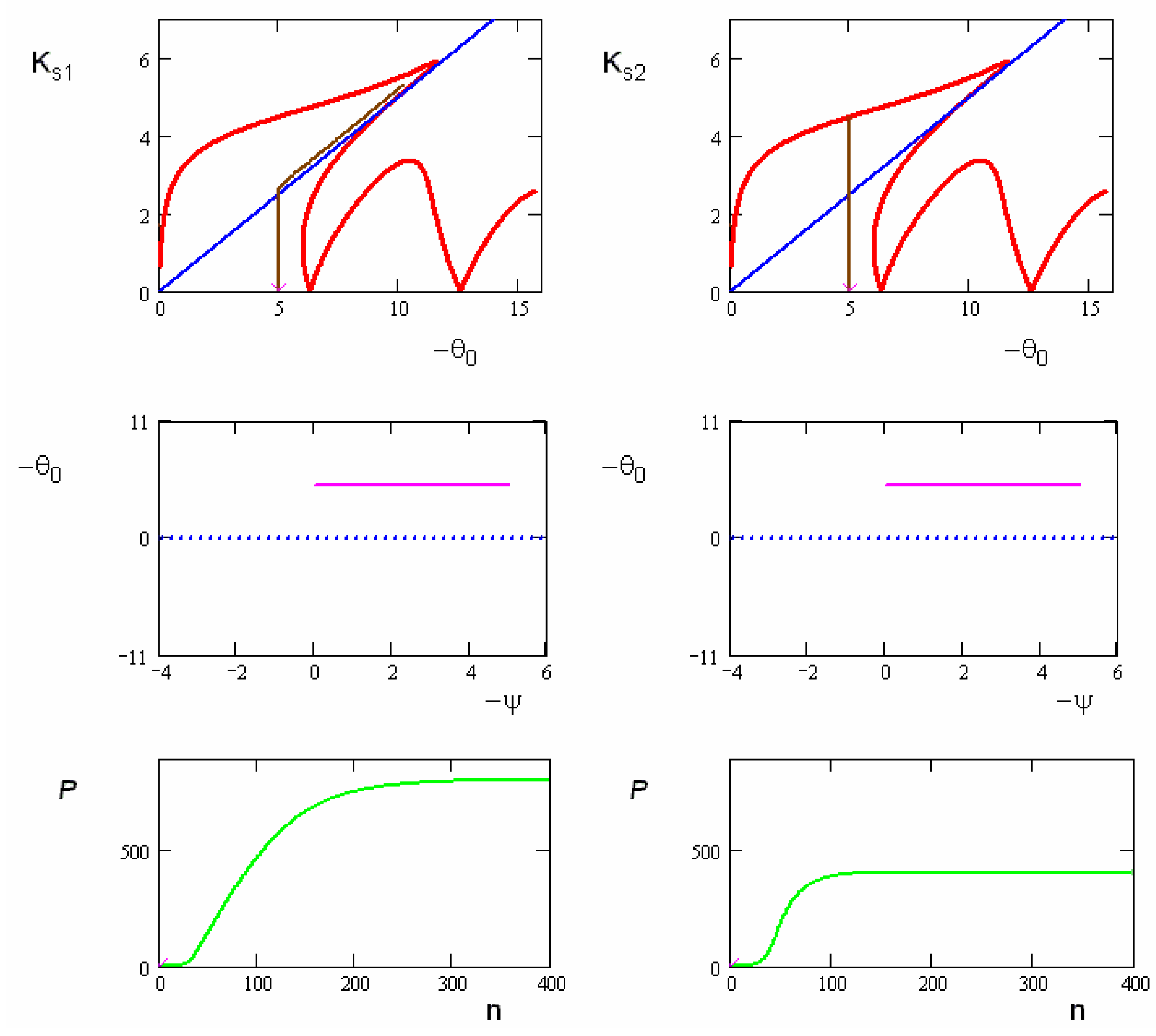
$R_{rt} = 0.97$

$G\text{-}1 = 0.005$



Oscillation build-up in Stimulated Superradiance FEL Oscillator

Post-Saturation Detuning Control | **Fixed Detuning**



FREE ELECTRON SUPERRADIANT SPIN–FLIP EMISSION OF RADIATION.

Excitation of modes
(Including emission from magnetic dipoles)

$$\mathbf{E}(\mathbf{r},\omega)=\sum_{\pm q} C_q(z,\omega)\tilde{\mathbf{E}}_q(\mathbf{r})$$

$$\mathbf{H}(\mathbf{r},\omega)=\sum_{\pm q} C_q(z,\omega)\tilde{\mathbf{H}}_q(\mathbf{r})$$

$$C_q^{out}(\omega)-C_q^{in}(\omega)=\sum_{j=1}^{N}\Delta C_{qj}=-\frac{1}{4P_q}\sum_{j=1}^{N}\Delta W_{qj}$$

$$\Delta W_{qj}=-\mathbf{e}\int_{-\infty}^{\infty}\mathbf{v}_j(t)\cdot\tilde{E}_q^{*}(r_j(t))\mathbf{e}^{i\omega t}dt+\int_{-\infty}^{\infty}\boldsymbol{\mu}_j(t)\cdot\tilde{H}_q^{*}(\underline{r}_j(t))\mathbf{e}^{i\omega t}dt$$

$$\frac{dW_q}{d\omega}=\frac{2}{\pi}P_q\left|C_q^{out}(\omega)\right|^2$$

$$\tilde{\mathbf{E}}_q=\tilde{\mathbf{E}}_{00}=\frac{\mathbf{w}_0}{\mathbf{w}(z)}\exp\left[-\frac{\mathbf{x}^2+\mathbf{y}^2}{\mathbf{w}^2(z)}-ik\frac{\mathbf{x}^2+\mathbf{y}^2}{2R(z)}+i\varphi+ikz\right]$$

Electron Spin Dynamics

Solve in the electron rest-frame or solve the BMT equation in the lab frame

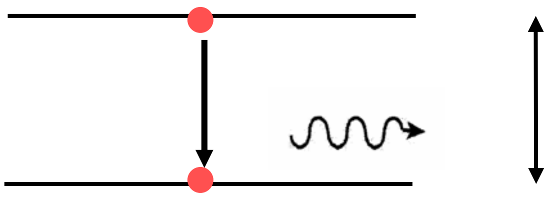
$$\left.\begin{aligned} \frac{d\langle S_x\rangle}{dt} &= -\frac{eg}{2m\gamma_z} B_z \langle S_y\rangle \\ \frac{d\langle S_y\rangle}{dt} &= \frac{eg}{2m\gamma_z} B_z \langle S_x\rangle \end{aligned}\right\} \Rightarrow \quad \begin{aligned} &\frac{d^2}{dt^2}\langle \mathbf{S}\rangle + \omega_s^2 \langle \mathbf{S}\rangle = 0 \\ &\omega_s = \frac{eg}{2m\gamma_z} B_z = \frac{\omega_{s0}'}{\gamma} \end{aligned}$$

$$\frac{d\boldsymbol{\mu}_j}{dt} = \omega_s \boldsymbol{\mu}_j \times \hat{e}_z$$

$$\textbf{Solution:}\ \boldsymbol{\mu}_j = \mathbf{Re}\left[\tilde{\boldsymbol{\mu}} e^{-i\omega_s\left(t-t_{0j}\right)}\right] \quad , \quad \tilde{\boldsymbol{\mu}} = |\boldsymbol{\mu}|\hat{\boldsymbol{\sigma}}_+ e^{i\varphi_{s0j}}$$

Electron Spin Resonance Emission Frequency

In the e-rest frame:



$$\hbar\omega_s' = g\mu_B B_z'$$

In the lab frame:

$$B_z' = B_z$$

$$\omega_s' = \gamma_z\left[\omega_s - \beta_z c k_z(\omega_s)\right]$$

For $\quad k_z = (\omega_s/c)\cos\Theta$:

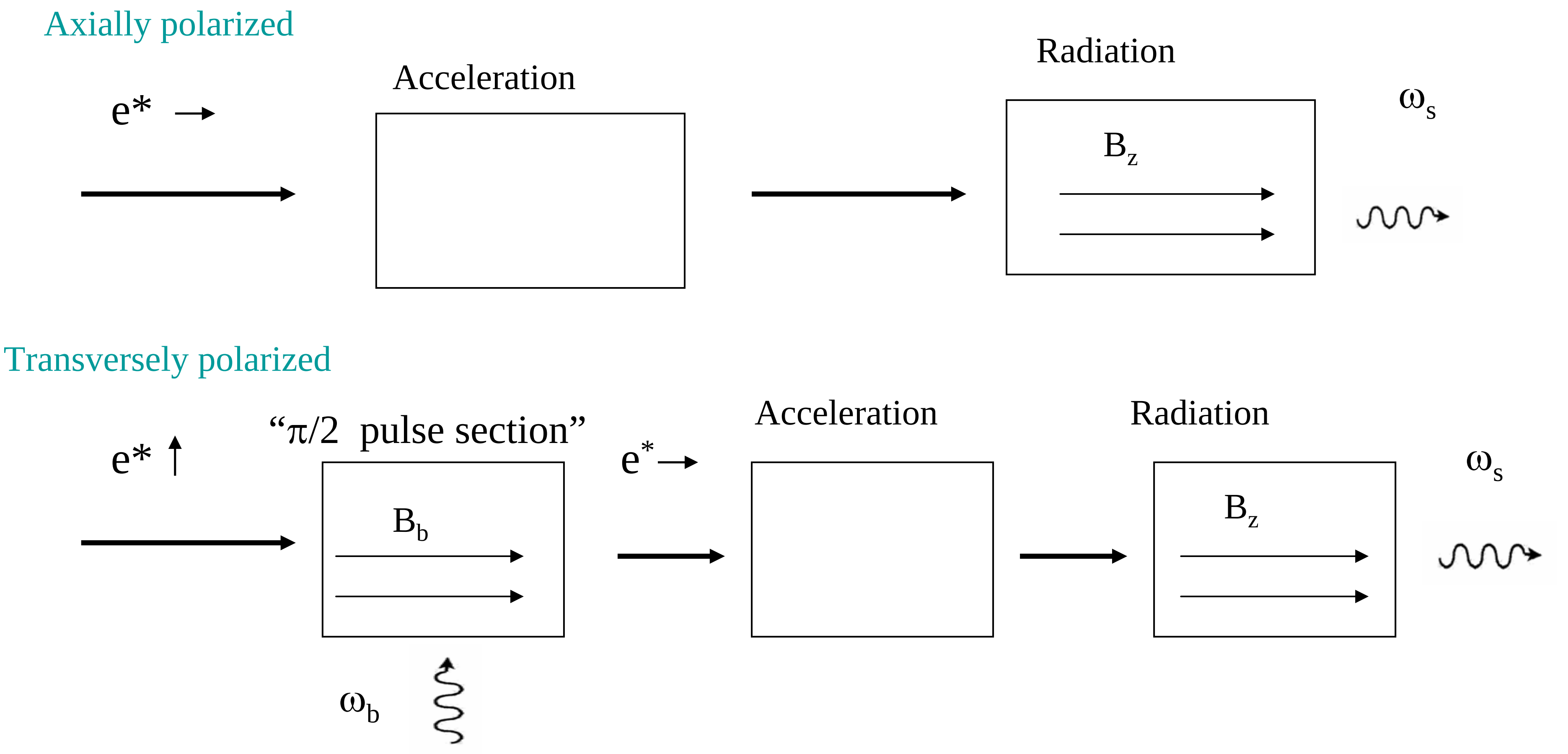
$$\omega_s = \frac{\omega_{s0}'/\gamma_z}{1-\beta_z\cos\Theta}$$

For $\quad \Theta = 0$:

$$\omega_s = (1+\beta_z)\gamma_z\omega_{s0}'$$

$B_z = 3.5\text{kGauss},\ \gamma = 100 \Rightarrow f_s' = 10\text{GHz},\ f_s = 2\text{THz}$

Free Electron Spin-Flip Emission of Radiation (FESFER)



$$P_{max} = \frac{I_b}{e}\hbar\omega_s \; ; \qquad \textbf{FOR} \quad f_s = 2\text{THz}, \quad \Rightarrow \quad P_{max}/I_b = 8\text{mW/A}$$

Spectral Power of Spontaneous FESFER

$$\Delta \mathbf{W}_{qj}^{m} = \int_{-\infty}^{\infty} \dot{\boldsymbol{\mu}}_j(t) \cdot \tilde{\mathbf{B}}_q^*(\mathbf{r}_j(t)) e^{i\omega t} dt$$

$$\Delta \mathbf{W}_{qj}^{m} = \frac{i\omega_s}{2} \int_{t_{0j}}^{t_{0j}+L/v_z} \tilde{\boldsymbol{\mu}}_j \cdot \mathbf{H}_q^*(\mathbf{r}_j) e^{-i\omega_s(t-t_{0j}) - ik_{zq}z_j(t) + i\omega t} dt$$

$$\left(\frac{dW_q}{d\omega}\right)_{sp} = \frac{N}{32\mathrm{P}_q}\left(\frac{\omega'_{s0} L}{v_z \gamma}\right)^2 \left|\tilde{\boldsymbol{\mu}}_j \cdot \tilde{\mathbf{H}}_q^*\right|^2 \mathbf{sin}\, c^2(\theta L/2)$$

$$\theta = \frac{\omega - \omega_s}{v_z} - k_{zq}$$

Cylotron Resonance Emission

$$\frac{d(\gamma m \mathbf{v}_\perp)}{dt} = -eB_z \mathbf{v}_\perp \times \hat{\mathbf{e}}_z$$

$$\mathbf{v}_{j\perp}(t) = \mathrm{Re}\left[\tilde{\mathbf{v}}_{j\perp} e^{-i(\omega_{c0} t + \varphi_{c0})}\right]$$

$$\tilde{\mathbf{v}}_{j\perp} = \sqrt{2} v_\perp \mathrm{e}^{-i\varphi_{c0j}} \hat{\boldsymbol{\sigma}}_+$$

$$\omega_c = \frac{eB_z}{\gamma m}$$

$$\Delta W_{qj}^c = -\frac{e\sqrt{2}}{2} \frac{v_\perp}{v_z} L \hat{\boldsymbol{\sigma}}_+ \cdot \tilde{\boldsymbol{\varepsilon}}_q^*(0) e^{i(\omega t_{0j} - \varphi_{c0j})} e^{i\theta L/2} \sin c(\theta L/2)$$

Combined ECR and FESFER Emission

$$\Delta \mathbf{W}_{qj} = \Delta W_{qj}^{c} + \Delta W_{qj}^{s}$$

$$\Delta W_{qj}^{c} = -\frac{L}{V_z}\hat{\boldsymbol{\sigma}}_{+} \cdot \tilde{\boldsymbol{\varepsilon}}_{q}^{*}(0) \quad \frac{e v_{\perp}}{\sqrt{2}} e^{-i\varphi_{c0j}} e^{i\theta_c L/2} \,\mathbf{sin}\, c(\theta_L L/2)$$

$$\Delta W_{qj}^{s} = -\frac{L}{v_z}\hat{\boldsymbol{\sigma}}_{+} \cdot \tilde{\boldsymbol{\varepsilon}}_{q}^{*}(0) \frac{\omega_{s0}^{'} \mu B}{\gamma c} \frac{Z_0}{Z_q} \frac{gs}{2} e^{-i\varphi_{s0i}} e^{i\theta_s L/2} \,\mathbf{sin}\, c(\theta_s L/2)$$

$$\frac{\omega_s - \omega_c}{\omega_0} = \frac{g-2}{2} = 1.16 \times 10^{-3}$$

$$\frac{\left|\Delta W_q^s\right|}{\left|\Delta W_q^c\right|} = \frac{1}{2\sqrt{2}} \frac{Z_0}{Z_q} \frac{1}{\beta_{\perp}\gamma_z} \frac{\hbar\omega_s}{mc^2} \cong \frac{1}{2\sqrt{2}} \frac{\pi \; r_b}{\varepsilon_n} \frac{\hbar\omega_s}{mc^2} \approx 10^{-6}$$

$$(FOR \quad \omega_s = 2\,THz, \quad r_b = 1\,mm, \quad \varepsilon_n = 6\,\pi \;\; mm \cdot mrad)$$

Ratio of SR-FESFER to SP-ECR

$$\left(\frac{dW_q^s}{d\omega}\right)_{SP/SR} = \frac{1}{4\pi}\frac{\sqrt{\mu_0/\varepsilon_0}}{A_{em}}\left(\frac{L}{v_z}\left|\hat{\boldsymbol{\sigma}}_+\cdot\hat{\mathbf{e}}_{\mathbf{q}}^*\right|\frac{Z_0}{Z_q}\frac{\omega_{s0}'\mu_B}{\gamma c}\frac{g_s}{2}\right)^2 \mathbf{sin}\,c^2(\theta_s L/2)\left[N(1-P_{s0})+N^2P_{s0}^2\right]$$

$$\frac{\left(dW_q^s/d\omega\right)_{SP/SR}}{\left(dW_q^c/d\omega\right)_{SP}} = \frac{1}{2\sqrt{2}}\frac{\pi\ r_b}{\varepsilon_n}\frac{\hbar\omega_s}{mc^2}\frac{N(1-P_{s0})+N^2P_{s0}^2}{N}$$

$$\approx \frac{1}{2\sqrt{2}}\frac{\pi\ r_b}{\varepsilon_n}\frac{\hbar\omega_s}{mc^2}NP_{s0}^2$$

CONCLUSIONS

- SUPERRADIANT EMISSION FROM femto/pico-SEC E-BEAM BUNCHES IS A PROMISSING HIGH POWER **THz RADIATION** SCHEME.

- FORMULATION FOR THE CALCULATION OF COHERENT SPECTRAL CHARACTERISTICS OF **ANY KIND** OF RADIATION SCHEMES WAS PRESENTED.

- A **STIMULATED-SUPERRADIANCE OSCILLATOR** SCHEME PROVIDES ULTIMATE RADIATIVE CONVERSION **EFFICIENCY** (WITH ENERGY RETRIEVAL SCHEMES).

- A RADIATION EMISSION SCHEME FROM ACCELERATED ELECTRONS IS PROPOSED: **FREE ELECTRON SPIN-FLIP EMISSION OF RADIATION (FESFER).**

- A SHORT BUNCH OF POLARIZED ELECTRONS CAN PROVIDE ENHANCED (SUPERRADIANT) **SR-FESFER** RADIATION, WITH APPRECIABLE INTENSITY RELATIVE TO **CRE RADIATION**.