A STUDY ON ION BEAM STABILITY IN ELIC

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There exist various collective modes which can become unstable for the beam in an ion storage ring which is currently being considered in our ELIC design. We have studied major beam stability problems most likely to affect beam in ELIC's highest energy ion ring. In the following we summarize results for proton beam to be more specific.

Longitudinal Microwave Instability Threshold:

$$I_p = \frac{2\pi|\eta|\left(\frac{E}{e}\right)\beta^2\sigma_\delta^2}{\left|\frac{Z}{n}\right|_{eff}} \qquad ☺ \qquad \left|\frac{Z}{n}\right|_{eff} = 133\Omega$$

Tune Spread due to Nonlinear RF Bucket:

$$\Delta\nu_s = \frac{1}{8}\left(\frac{\sigma_z}{R}\right)^2 h_{rf}{}^2\nu_s \qquad ☺ \qquad \Delta\nu_s = 1.8\times10^{-4}$$

Longitudinal Coupled Bunch Instability:

$$I_p = \frac{\pi|\eta|^3\left(\frac{E}{e}\right)h_{rf}^2\beta^2\sigma_\delta^4}{4\,\mathrm{Im}(\frac{Z}{n})_{eff}\,\nu_s^2} \qquad ☺ \qquad \mathrm{Im}(\frac{Z}{n})_{eff} = 1.5\Omega$$

Transverse Microwave Instability Threshold:

$$I_b = \frac{4\left(\frac{E}{e}\right)\nu_s b}{|Z_T|_{eff}\,\beta_{av}R} \qquad ☺ \qquad |Z_T|_{eff} = 2.5G\Omega/m$$

Transverse Mode Coupling Instability Threshold:

$$I_p = \frac{4\left(\frac{E}{e}\right)\nu_s}{\mathrm{Im}(Z_T)_{eff}\,\beta_{av}}\frac{4\sqrt{2}}{3}\pi\beta \qquad ☺ \qquad \mathrm{Im}(Z_T)_{eff} = 5.4G\Omega/m$$

Strong Head-Tail Instability Threshold due to Beam-Beam Interaction:

$$\xi_e\xi_p \le \frac{\beta_e\nu_s}{\pi^2\sigma_z} \qquad ☺ \qquad \text{Safe}$$

Head-Tail Instability Growth Rate due to Beam-Beam Interaction:

$\tau^{-1} = \frac{f_{rev} D_e D_p}{16} \frac{\sigma_{pz}}{\sigma_{ez}}$ ☺ $\tau = 195(12)\mu s$ About 50(3) turns – a big problem!

Intrabeam Scattering Growth Rates:

($\log_c = 10$ is assumed)

$$\tau_s^{-1} = \frac{e^4 N_p \log_c}{8 m_p^2 c^3 \beta^3 \gamma^3 \sigma_z \varepsilon_x^{3/2} \sigma_\delta^2} \sqrt{\frac{\nu_x}{R}} \quad ☺ \quad \tau_s = 180s$$

$$\tau_x^{-1} = \frac{e^4 N_p \log_c}{16 m_p^2 c^3 \beta^3 \gamma^3 \sigma_z \varepsilon_x^{5/2}} \sqrt{\frac{R}{\nu_x}} \left(\frac{2}{\nu_x^2} - \frac{1}{\gamma^2} \right) \quad ☺ \quad \tau_x = 2.3s$$ (what does this mean?)

Electron Cooling Times:

($\log_c = 2$ and $\eta = 0.01$is assumed)

$$\tau_s = \frac{3 m_p m_e c^3 \beta^3 \gamma^5}{16 \pi n_e e^4 \eta \log_c} \left(2\sigma_{x'}^2 + 0.66 \left(\frac{\sigma_\delta^2}{\gamma^2} \right) + 1.6 \sigma_{ex'}^2 \right) \sqrt{\left(\frac{\sigma_\delta^2}{\gamma^2} \right) + 2.4 \sigma_{ex'}^2}$$

This is basically same as Derbenev's expression

$\tau_s = \frac{8\gamma\Gamma}{r_e r_p c N_e \eta \log_c} \frac{S_e}{S_{ion}}$ ☺ $N_e^{critical} = 6.7 \times 10^9$ (assuming $\eta = 0.01$ and $S_e/S_{ion} = 10$)

Synchrotron Radiation:

For Proton in Storage

P_s = 19 mW for B = 5 Tesla ☺ negligible

Number of emitted photons per bunch per revolution = 2×10^{10}

Mean energy of photons = 5 meV

For Electron in Circulator Ring

P_s = 5.1 MW for B = 2.34 kG ☺ a huge factor

Electron beam damps after 13000 (3500?) turns and requires minimum 2.2 MV just to keep electrons in the ring with no phase focusing at all.

Electron Cloud Instability:

Single bunch head-tail instability

$\rho^{th} = \frac{\gamma Q_s}{\pi^2 r_p R \beta_{av}}$ ☺ $\rho^{th} = 2.6 \times 10^{14} / m^3$ safe

Average volume density of ELIC proton beam $= 1.0 \times 10^{13} / m^3$

Coupled bunch instability

Instability due to Beam-Beam Interaction in Cooling Section:

Is this a problem? There is a possibility of instability in principle if we use a circulator ring concept for cooler.

Incoherent Space Charge Tune Shift:

$$\Delta Q_y^L = \frac{r_p R}{\sqrt{2\pi}\beta^2\gamma_p^3}\frac{\beta_y N_B}{\sigma_y(\sigma_x+\sigma_y)\sigma_z} \quad ☺ \quad \Delta Q_y^L = 0.017$$

We conclude that energy recovering linear collider has a potential for making high energy experiments demanding an extremely large luminosity possible.

APPENDIX

A consistent set of ELIC design machine and beam parameters is presented in the Table below.

A List of ELIC Parameters as of 2/23/04	
γ_p / γ_e	160/13700
$\sigma_\delta \equiv \Delta\gamma / \gamma$	3×10^{-4} (relative energy spread)
$\varepsilon_{nx}^p / \varepsilon_{nx}^e$	1/86 μm (normalized horizontal emittance)
$\varepsilon_{ny}^p / \varepsilon_{ny}^e$	0.01/0.86 μm (normalized vertical emittance)
σ_z^p / σ_z^e	5/1 mm (bunch length)
N_p / N_e	$2 \times 10^9 / 10^{10}$
β^*	5 mm (beta at interaction point)
σ_y^*	0.56 μm (vertical beam size at interaction point)
R	191 m (mean radius of ring)
ν_x, ν_y	15 (betatron tunes)
β_{av}	12.7 m (average beta in ring)
σ_y	2.82×10^{-5} m
θ_y	2.22×10^{-6}
I_b	0.08 mA (bunch current)
I_p	3.1 A (bunch peak current)
I_{av}	480 mA
M	6000 (number of bunches)
U	300 kJ (stored beam energy)
V_{rf}	1.0×10^8 V
h_{rf}	6000 (harmonic number)
f_{rev}	0.25 MHz
η	4×10^{-3} (frequency slip parameter)
ν_s	0.06 (synchrotron tune)
b	1.74 cm (beam pipe radius)
$c / 2\pi b$	2.75 GHz
ξ_{ex} / ξ_{ey}	0.0095/0.095 (tune shift per interaction)
ξ_{px} / ξ_{py}	0.0022/0.022 (tune shift per interaction)
D_{ex} / D_{ey}	0.12/1.2 (disruption per interaction)
D_{px} / D_{py}	0.0055/0.055 (disruption per interaction)
r_e	2.818×10^{-15} m
r_p	1.535×10^{-18} m (proton radius)