Muon Acceleration in FFAG Rings

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CASA Seminar at JLab

26 April 2004

My WWW home directory:

`http://keil.home.cern.ch/keil/MuMu/Doc/JLab_Apr04/talk.pdf`

Motivation

- Neutrino factory studies in US and Europe assumed muon acceleration in recirculating linear accelerators ”similar” to CEBAF with
  - only 4 or 5 passes
  - 7 or 9 arcs
  - 4 spreaders and combiners
  - no kickers for injection and ejection
  - 37.5% and 20% of total cost of neutrino factory in studies I and II
- FFAG rings promise
  - more passes
  - fewer arcs
  - no spreaders and combiners
  - fun with kickers for injection and ejection

Styles of FFAG Accelerators

- Scaling FFAG rings
  - have similar orbits at different momenta
  - have tunes independent of momentum
  - have nonlinear fields
  - radial or spiral sectors
  - are part of the Japanese neutrino factory design
- Non-scaling FFAG rings
  - are essentially alternating-gradient lattices with small dispersion and controlled values of slip factors η_0 and η_1
  - have tunes that vary with momentum
  - have linear fields
  - are considered for US neutrino factory design

Actors and References

- *C.J. Johnstone* and *S. Koscielniak*, Recent Progress on FFAGs for Rapid Acceleration, APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) T508.
- *D. Trbojevic* et al., Fixed Field Alternating Gradient Lattice Design without Opposite Bend, EPAC 2002, Paris, France, 1199.
- C.J. Johnstone and S. Koscielniak, Recent Progress on FFAGs for Rapid Acceleration, EPAC 2002, Paris, France, 1261.
- C. Johnstone and S. Koscielniak, FFAGS for Rapid Acceleration, accepted for publication in NIM-A Nov 2002.
- *E. Keil* and *A.M. Sessler*, Muon Acceleration in FFAG Rings, PAC 2003, 414.
- D. Trbojevic et al., FFAG Lattice for Muon Acceleration with Distributed RF, PAC 2003, 1816.
- *J.S. Berg* and C. Johnstone, Design of FFAGs Based on a FODO Lattice, PAC 2003, 2216.
- J.S. Berg et al., FFAGs for Muon Acceleration, PAC 2003, 3413.

Longitudinal Dynamics

- Longitudinal Hamiltonian for stationary buckets

$$H_1(p_t,\varphi) \quad = \quad \frac{2\pi h\beta_0^2 E_0}{eVN_c}\left(\frac{\eta_0 p_t^2}{2} + \frac{\eta_1 p_t^3}{3} + \ldots\right) + \sin^2 \pi\varphi$$

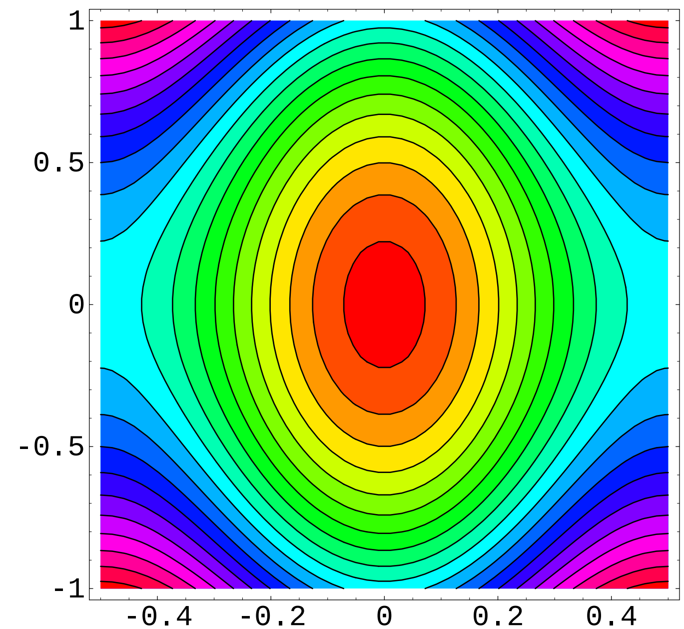
  - p_t momentum error relative to reference particle with total energy E_0 and speed $\beta_0 c$
  - φ phase measured in cycles with origin at stable fixed point and $-1/2 \leq \varphi \leq +1/2$
  - h harmonic number, V peak accelerating voltage, N_c number of RF cavities

- Consider 3 cases:
  - Linear motion with $\eta_0 \neq 0$ and $\eta_1 = \eta_2 = 0$
  - Nonlinear motion with $\eta_0 \neq 0$, $\eta_1 \neq 0$ and $\eta_2 = 0$
  - Motion near transition with $\eta_0 = 0$, and $\eta_1 \neq 0$

Linear Longitudinal Motion

- Measure momentum offset y in units of half linear bucket height
- For stationary buckets in FFAG rings
  - Stable fixed point at $\varphi = y = 0$
  - Unstable fixed points at $\varphi = \pm 1/2$ and $y = 0$
  - Hamiltonian

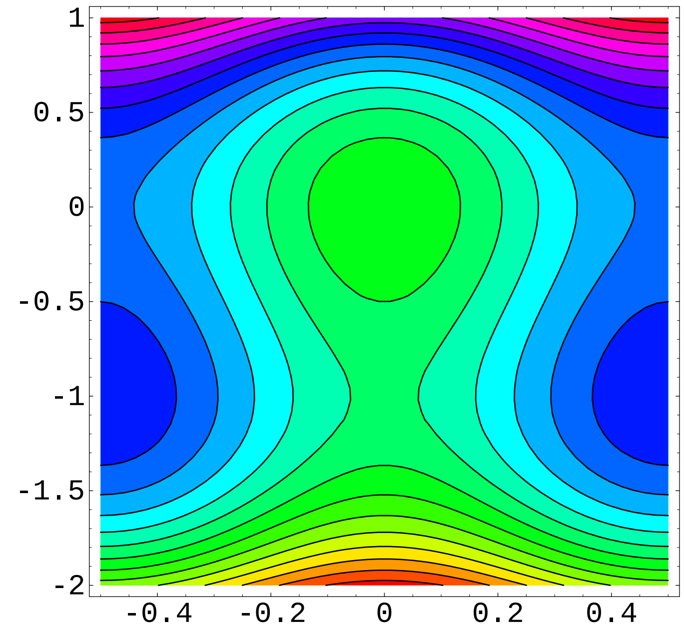
$$H(\varphi, y, a) = y^2 + \sin^2 \pi\varphi$$



Contour plot of Hamiltonian for linear motion. Muons move along level lines.

Effect of $\eta 1 \neq 0$ on Longitudinal Hamiltonian

- $a = \eta_1 p_b/\eta_0$ with half bucket height p_b
- New stable fixed points at $\varphi = \pm 1/2$ and $y = -1/a$
- New unstable fixed point at $\varphi = 0$ and $y = -1/a$
- Ω-shaped trajectories start below fixed point at $\varphi = \pm 1/2$ and $y = -1/a$, circle around fixed point at $\varphi = 0$ and $y = 0$, and reach maximum y above it
- Acceleration in FFAG rings along light blue Ω-shaped trajectories
- Find limit on a for Ω-shaped trajectories



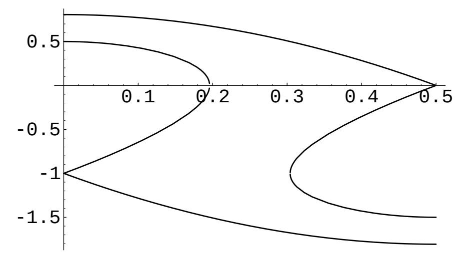
Contour plot of Hamiltonian at $a = 1$.

Separatrices

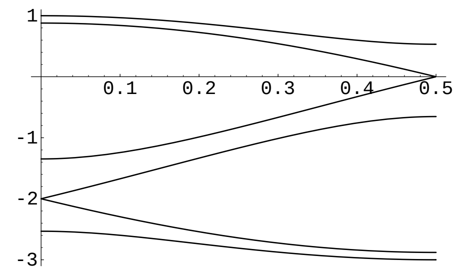
- Separatrices pass unstable fixed points
- 2 unstable fixed points and 2 separatrices when $a \neq 0$
- Find separatrices by solving for y:

$$
\begin{aligned}
H(\varphi, y, a) &= H(-1/2, 0, a) \\
H(\varphi, y, a) &= H(0, -1/a, a)
\end{aligned}
$$

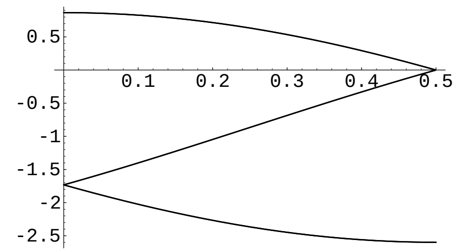
- Use symmetry and plot for $0 \leq \varphi \leq 1/2$
- Acceleration along trajectories in S-shaped channel between islands starts between separatrices in lower right corner below $y = -3/2a$, and ends between separatrices in upper left corner above $y = 1/2a$
- At $a = 1/2$ regular bucket centred at $\varphi = y = 0$ blocks acceleration across $y = 0$
- At $a = 1/\sqrt{3}$ buckets centred at $\varphi = y = 0$ and at $\varphi = 1/2$ and $y = -\sqrt{3}$ just touch, and channel of acceleration has width zero, agreeing with K.Y.Ng's result



$a = 1$



$a = 1/2$



$a = 1/\sqrt{3}$

Longitudinal Motion Near Transition

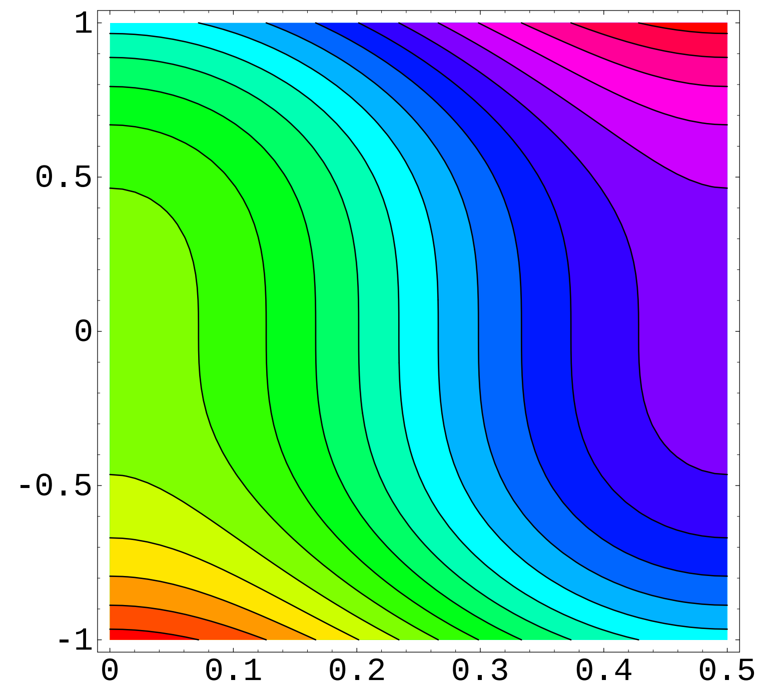
- Introduce scaled momentum variable y

$$y = p_t \left(\frac{2\pi\beta_0^2 E_0 h \eta_1}{3eVN_c} \right)^{1/3}$$

- Scaled Hamiltonian $H_5(y, \varphi)$

$$H_5(y, \varphi) = y^3 + \sin^2 \pi\varphi$$

- Acceleration in FFAG rings happens along light blue S-shaped trajectory, which starts at $\varphi = 1/2$ and $y = -1$, and reaches maximum $y = 1$ at $\varphi = 0$
- Equation relates range $\pm p_t$ and ring parameters at $y = \pm 1$, cf. next page
- Discuss later two FFAG rings operating near transition, doublet lattice for muons, and model for electrons



Contour plot of $H_5(y, \varphi)$

Parameters and Scaling Laws

- Calculate RF cavity voltage V from accelerating range p_t and ring parameters:

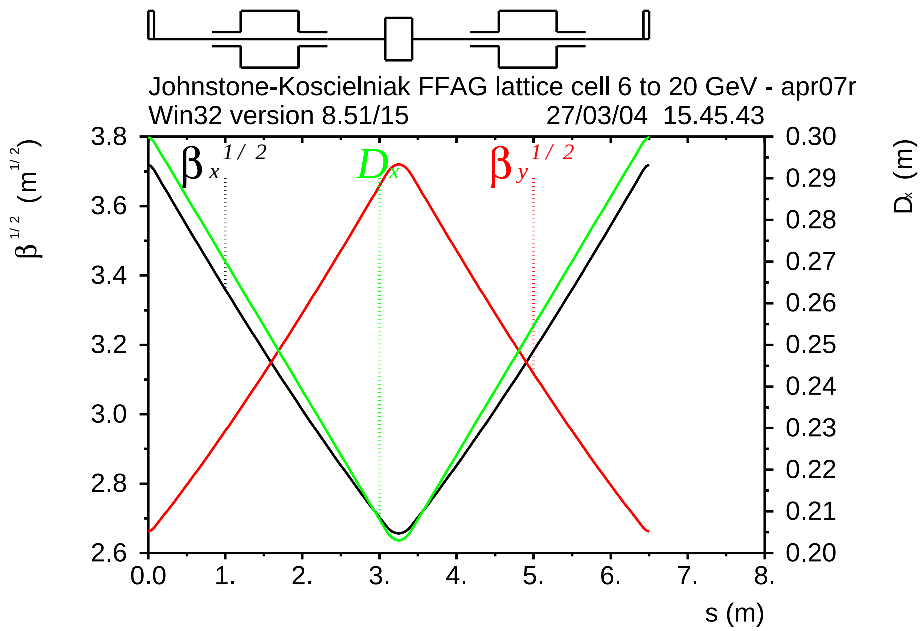
$$V = \frac{2\pi\beta_0^2 E_0}{3e}\left(\frac{h\eta_1}{N_c}\right)p_t^3$$

- Scaling with energy E_0 in first term, with range p_t in third term
- Scaling with N lattice periods of length L in brackets:
  - h and circumference C at given RF frequency $\propto LN$
  - $N_c \propto N$
  - $\eta_1 \propto 1/N^2$ derived analytically by K.Y. Ng for FODO lattice with $N \gg 1$; I believe from numerical studies that it holds for any lattice style
- $V \propto E_0 L p_t^3/N^2$ and $N_c V \propto E_0 L p_t^3/N$
- Assuming that cost of magnets, vacuum, tunnel is $C_\mathrm{M} LN$, that cost of RF cavities and power installation is $C_\mathrm{RF} E_0 L p_t^3/N$ yields cost optimum at eqal cost components

$$C = 2L\sqrt{C_\mathrm{M} C_\mathrm{RF} E_0 p_t^3}$$

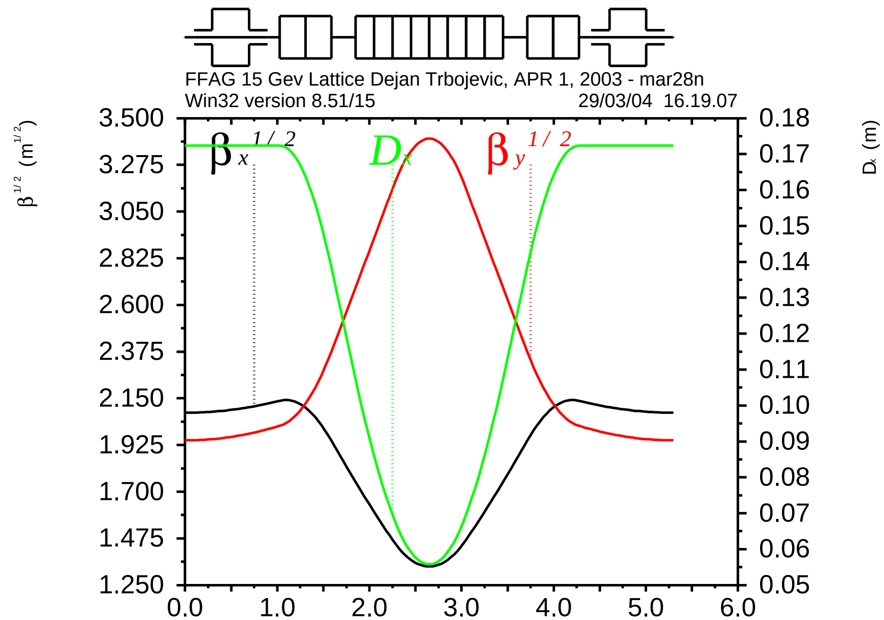
Johnstone-Koscielniak FODO Lattice JK

- Focusing quadrupoles
- Defocusing gradient dipoles
- FODO lattice with $Q_x \approx Q_y$
- Number of cells $N = 314$
- Circumference $C = 2041$ m
- Space for two superconducting RF cavities in cell
- Accelerating voltage $V = 2.5$ MV



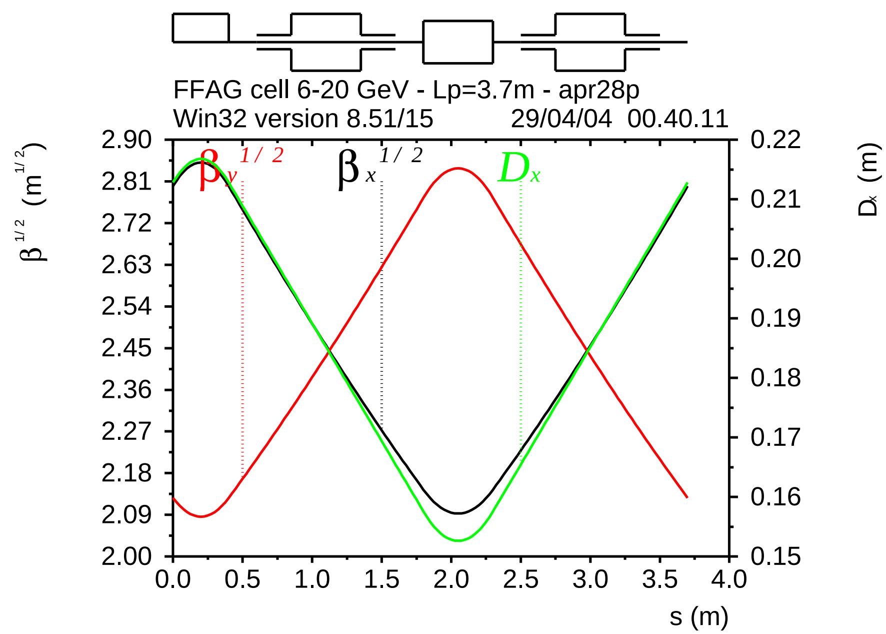
Trbojevic Triplet Lattice T

- Focusing gradient dipoles
- Defocusing gradient dipoles
- Triplet lattice with $Q_x \neq Q_y$
- Number of cells $N = 60$
- Circumference $C = 318$ m
- Space for super-conducting RF cavity
- Accelerating voltage $V = 10$ MV



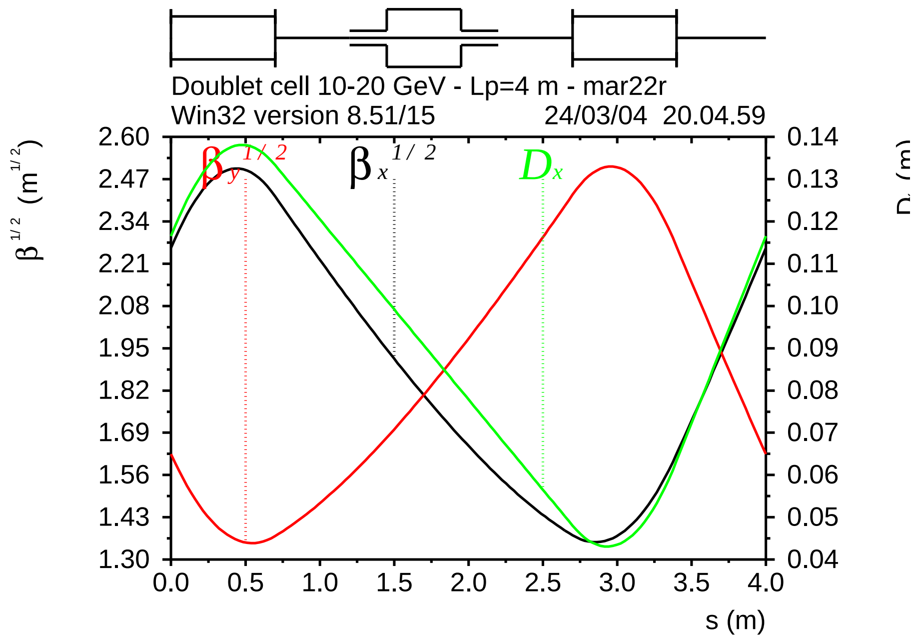
Keil-Sessler FODO Lattice KS-F

- Focusing quadrupoles
- Defocusing gradient dipoles
- FODO lattice with $Q_x \approx Q_y$
- FODO lattice with $Q_x \approx Q_y$
- Number of cells $N = 2800$
- Circumference $C = 1036$ m
- Space for two room-temperature RF cavities in cell
- Accelerating voltage $V = 3$ MV



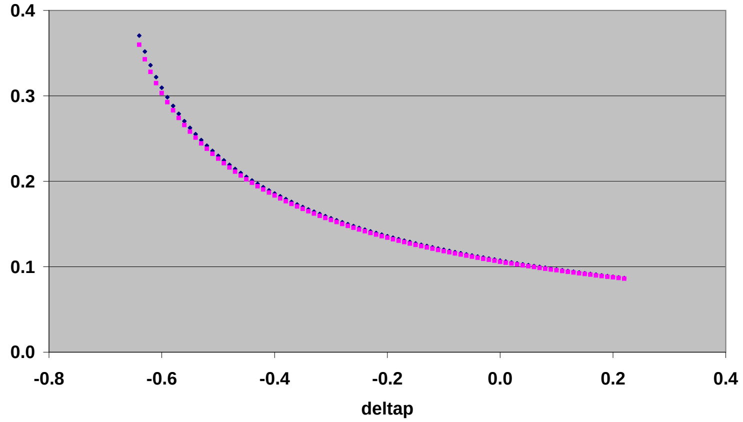
Keil-Sessler Doublet Lattice KS-D

- Focusing gradient dipoles
- Defocusing gradient dipoles
- FODO lattice with $Q_x \approx Q_y$
- Number of cells $N = 100$
- Circumference $C = 400$ m
- Space for super-conducting RF cavity
- Accelerating voltage $V = 13.5$ MV

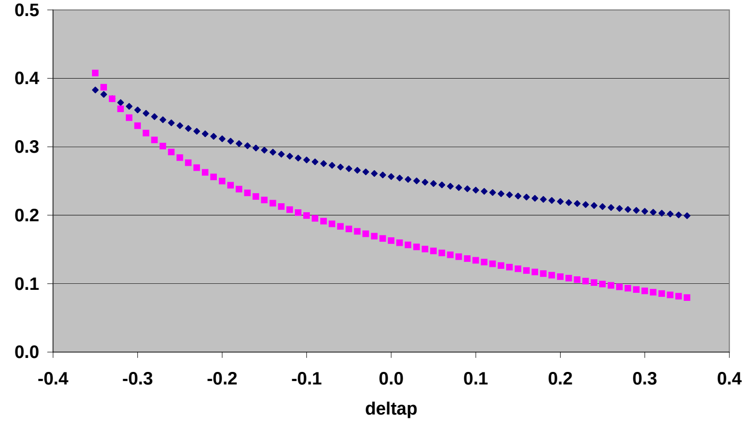


Tunes q_x and q_y vs. $\delta p/p$

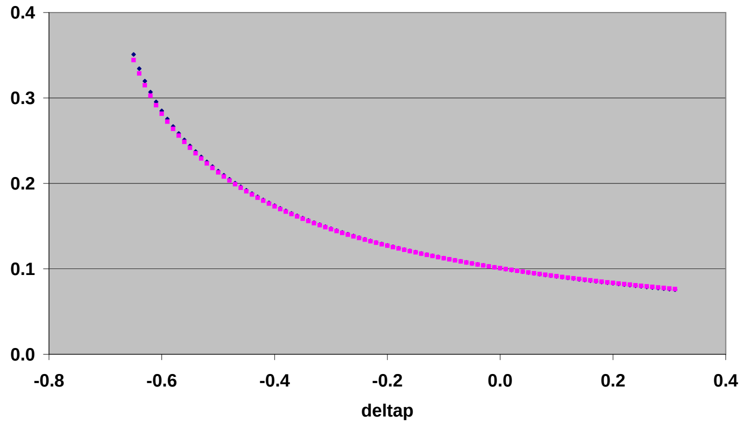
Johnstone-Koscielniak lattice



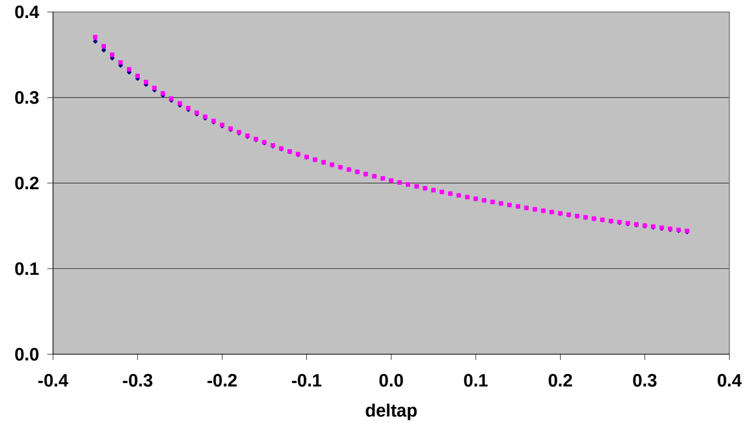
Trbojevic triplet lattice



Keil-Sessler FODO lattice

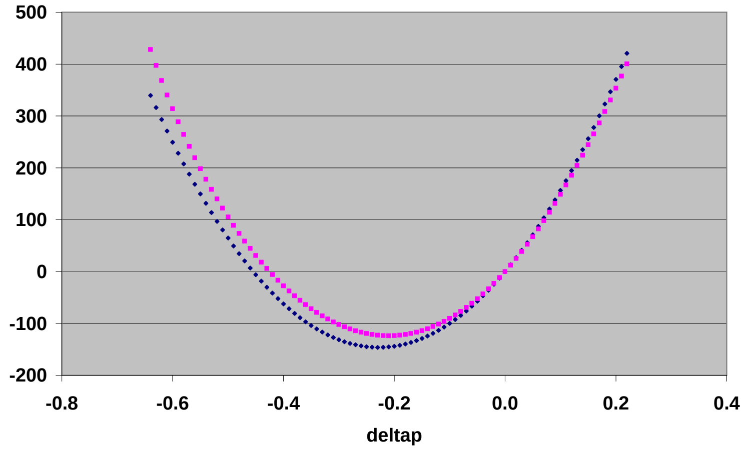


Keil-Sessler doublet lattice

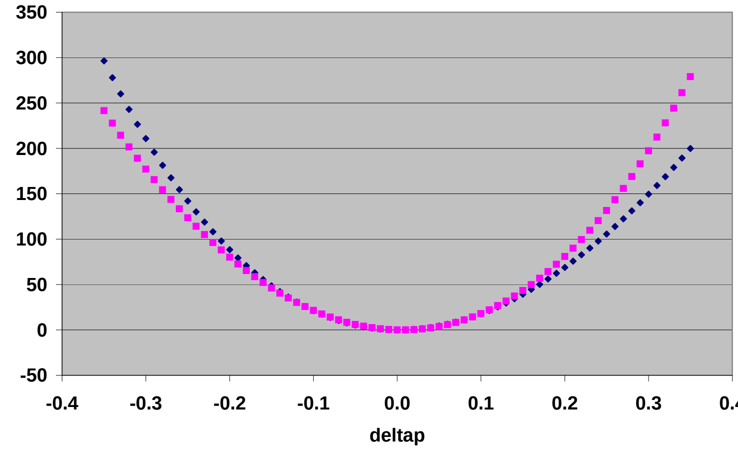


Path length $\delta(s)$ and travel time ct in mm vs. $\delta p/p$

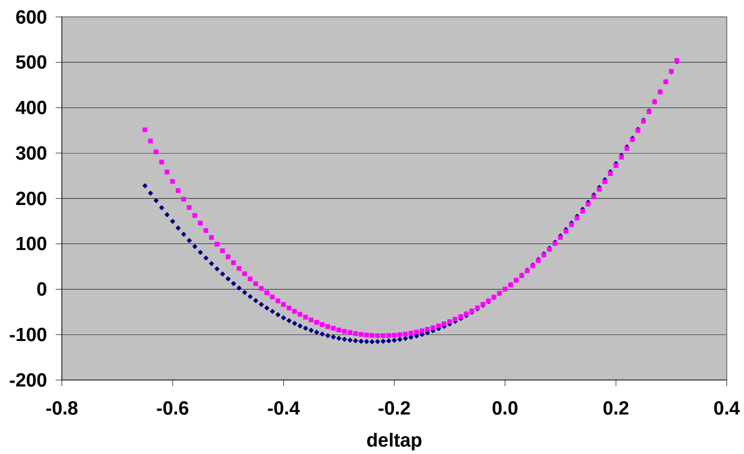
Johnstone-Koscielniak lattice



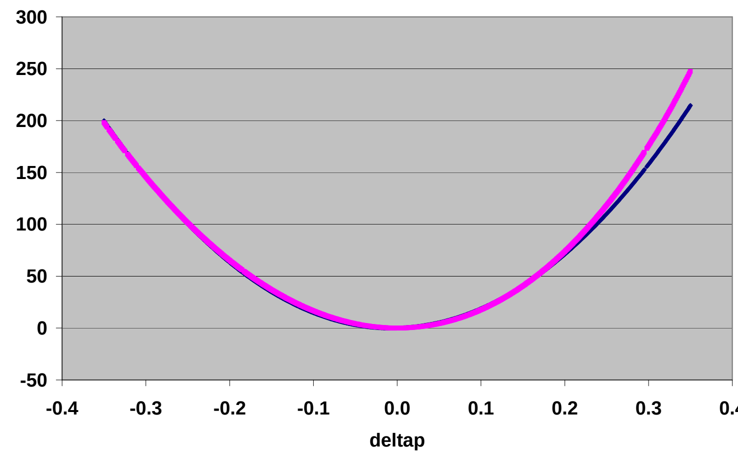
Trbojevic triplet lattice



Keil-Sessler FODO lattice

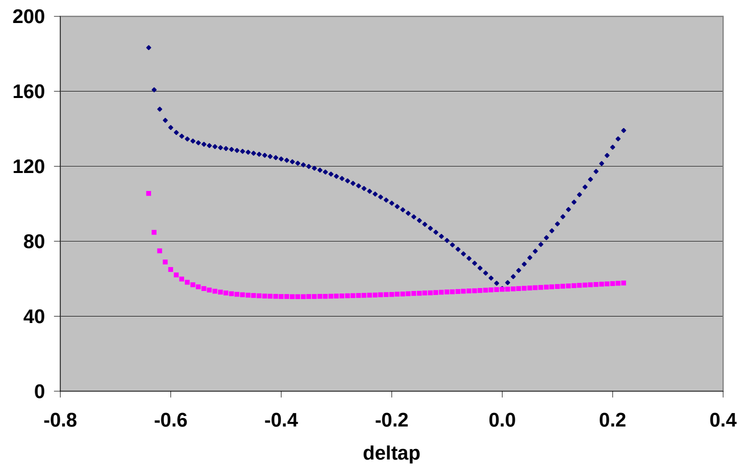


Keil-Sessler doublet lattice

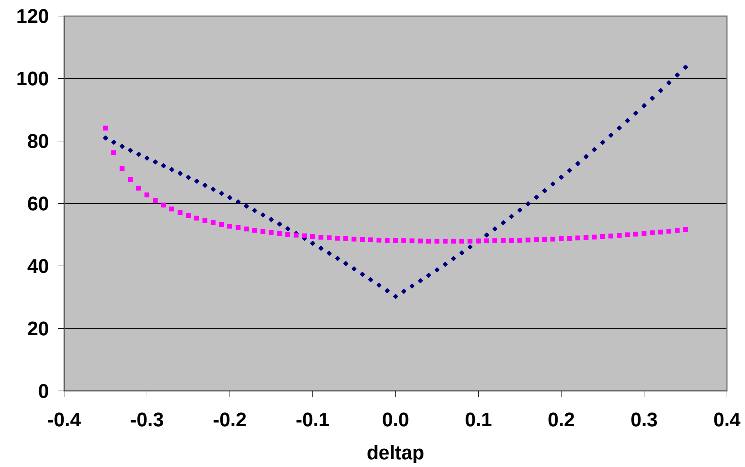


Horizontal aperture A_x and vertical aperture A_y in mm vs. $\delta p/p$

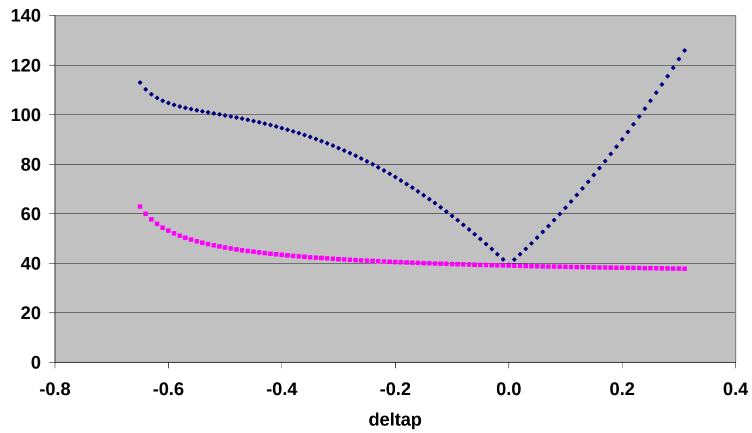
Johnstone-Koscielniak lattice



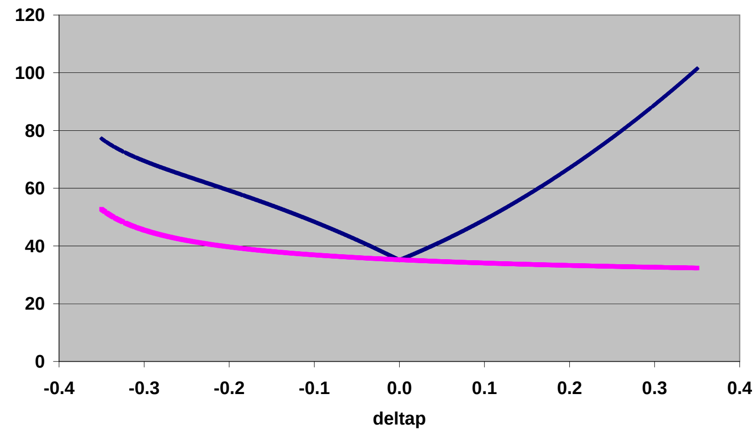
Trbojevic triplet lattice



Keil-Sessler FODO lattice



Keil-Sessler doublet lattice

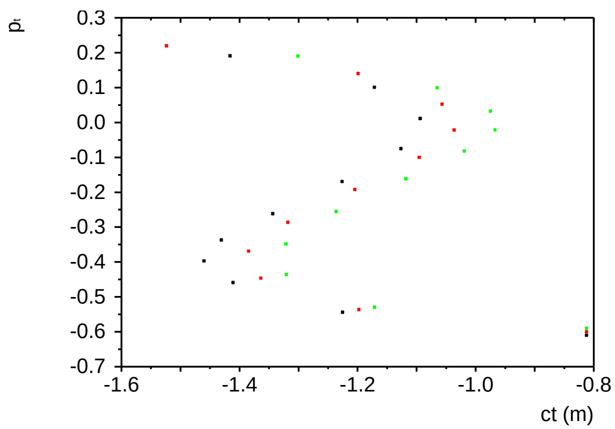


Lattice Parameters of FFAG Rings for Muons

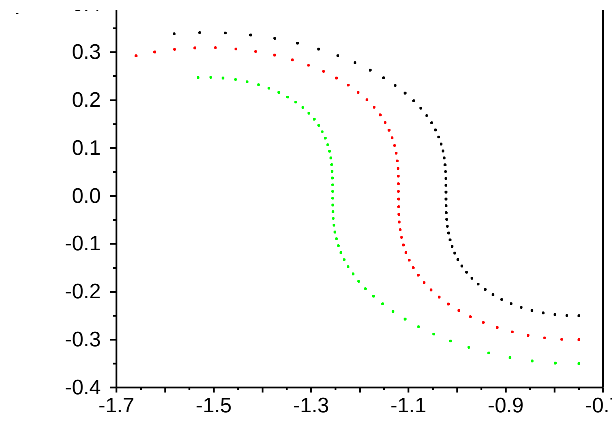
	JK	T	KS-F	KS-D
Total ref. energy E/GeV	16.5	15	16	15
Energy range/GeV	$6 \ldots 20$	$10 \ldots 20$	$6 \ldots 20$	$10 \ldots 20$
Offset in F magnet x/mm	$-76 \ldots 79$	$-42 \ldots 76$	$-55 \ldots 69$	$-25 \ldots 69$
Period length L_p/m	6.5	5.3	3.7	4
Periods N_p	314	60	280	100
Circumference C/m	2041	318	1036	400
Gradients G_F/G_D/T/m	$75/-32$	$40.4/-45.5$	$49/-39$	$52.6/-52.6$
Dipole field B_F/B_D/T	0/3.1	$-4.0/6.1$	0/2.4	$-2.4/6.9$
Path length spread/mm	535	279	473	221
Slip factor η_0	0.000586	0	0.000892	0
Slip factor η_1	0.001436	0.006206	0.002040	0.004233

Acceleration in FFAG Rings for Muons

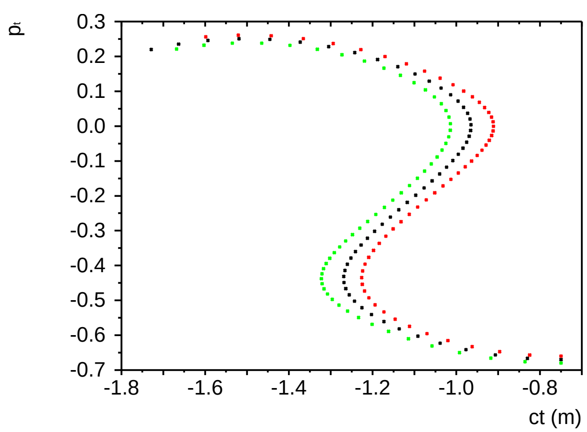
Johnstone-Koscielniak lattice



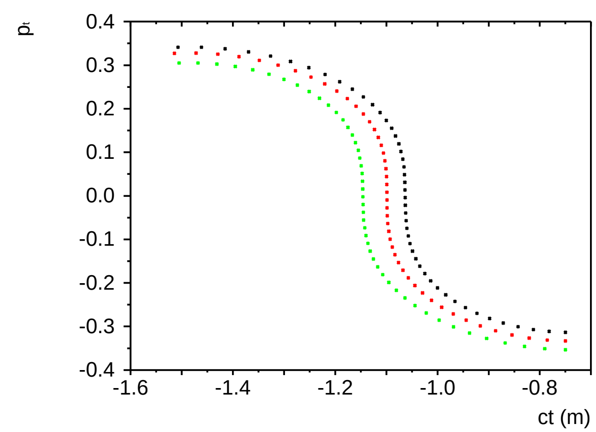
Trbojevic triplet lattice



Keil-Sessler FODO lattice



Keil-Sessler doublet lattice



RF System Parameters of FFAG Rings for Muons

	JK	T	KS-F	KS-D
Range of p_t	$-0.636\ldots0.212$	$\pm1/3$	$-0.625\ldots0.25$	$\pm1/3$
RF frequency/MHz	184.5	198.0	199.7	202.4
Number of RF cavities	628	120	560	100
RF cavity voltage V/MV	2.5	10	3	13.5
Circumf. RF accel. voltage	1570	1200	1680	1350
Number of turns	10	9	12	9

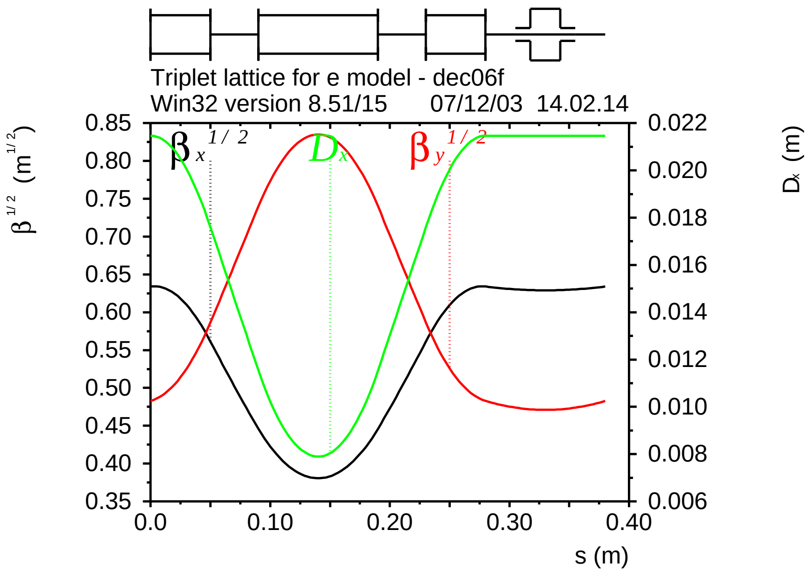
Motivation of Electron Model

- Demonstrate novel features of acceleration in non-scaling FFAG rings
  - Acceleration outside buckets
  - Crossing of many integral and half-integral resonances

  at fraction of cost of FFAG rings in neutrino factory
- Electron model
  - accelerates from about 10 to about 20 MeV
  - fits into a small hall
  - is constructed next to a suitable electron linac

Electron Model Lattice

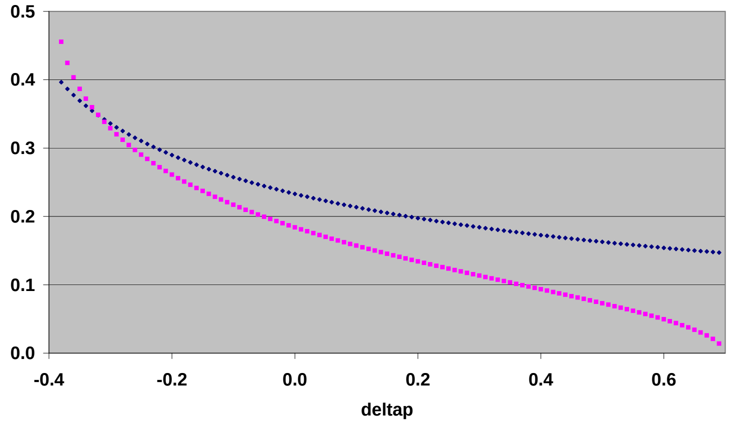
- FDF triplet lattice
- Displaced F and D quadrupoles
- F quadrupoles have reversed field and bend away from ring centre
- Space for room-temperature single-cell RF cavity at 3 GHz, similar to buncher cavity in S-band linac
- Space for coils between magnets
- Aperture and effective length comparable
- Magnetic field within reach of permanent magnets



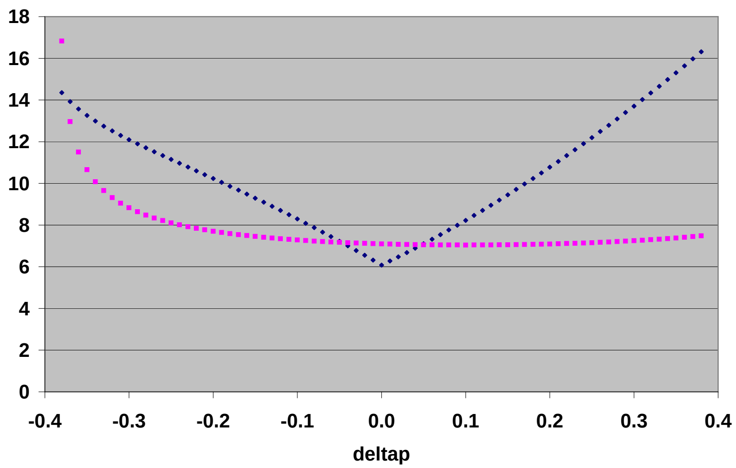
Layout and orbit functions in a cell of the electron model

Electron Model Figures

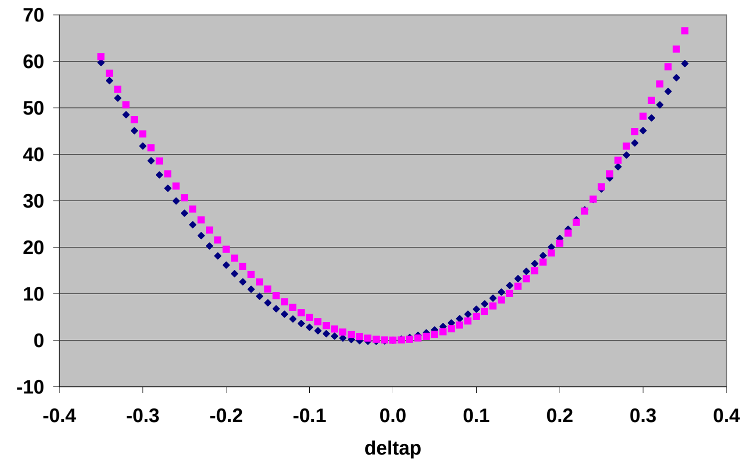
Tunes q_x and q_y vs. $\delta p/p$



Half apertures A_x and A_y vs. $\delta p/p$



Path length $\delta(s)$ and travel time ct



For $-0.35 \leq \delta p/p \leq +0.35$

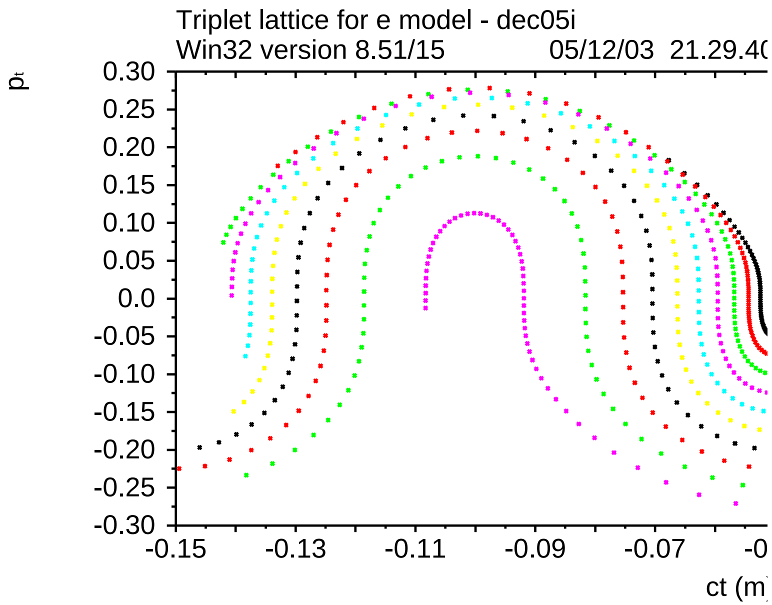
- Stable tunes q_x and q_y
- $A_x < 20$ mm and $A_y < 10$ mm
- Fit to ct yields $\eta_1 = 0.0149$

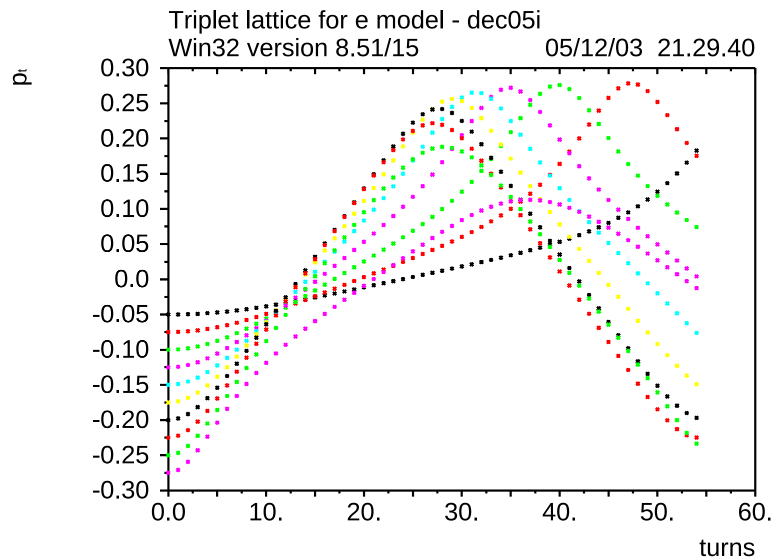
Lattice Parameters at 15 MeV

- Char. length $X = B/G =$ quadrupole displacement
- $X_F < A_x \rightarrow$ displaced F quadrupoles
- $X_D > A_x \rightarrow$ displaced half D quadrupoles
- Obtain pole tip field assuming that hyperbolic pole tip passes through corner of rectangular aperture
- Good field extends further between poles than usual

Number of cells	45	
Cell length	0.38	m
F/D magnet length	50/100	mm
F/D magnet bore radius	25/32	mm
F magnet angle	-37.459	mrad
F magnet gradient	5.638	T/m
F magnet central field	-37.464	mT
F magnet pole tip field	0.14	T
F magnet char. length	-6.64	mm
D magnet angle	214.545	mrad
D magnet gradient	-4.746	T/m
D magnet central field	107.285	mT
D magnet pole tip field	0.15	T
D magnet char. length	-22.6	mm

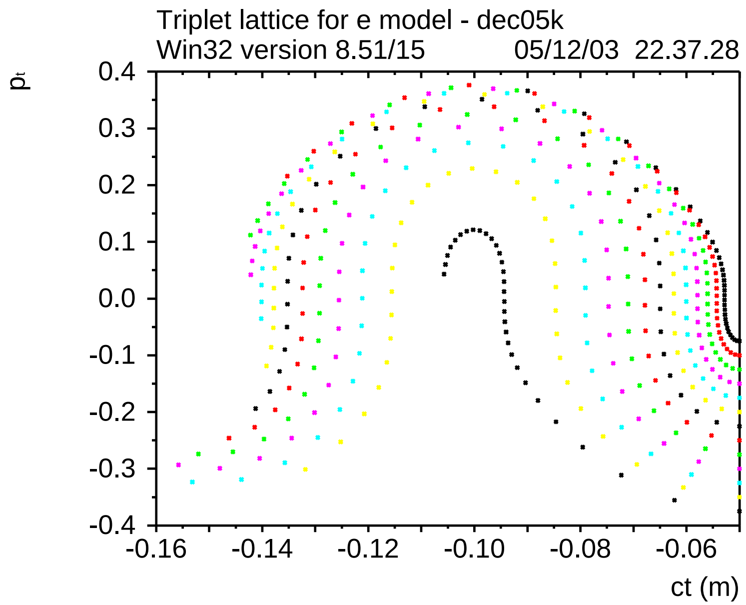
Acceleration at $V = 20$ kV

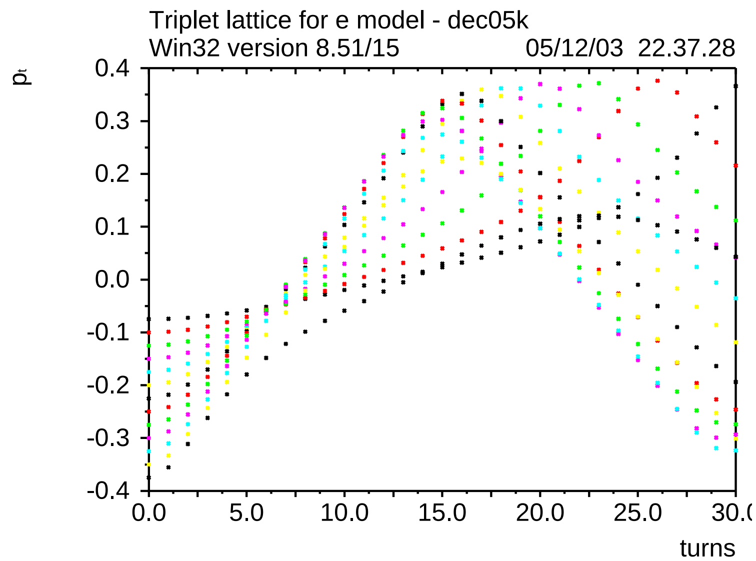




- (ct, p_t) phase space on the left, p_t vs. turn on the right
- Coordinates recorded every 1/5 turn
- Particles launched near $p_t = -0.2$ accelerated to $p_t \approx 0.2$

Acceleration at $V = 50$ kV





- (ct, p_t) phase space on the left, p_t vs. turn on the right
- Coordinates recorded every 1/5 turn
- Particles launched near $p_t = -0.3$ accelerated to $p_t \approx 0.3$

Beam Loading

- Compare energy extracted by beam W_e to stored energy W_s in cavity:

$$W_s = \frac{U^2}{4\pi f_{\mathrm{RF}}(R/Q)}$$

- Peak cavity voltage $U = V\pi/2$, frequency of RF system $f_{\mathrm{RF}} \approx 3$ GHz, intrinsic impedance $R/Q = 121\Omega$ in pillbox cavity
- With beam current I, acceleration in n turns, W_e becomes with circumference C:

$$W_e = \frac{ICVn}{c\beta_0}$$

- Taking $W_e/W_s \ll 1$ yields upper limit for I with harmonic number h of RF system:

$$I \ll \frac{V}{16nh(R/Q)}$$

- Accurate calculation of transient beam loading should take into account variation of phase and acceleration.
- Beam observation system must work to expected accuracy at beam current I

RF System Parameters for two Accelerating Voltages

- RF power

$$P = \frac{U^2}{2Q(R/Q)}$$

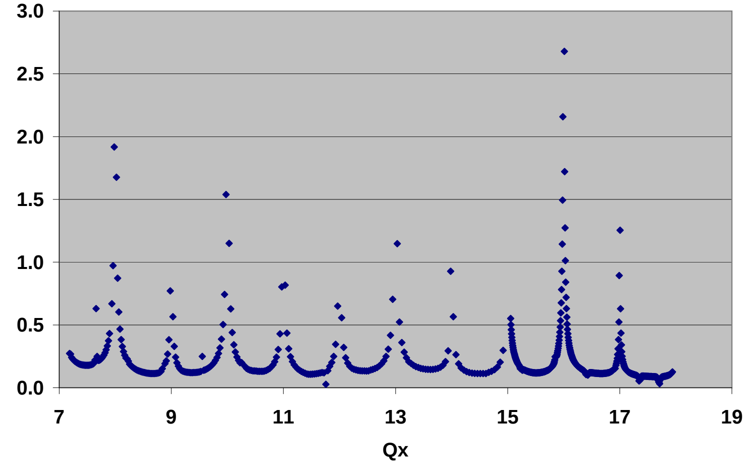
- Quality factor $Q = 17700$ for Cu cavities with $\sigma = 5.7 \times 10^{-7}\Omega^{-1}\mathrm{m}^{-1}$
- Feed with one TWT and one waveguide, tapping off power

Slip factor η_1	0.0149	0.0149	
Number of cavities N_c	45	45	
Harmonic number h	171	171	
Cavity accel. voltage V	20	50	kV
Peak cavity voltage U	31.4	78.5	kV
RF cavity power P	230	1440	W
Stored energy W_s	0.216	1.35	mJ
Range p_t in formula	0.224	0.304	
Initial p_t	-0.2233	-0.3011	
Final p_t	0.2234	0.3012	
Number of turns n	9	5	
Beam current I	$\ll 21.1$	$\ll 94.9$	mA

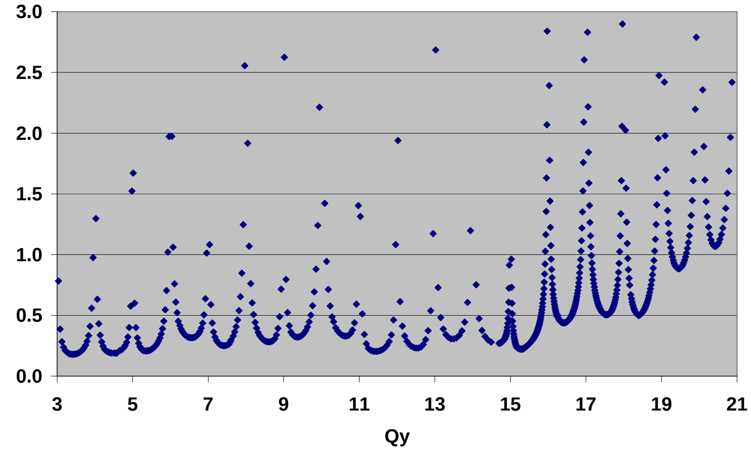
Misalignments

- Triplets on girders without internal errors
- Small RMS displacement of girders 0.03 mm, achieved by survey with central monument
- Misalignments also strongly drive D_x and D_y at natural chromaticity, find $D_x = 23.6 \pm 0.7$ mm and $D_y = 8.9 \pm 4.2$ mm at $\delta p/p = 0$
- With gradient errors would get half-integral resonances
- Beam does not circulate at constant $\delta p/p$ close to integral resonances
- Will beam be accelerated across resonances?

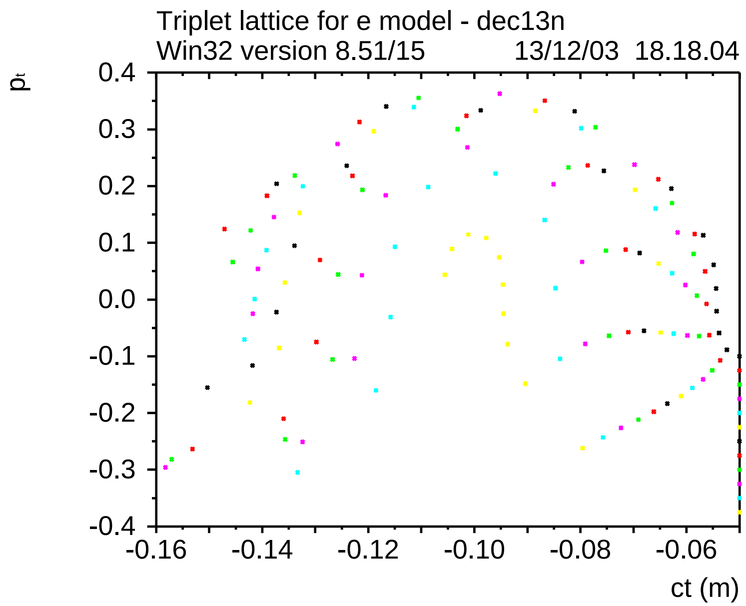
Maximum horizontal orbit offset in mm

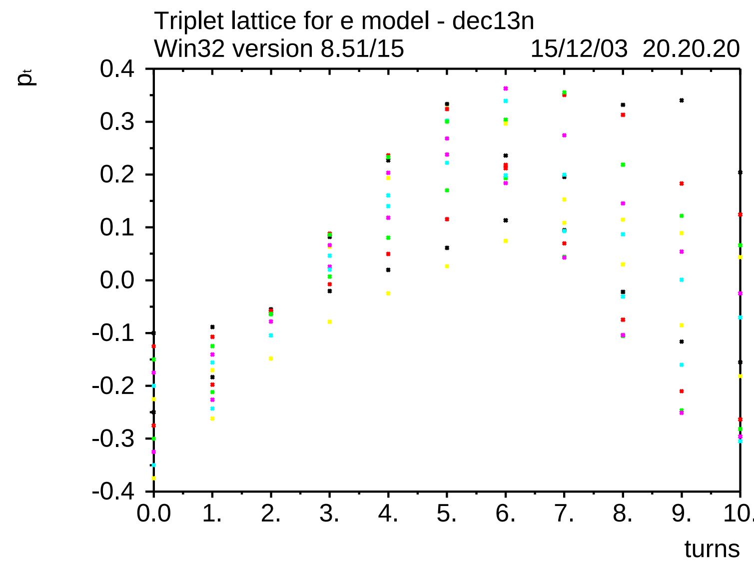


Maximum vertical orbit offset in mm



Acceleration with Misalignments at $V = 50$ kV





- (ct, p_t) phase space on the left, p_t vs. turn on the right
- Particles launched near $p_t = -0.15$ and $p_t = -0.3$ are lost

Conclusions

- Replace RLA's by FFAG rings for muon acceleration to reduce cost
- FFAG rings have circumferences between 0.3 and 2 km
- Circumferential RF accelerating voltage about 1500 MV
- Longitudinal dynamics of acceleration outside buckets close to transition shows effect of acceleration range $\pm p_t$
- Palmer et al. propose several FFAG rings in cascade
- Demonstrate concepts in electron model
- Unanswered questions:
  - Longitudinal acceptance
  - Design of injection and ejection kickers with sum of fall time and beam pulse length smaller than revolution period