Recent Developments in Coherent Synchrotron Radiation

Robert Warnock

SLAC and U. of New Mexico

in collaboration with

M. Venturini, J. Ellison

with help from R. Ruth, K. Bane

Seminar at Jefferson Laboratory
January 17, 2003

TOPICS – Part I

1. Introduction and motivation for theory
2. Dynamical scheme – Vlasov-Fokker-Planck (VFP) equation, and its numerical solution
3. VFP and sawtooth mode in SLC damping rings
4. Instability from CSR in a compact storage ring
5. Results for bursts of CSR in NSLS-VUV

TOPICS – Part II

1. Single-pass CSR, in bunch compressors, etc.
2. Motivation for Fourier analysis of fields
3. Solution of wave equations
4. Treatment of fast oscillations in inverse FT
5. Preliminary numerical tests

"It is best to confuse only one issue at a time" (Kernighan and Ritchie).

"There is no use in telling more than you know, no, not even if you do not know it" (Gertrude Stein).

Incoherent and Coherent Synchrotron Radiation

N particles moving on circle of radius R with angular velocity $\omega_0 = \beta c/R$. Line density of discrete particles:

$$\Lambda(\theta, t) = \frac{1}{N} \sum_{i=1}^{N} \delta_P(\theta - \omega_0 t - \theta_i)$$

The radiated power is ($P = RI^2$)

$$P = (eN\omega_0)^2 \sum_n \mathrm{Re} Z(n) |\Lambda_n|^2 \, , \quad \Lambda_n = \frac{1}{2\pi} \int e^{-in\theta} \Lambda(\theta, 0) d\theta \, ,$$

hence

$$P = \Big(\frac{e\omega_0}{2\pi}\Big)^2 \sum_n \mathrm{Re} Z(n) \sum_{i,j} e^{in(\theta_i - \theta_j)}$$

Incoherent and Coherent Synchrotron Radiation – cont'd

Assume that the offsets θ_i are independent, identically distributed random variables with probability density $\lambda(\theta)$. Then

$$< P >= (e\omega_0)^2 \sum_n \mathrm{Re} Z(n) \left[\frac{N}{(2\pi)^2} + N(N-1)|\lambda_n|^2 \right] ,$$

with variance $\Delta P =< P > \mathcal{O}(N^{-1/2})$.

Incoherent radiation (from $i = j$) is $\mathcal{O}(N)$.

Coherent radiation (from $i \neq j$) is $\mathcal{O}(N^2)$.

Shielded Coherent Synchrotron Radiation

For a Gaussian of r.m.s. width σ,

$$|\lambda_n|^2 = \frac{1}{(2\pi)^2} \exp\left[-\left(\frac{n\sigma}{R}\right)^2\right]$$

Coherent radiation of wave length $2\pi R/n$ can be excited only if $R/n > \sigma$; (one sometimes hears "only if the wave length is bigger than the bunch size" – wrong by 2π).

However, shielding due to the vacuum chamber exponentially suppresses $\mathrm{Re}Z(n)$ for

$$\frac{R}{n} > \frac{h}{\sqrt{2}}\left(\frac{h}{R}\right)^{1/2}, \quad h = \text{chamber height}$$

(estimate for parallel plate model)

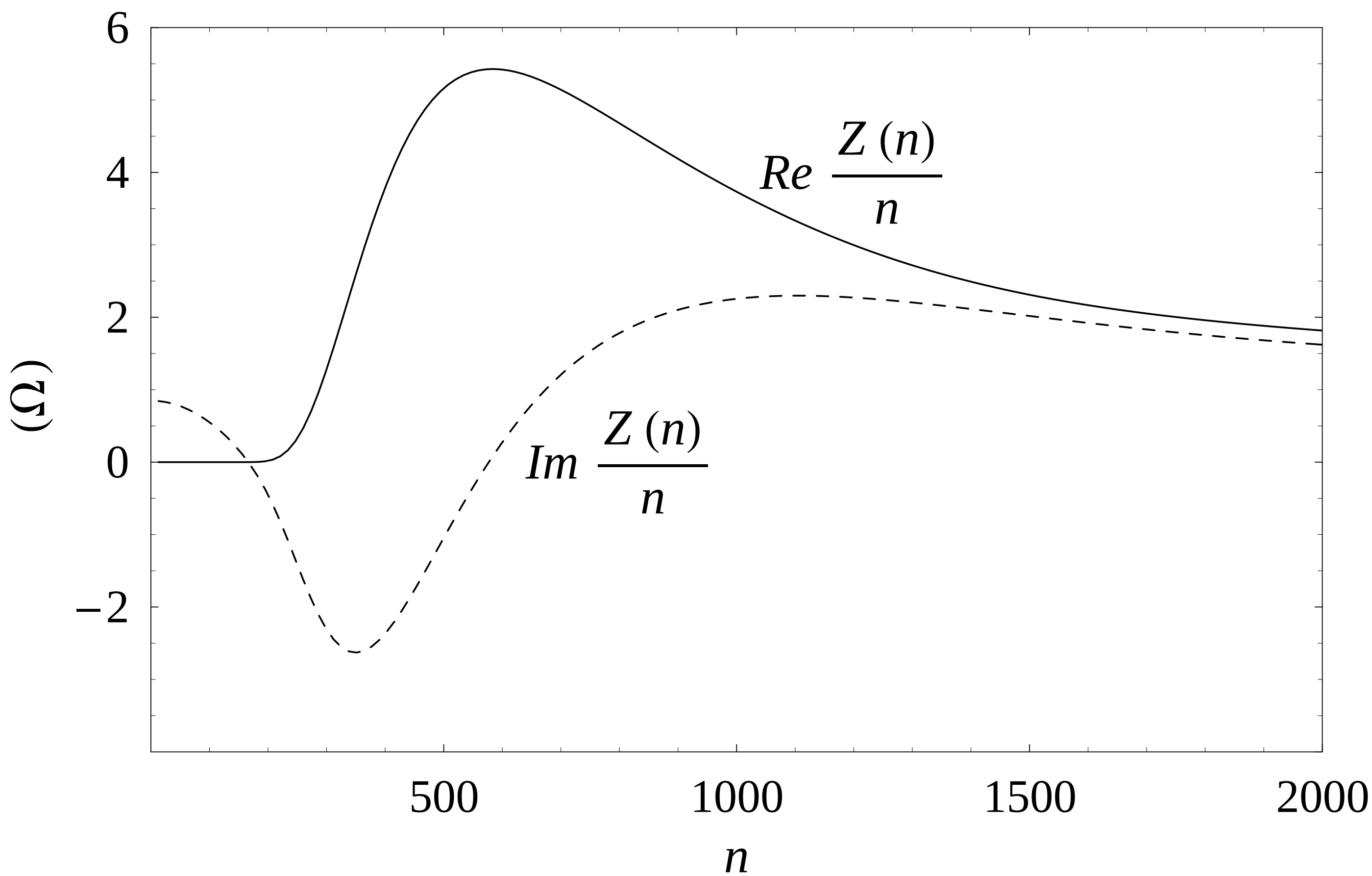


Figure 1: Impedance for parallel plate model, $h = 1$ cm , $R = 25$ cm , $E_0 = 25$ MeV

Microbunching can overcome shielding

CSR of wavelength $2\pi R/n$ is excited and unshielded if and only if

$$\sigma < \frac{R}{n} < \frac{h}{\sqrt{2}}\left(\frac{h}{R}\right)^{1/2}$$

If σ is the nominal bunch length, this is usually impossible for all n in normal storage rings.

However, if σ is interpreted as the size of a microstructure on the bunch, formed through an instability, then we may satisfy both inequalities.

Microbunching can overcome shielding – cont'd

More exactly, if

$$n > \sqrt{2}\left[\frac{R}{h}\right]^{3/2} = \text{ shielding cutoff}$$

and $|\lambda_n|^2$ is sufficiently large (through ripples or sharp edges in the bunch form), we can have substantial CSR.

We try to show that recent observations of CSR in storage rings arise in this way, the ripples coming from an instability induced by the CSR force itself and/or geometric impedances.

Experimental Observation of CSR

1. 1989 – Nakazato *et al.* – linac and bending magnet. Apparently overcame shielding through high Fourier components in bunch.

2. 2000 – 2002 – Semi-periodic bursts of IR radiation at light source storage rings (NSLS-VUV, NIST, BESSY, MAX-LAB, ALS). N^2 enhancement, polarization characteristic of CSR. Wave length $\ll \sigma(\text{nominal})$. Time between bursts is fraction of damping time.

3. 2002 –Steady CSR at BESSY in setup with low momentum compaction.

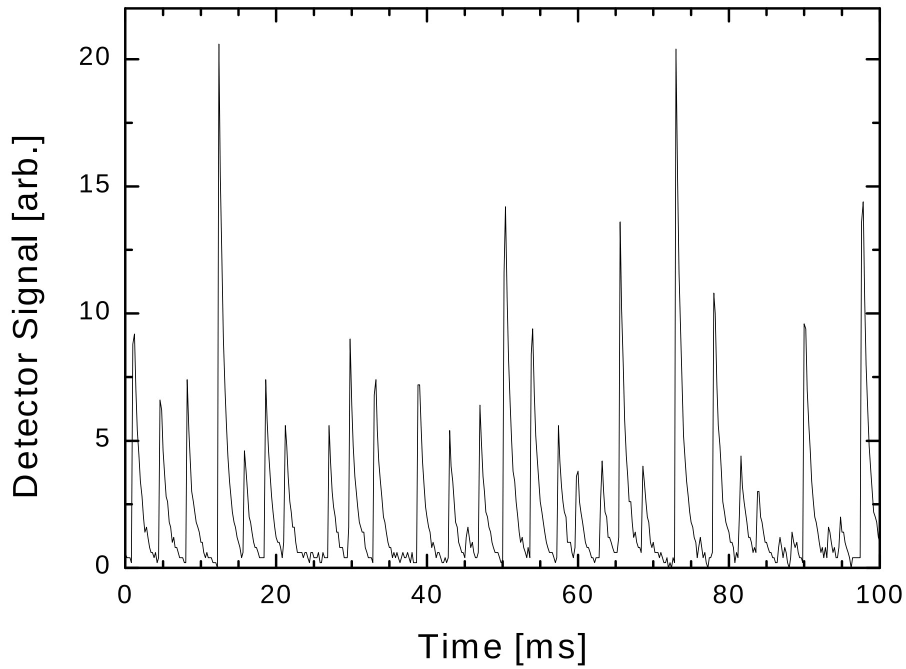


Figure 2: Far infrared detector output at NSLS VUV (Courtesy of G. Carr) Damping time $\tau_\epsilon = 7$ ms

Equations of Longitudinal Motion

$$\frac{dq}{d\tau} = p \ , \qquad \frac{dp}{d\tau} = -q + I_c F(q, f, \tau) \ ,$$

where (q, p) are normalized phase space coordinates:

$$q = \frac{z}{\sigma_z} \ , \quad p = -\frac{E - E_0}{\sigma_E} \ , \quad \tau = \omega_s t \qquad \left(\frac{\omega_s \sigma_z}{c} = \frac{\alpha \sigma_E}{E_0}\right)$$

The Collective Force , $I_c F(q, f, \tau)$, is a functional of

$f(q, p, \tau)$ = phase space distribution function

I_c = current parameter

Collective Force from Wake Potential or Impedance

$$\text{Charge density} = \rho(q,\tau) = eN\int f(q,p,\tau)dp$$

$$= (eN\sigma_z/R)\lambda(\theta,t)\ . \quad \left(\theta = q\sigma_z/R\ . \quad t = \tau/\omega_s\right)$$

$$F(q,f,\tau) = \int W(q-q')\rho(q',\tau)dq' = \omega_0\sum_n Z(n)e^{in\theta}\lambda_n(t)$$

This representation of the collective force F is an approximation. No retardation!

Vlasov-Fokker-Planck Equation

$$\frac{\partial f}{\partial \tau} + p\frac{\partial f}{\partial q} - \frac{\partial f}{\partial p}\left[q + I_c F(q, f, \tau)\right] = \frac{2}{\omega_s t_d}\frac{\partial}{\partial p}\left(pf + \frac{\partial f}{\partial p}\right) . \quad (1)$$

$$\frac{\partial f}{\partial \tau} + Vf = FPf$$

$V = \text{Vlasov operator} \leftrightarrow$

$\text{nonlinear self} - \text{consistent Hamiltonian dynamics}$

$FP = \text{Fokker} - \text{Planck operator} \leftrightarrow$

$\text{damping and diffusion from incoherent radiation}$

Numerical Solution of the VFP Equation

Operator Splitting: $V \to FP \to V \to FP \to \cdots$

(1) Propagate over time step $\Delta\tau$ by (nonlinear) Vlasov operator alone

(2) Propagate over time step $\Delta\tau$ by (linear) Fokker-Planck operator alone.

Vlasov integration by Method of Local Characteristics

Fokker-Planck integration by finite-difference approximation of p-derivatives and simple Euler step in time.

Method of Local Characteristics

Set of Characteristics given by map

$$M(z) = M(\tau + \Delta\tau, \tau, f)(z)$$

which propagates any phase space point $z = (q, p)$ over a time step $\Delta\tau$:

$$M(z(\tau)) = z(\tau + \Delta\tau)$$

In principle, M depends on the distribution f at all times previous to $\tau + \Delta\tau$, but for small $\Delta\tau$ we ignore changes in M due to changes in f during $(\tau, \tau + \Delta\tau)$. We then speak of Local Characteristics, determined by history up to time τ, valid over a small time step $\Delta\tau$.

Method of Local Characteristics – cont'd

Conservation of probability (for volume preserving map):

$$f(M(z), \tau + \Delta\tau) = f(z, \tau)$$

hence

$$f(z, \tau + \Delta\tau) = f(M^{-1}(z), \tau)$$

Numerically we realize this equation by defining f through its values on a Cartesian grid, with polynomial interpolation for off-grid points. The "unknowns" to be propagated are $f(z_i, \tau)$ for N grid points z_i.

The map is symplectic, a composition of a wake field kick and a rotation.

Application to SLC damping ring

See R. W. and J. Ellison, in *Physics of High Brightness Beams* (World Scientific, 2000)

- Apply Karl Bane's wake potential, for now without CSR.
- Starting with Haïssinski equilibrium, integrate VFP for several damping times.
- At small current the equilibrium is stable, invariant under the numerical time evolution.
- At a current threshold the equilibrium goes unstable, with constant-amplitude quadrupole-like oscillations in bunch length or energy spread.

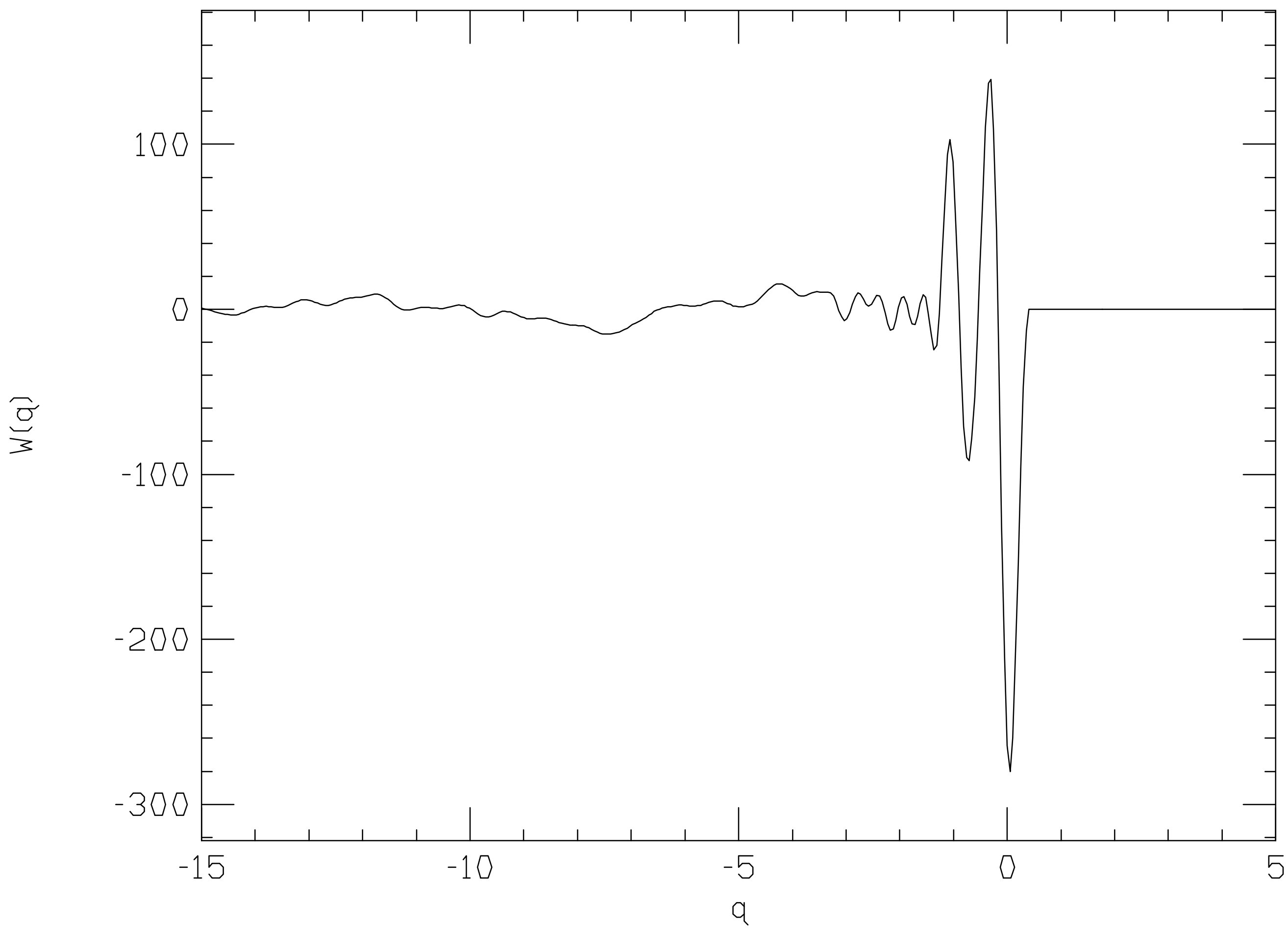
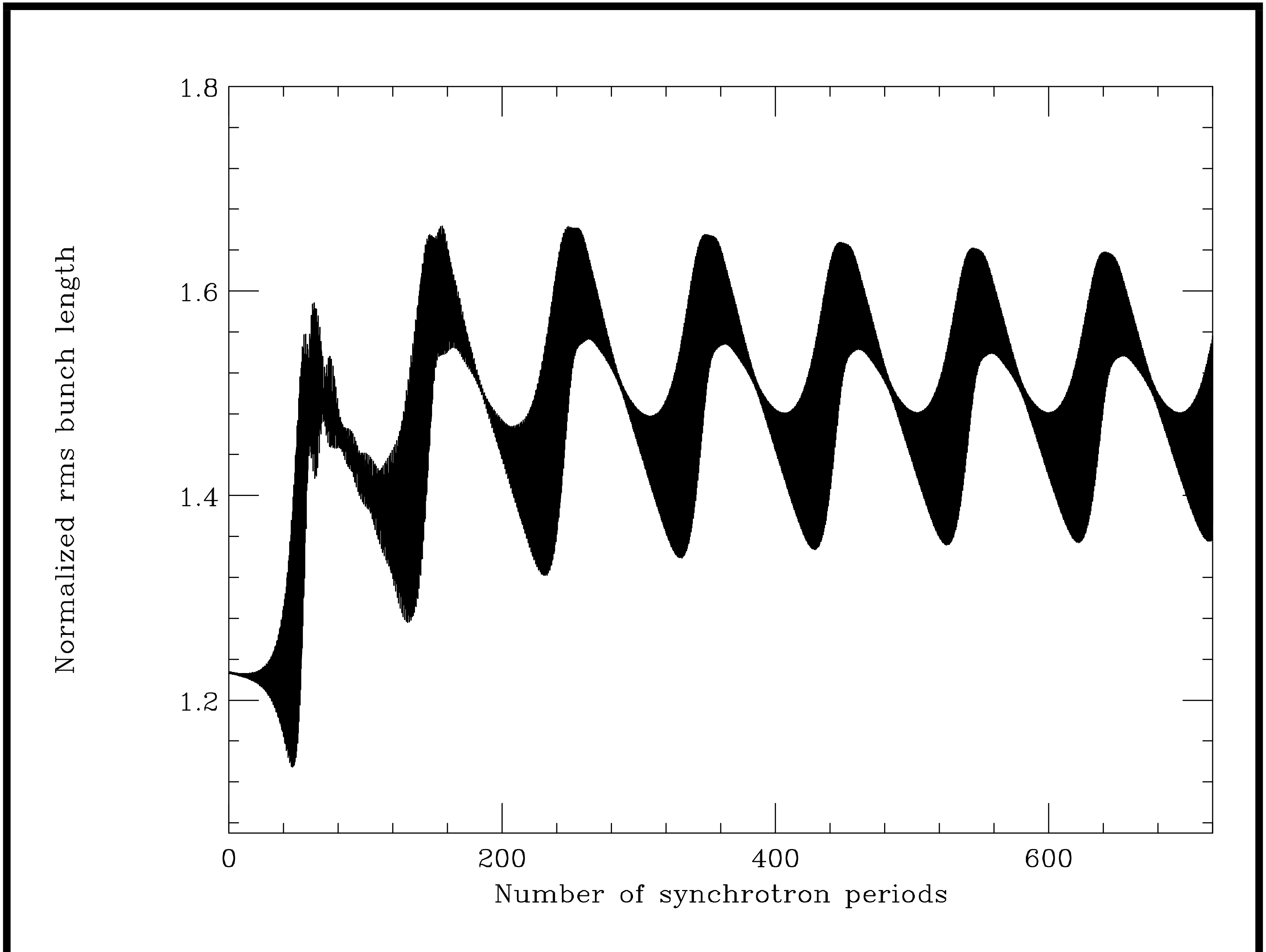


Figure 3: Bane's wake potential for SLC damping ring

Application to SLC damping ring–cont'd

- At a still higher current, there is a sawtooth modulation of the amplitude of quadrupole oscillations, with a period equal to a fraction of the damping time.
- Good agreement with experiment for thresholds of instability and sawtooth behavior, frequency of quadrupole oscillations (e.g., $\omega = 1.84\omega_s$), and period of sawtooth. Transition to constant-amplitude sextupole oscillations, seen in experiments, does not appear.


1.8
1.6
1.4
1.2
Normalized rms bunch length
0
200
400
600
Number of synchrotron periods

CSR in a compact storage ring

- Small 25 MeV storage ring to produce X-rays by Compton scattering on laser pulse stored in optical cavity (R. Lowen, R. Ruth.)
- Small circumference (6.3 m) to maximize collision frequency.
- Because of small bending radius, effect of CSR on beam stability is an issue.
- Because of low energy, damping time $\gg$ storage time.

CSR in a compact storage ring – cont'd

Typical relevant parameters:

Bending radius $= R = 25$ cm

Energy $= E_0 = 25$ MeV

Energy spread $= \sigma_E/E_0 = 3 \times 10^{-3}$

Bunch length $= \sigma_z = 1$ cm

Bunch population $= N = 6.25 \times 10^9 = 1$ nC

Synchrotron tune $= \nu_s = 0.018$

Damping time $= \tau_d = \infty$

Vacuum chamber height $= h = 1$ cm

CSR in a compact storage ring – cont'd

- Compute collective force from parallel-plate impedance and current value of FT of charge distribution. Use of wake potential (or integral of wake potential) proved to be impractical. Besides, it is informative to follow the bunch spectrum in time.
- Start run with Haïssinski equilibrium, even though injected beam is far from equilibrium. "Best case" regarding stability.
- Compare threshold of instability with coasting beam theory.

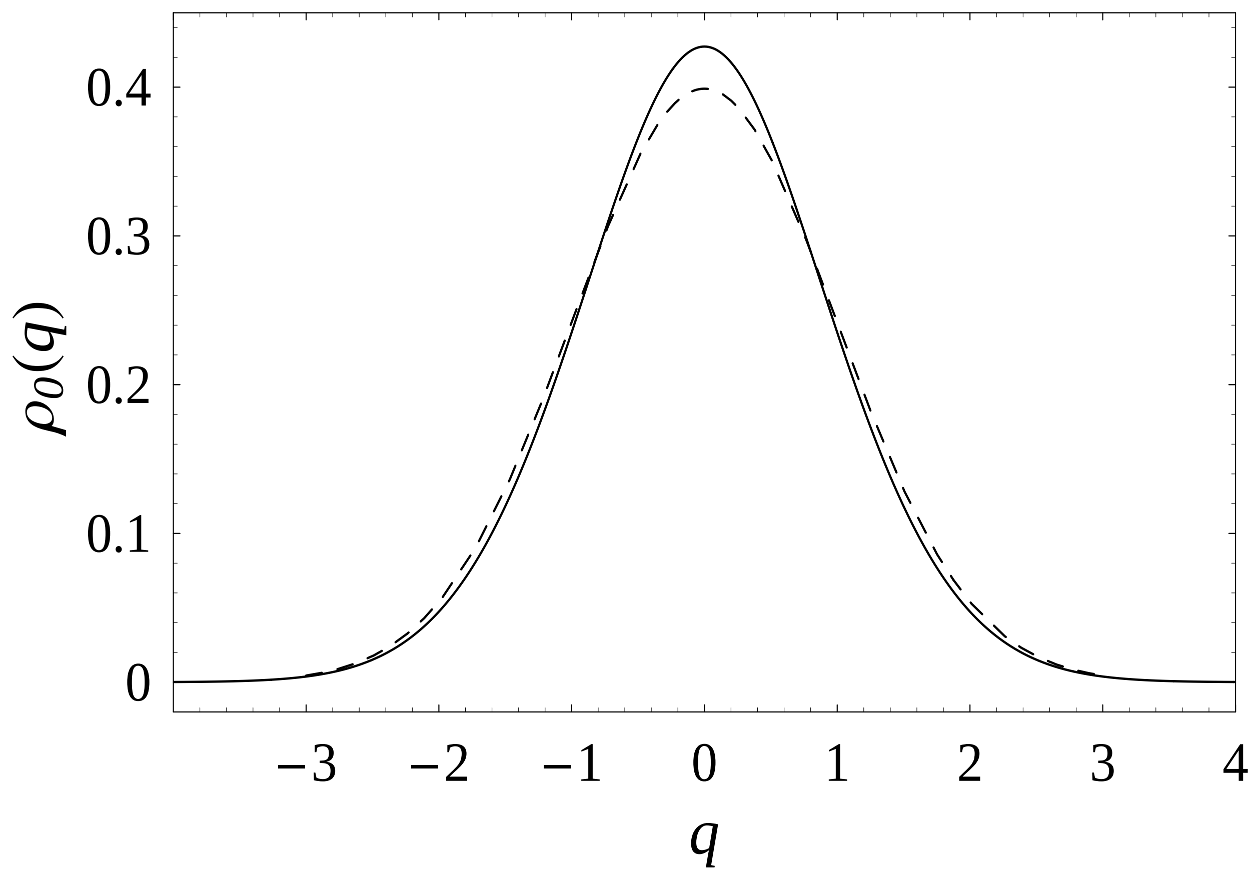


Figure 4: Equilibrium for compact storage ring. Dashed = unperturbed Gaussian

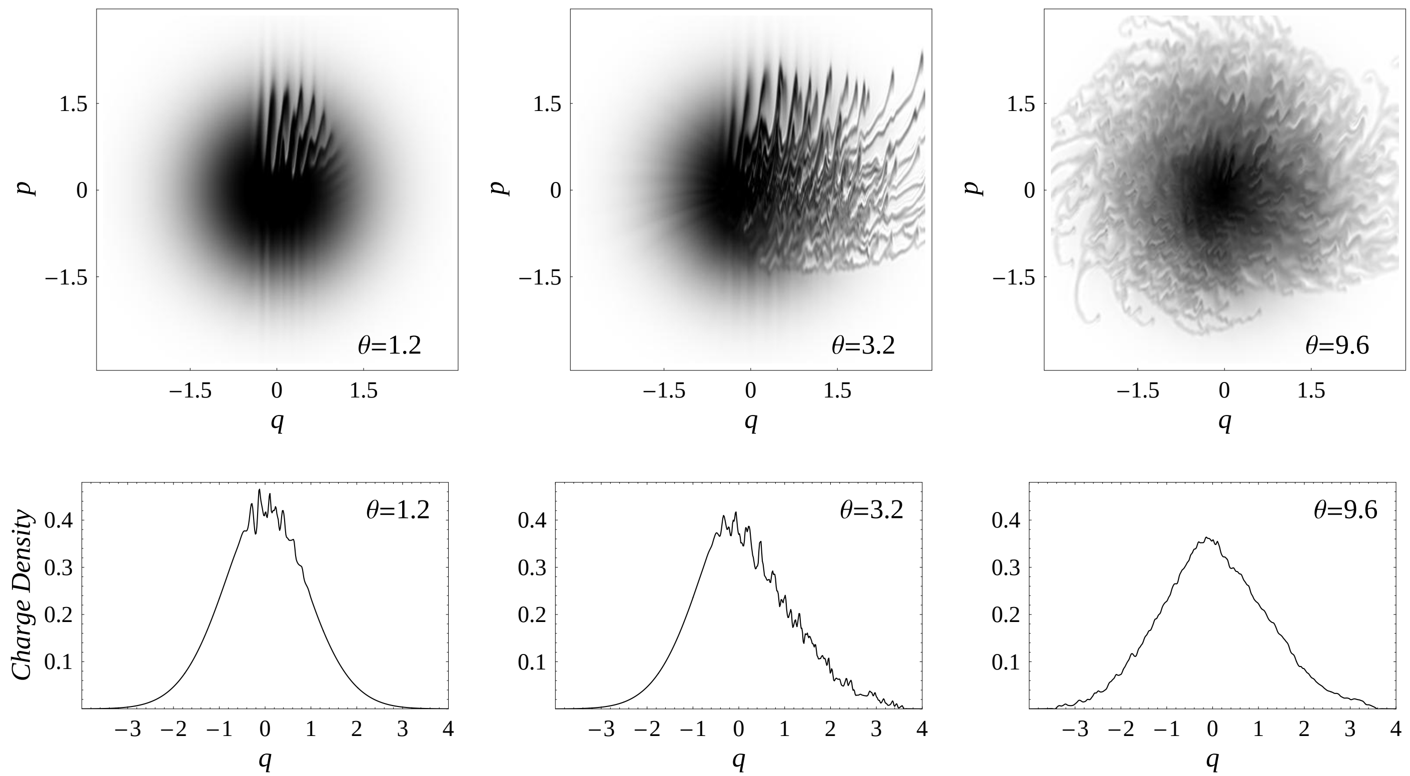


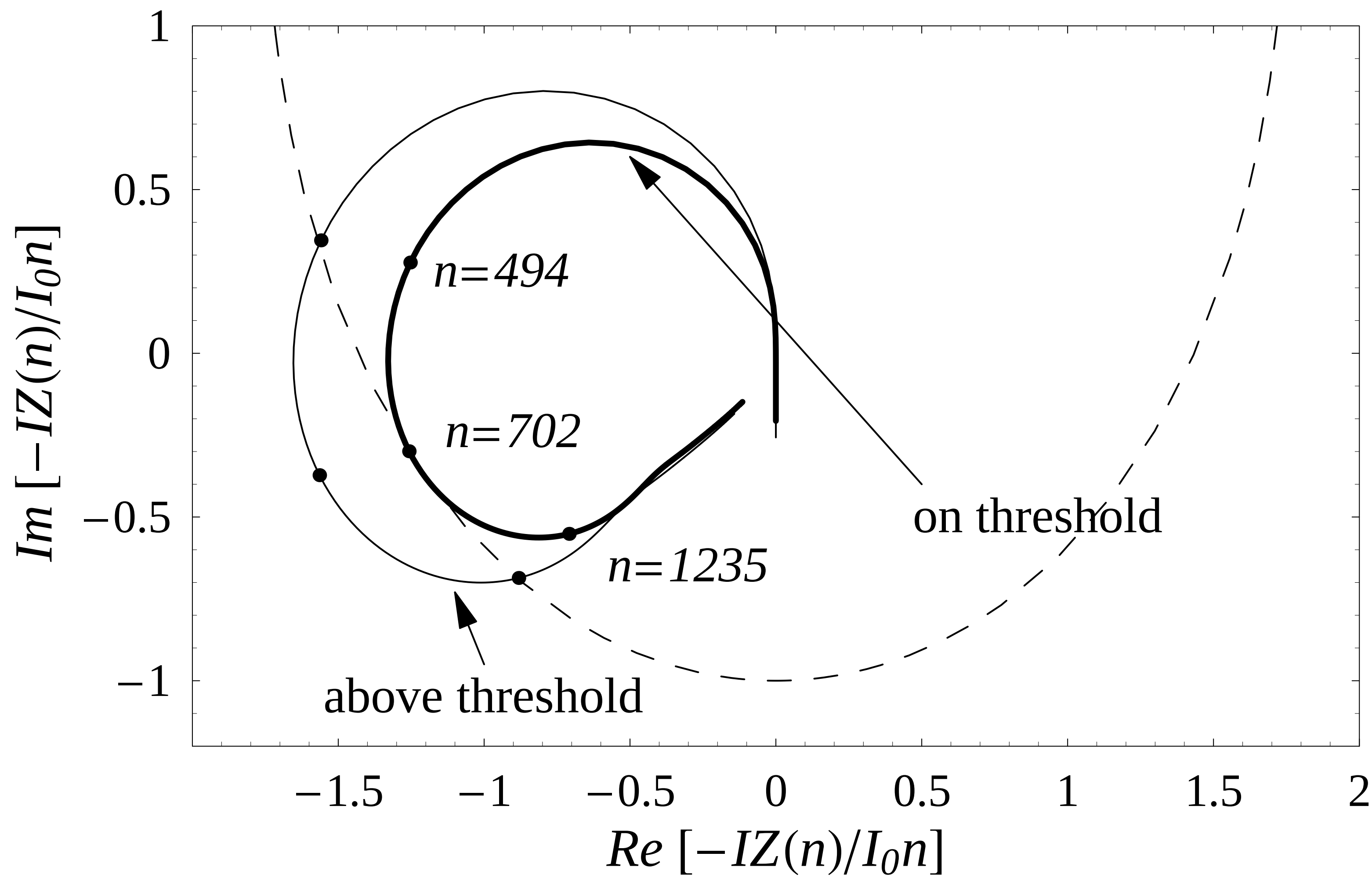
Figure 5: Phase space and charge densities, $\omega_s t$ = 1.2, 3.2, 9.6. Unit of q is 1 cm

Stability by linearized Vlasov equation

Linearize Vlasov equation about the equilibrium distribution (Gaussian in p)

If wavelength of an unstable mode is small compared to the bunch length, the Coasting Beam Approximation is valid (ignore r.f. focusing)

Then FT of Vlasov equation is a soluble integral equation, by which we find the first mode to become unstable ($\text{Im}\ \omega > 0$) as the current is increased from zero.


1
0.5
0
−0.5
−1
−1.5
−1
−0.5
0
0.5
1
1.5
2
$Im\ [-IZ(n)/I_0 n]$
$Re\ [-IZ(n)/I_0 n]$
n=494
n=702
n=1235
on threshold
above threshold

Bursts of CSR in the NSLS-VUV Ring

M. Venturini and R.W., Phys. Rev. Lett. **89**, 224802 (2002); G. Carr *et al.*, Nucl. Instr. Meth. Phys. Res. A **463**, 387 (2001).

Typical relevant parameters:

Bending radius = $R = 1.9$ m

Energy = $E_0 = 737$ MeV

Energy spread = $\sigma_E/E_0 = 5 \times 10^{-4}$

Bunch length = $\sigma_z = 5$ cm

Synchrotron tune = $\nu_s = 0.0020$

Damping time = $\tau_d = 10$ ms

Vacuum chamber height = $h = 4.2$ cm

Bursts of CSR: course of computation

- Again, the collective force is from the parallel-plate radiation impedance alone (but it is suspected that a certain bellows impedance is also important).
- Include Fokker-Planck terms to account for damping and diffusion due to incoherent synchrotron radiation.
- Integrate VFP for several damping times, starting with equilibrium (now essentially Gaussian), with a small sinusoidal perturbation with wavelength of the "most unstable mode" of linearized coasting beam theory.

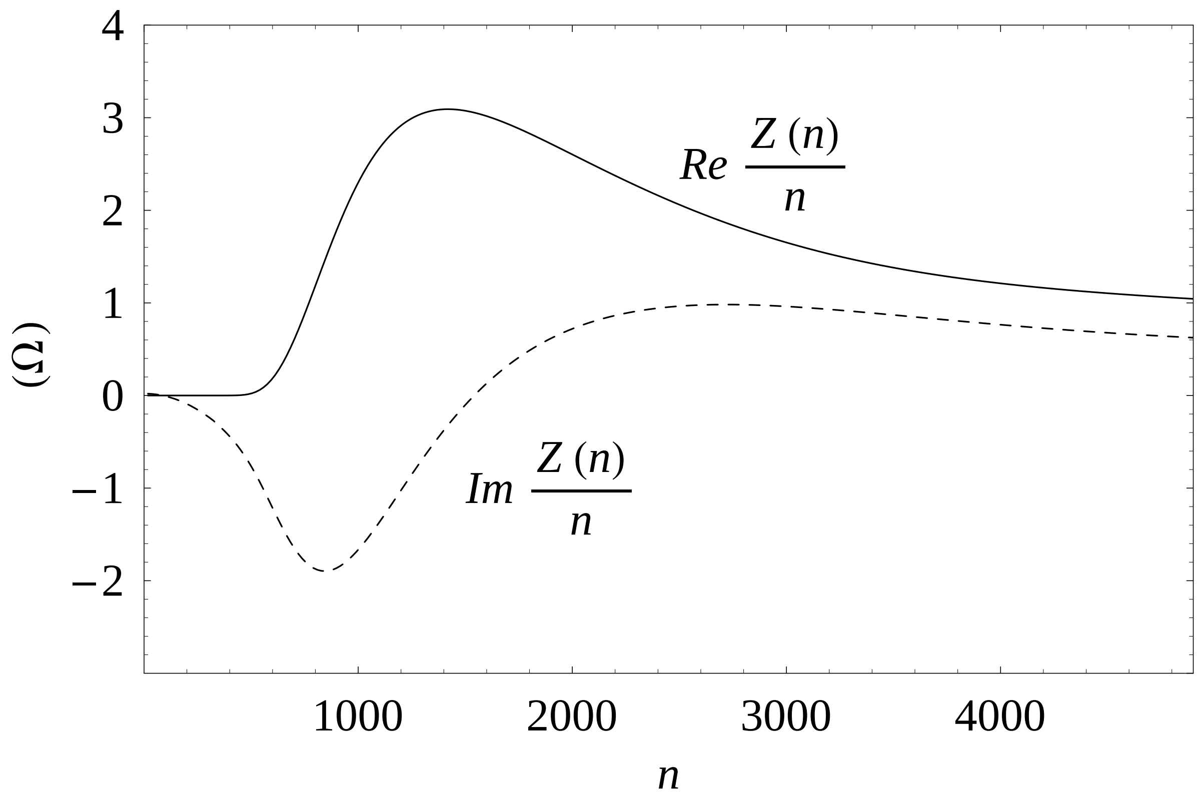


Figure 6: Parallel-plate radiation impedance for VUV parameters: $R = 1.9$ m, $h = 4.2$ cm, $E_0 = 737$ MeV.

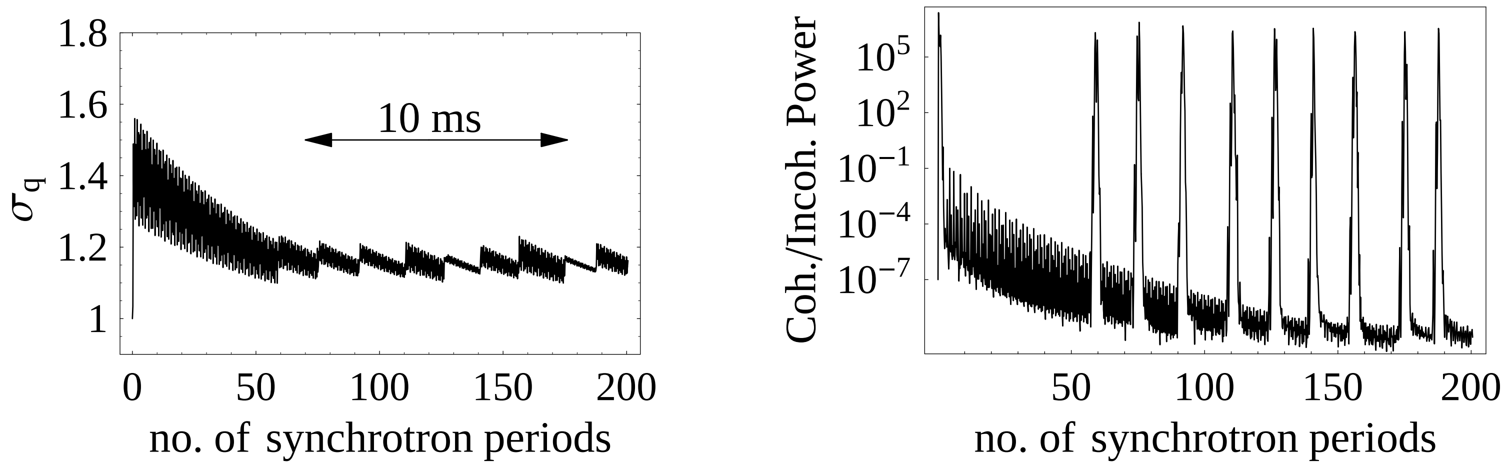


Figure 7: Normalized bunch length and ratio of coherent to incoherent power, versus time.

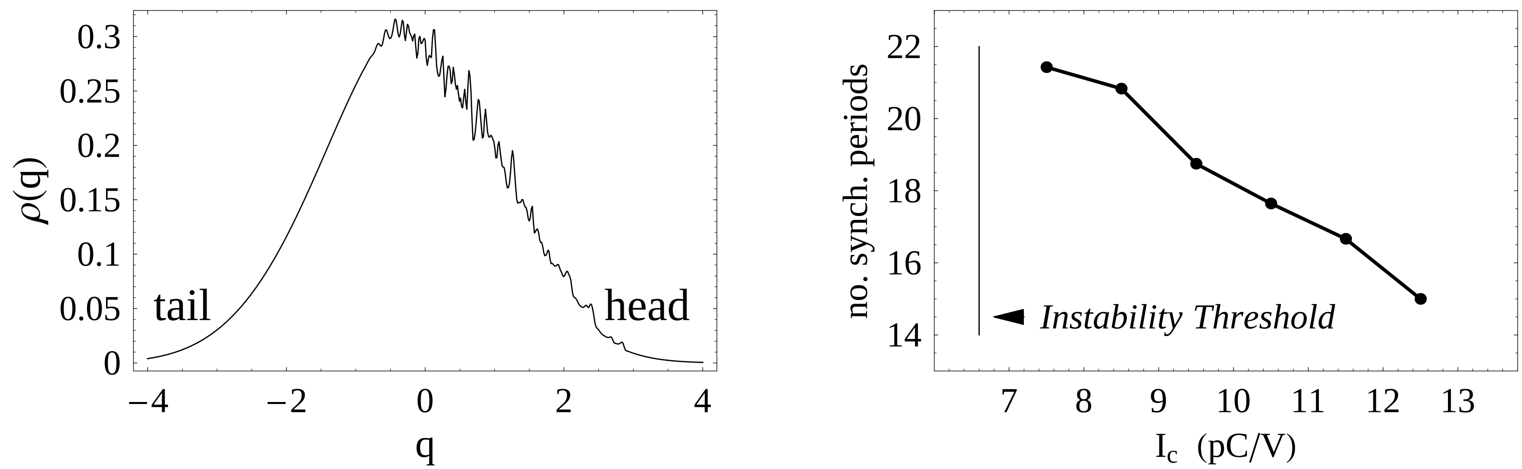


Figure 8: Charge density at peak of burst (left); burst separation versus vs. current (right)

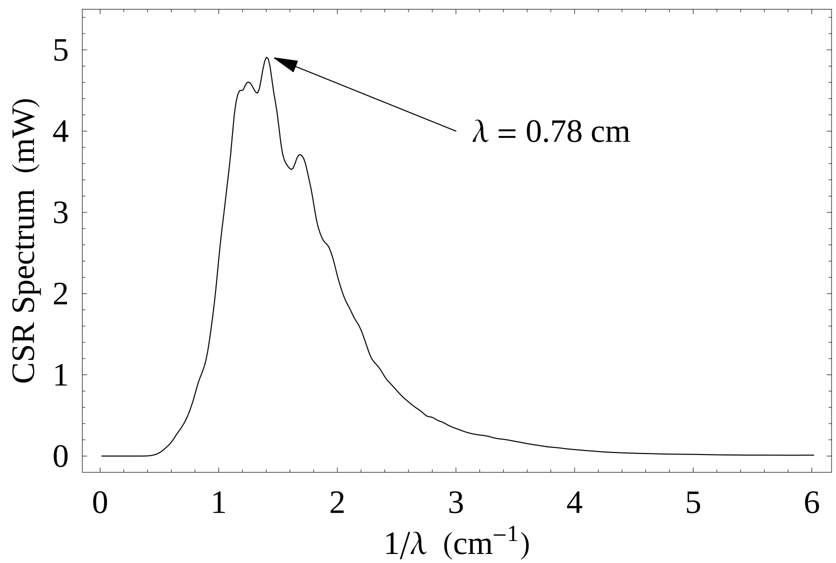


Figure 9: Spectrum of bunch, averaged in time over the burst. Peaks near "most unstable mode".

Bursts of CSR: the qualitative picture

(1) Rapidly growing instability with mode spectrum peaked near the most unstable mode of linear coasting beam theory. Attendant ripples in phase space density, and a burst of radiation.

(2) Quick phase mixing, which smooths and broadens phase space distribution in less than one synchrotron period. Removes conditions for instability, and accounts for short duration of the burst.

Bursts of CSR: the qualitative picture – cont'd

(3) Slow damping and diffusion due to incoherent radiation restore conditions for instability, causing another burst after a fraction of the longitudinal damping time.

(4) At high current the conditions for instability are less stringent, so it takes less damping to restore them. In agreement with experiment, the burst spacing decreases with increasing current.

(5) Notches in the sawtooth pattern are correlated with bursts.

Considerations for improved calculations

Effect of non-circular orbits : bend–to–straight transitions.

Effect of "geometric wake fields" from usual vacuum chamber corrugations.

Effect of non-zero transverse emittance.

Corrections to the collective force, even in our model with parallel plates and zero transverse emittance.

Simulation of steady-state CSR in ring with low momentum compaction (BESSY).

Corrections to collective force

With a deforming bunch the complete impedance defined by $\hat{V}(n,\omega) = Z(n,\omega)\hat{I}(n,\omega)$ is in principle required:

$$V(\theta,t) = eN\omega_0 \sum_n e^{in\theta} \int_{\mathrm{Im}\omega=v} d\omega e^{-i\omega t} Z(n,\omega)$$

$$\frac{1}{2\pi}\int_0^\infty dt' e^{i(\omega-n\omega_0)t')}\lambda_n(t') \ , \quad v > 0 \ . \qquad (2)$$

From causality, expressed by $Z(n,\omega)$ being analytic in upper half ω-plane), we expect no contribution for $t' > t$.

Corrections to collective force – cont'd

A careful analysis shows that the t'-integral, rather than merely being truncated at $t' = t$, is to be replaced by

$$i\lambda_n(t)\frac{e^{i(\omega - n\omega_0 t)}}{\omega - n\omega_0} + \int_0^t dt' e^{i(\omega - n\omega_0)t')}\lambda_n(t') \ .$$

A surprising extra term with a pole singularity, which in retrospect is not surprising: we have $\delta(n - n\omega_0)$ for a rigid bunch.

Can we approximate the resulting complicated expression for the force?

Collective force with leading effects of retardation

The t'-integral is concentrated near the synchronous point $\omega = n\omega_0$ (phase velocity = particle velocity). Expand $Z(n, \omega)$ in ω about that point, except near wave-guide cutoffs where Z has poles. Then some analysis gives

$$
\begin{aligned}
&V(z,t) = \\
&Q\omega_0 \sum_n e^{inz/R} \Big[\tilde{Z}(n, n\omega_0)\lambda_n(t) + i\frac{\partial \tilde{Z}}{\partial \omega}(n, n\omega_0)\lambda_n'(t) + \cdots \\
&+ \frac{Z_0 \pi R}{2\beta h} \sum_p \Lambda_p \int_{-t}^{0} \lambda_n(t+u)du \Big((n\omega_0 - \alpha_p c) e^{-i(n\omega_0 - \alpha_p c)} \\
&+ (p \to -p) \Big) \Big] , \quad \alpha_p = \pi p/h \ .
\end{aligned}
$$

Conclusions and Outlook

- Bursts of CSR explained by micro-bunching overcoming shielding, the microbunching induced by CSR itself (or CSR plus geometric wake). Duration of bursts is time for phase space mixing. Spacing is determined by interplay of incoherent radiation damping and the instability to microbunching.
- Time domain integration of Vlasov-Fokker-Planck equation has proved to be a successful and exciting development. We anticipate many more applications, especially for coherent radiation in bunch compressors, undulators, etc.

Conclusions and Outlook - cont'd

- Modeling of steady state CSR in BESSY is already underway. Seems to be caused by extreme potential well distortion from CSR, under small momentum compaction.

- Much interesting hard work to refine the modeling lies ahead. Recall Nietzsche :

"Aus der Kriegsschule des Lebens. -Was mich nicht umbringt, macht mich stärker"

(" From the Military School of Life: what doesn't kill me, makes me stronger").