April 2, 2002

LUMINOSITY LIMITS OF AN ELECTRON-ION COLLIDER

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The luminosity of an electron-ion collider, in the limit of equal electron and ion bunch sizes at the interaction point (IP), can be written as:

$$L = \frac{N_e N_i}{4\pi\sigma^{*2}} f_c$$

and can be expressed in terms of the electron beam-beam tuneshift parameter, ξ_e:

$$L = \frac{N_e \xi_e \gamma_e}{r_e \beta_e^*} f_c \qquad (1)$$

where γ_e is the relativistic factor of the electrons, r_e is the classical radius of the electron, β^*_e is the electron beta function at the IP and ξ_e is equal to

$$\xi_e = \frac{N_i r_e}{4\pi\varepsilon_e^n} \qquad (2)$$

in the limit of equal electron and ion bunch sizes. From eq. (2) one can obtain maximum luminosity at the maximum value of ξ_e:

$$L_{\max} = \frac{N_e \xi_e^{\max} \gamma_e}{r_e \beta_e^*} f_c \qquad (3)$$

The maximum electron beam intensity and therefore the maximum attainable value of the ξ_e parameter is either limited by the beam-beam interaction or by the intra-bunch space-charge forces which imposes a limit on the Laslett tuneshift:

$$\Delta\nu_L = \frac{N_i r_i}{4\pi\gamma_i^3} \frac{C}{\sqrt{2\pi}} \frac{1}{\varepsilon_i l_i} \quad . \qquad (4)$$

By re-expressing the Laslett tuneshift $\Delta\nu_L$ in terms of ξ_e, one can define a space-charge-dependent beam-beam parameter given by:

$$\xi_e^{(\Delta\nu_L)} = (\Delta\nu_L)\sqrt{2\pi}\gamma_i^2 \frac{l_i}{C} \frac{E_i}{E_e} \frac{\beta_e^*}{\beta_i^*} \qquad (5)$$

Thus:

$$\xi_e^{\max} = \min\left\{\xi_{e,\max}^{bb},\ \xi_e^{(\Delta\nu_L)}\right\} \qquad (6)$$

and the ratio of ξ_e^{max}/l_i , which enters the luminosity formula, is then equal to:

$$\frac{\xi_e^{\max}}{l_i} = \min\left\{\frac{\xi_{e,\max}^{bb}}{l_i},\ (\Delta\nu_L)\sqrt{2\pi}\gamma_i^2 \frac{1}{C}\frac{E_i}{E_e}\right\} , \qquad (7)$$

where equal beta functions for the electrons and ions at the IP are assumed.

An optimized point design may be defined as one in which the two terms in the equation above are equal. The bunch length, which sets these two terms equal, is then given by:

$$l_i = \frac{\xi_{e,\max}^{bb}}{(\Delta\nu_L)} \frac{C}{\sqrt{2\pi}\gamma_i^2} \frac{E_e}{E_i} \quad . \tag{8}$$

A plot of l_i vs. E_i is shown in Fig. 1, for energies from 25 GeV to 250 GeV, ξ_e^{max} =0.2 (circulator ring option) and two values of $\Delta\nu_L$, the more conservative 0.004 and the more challenging 0.04, circumference = 3833 m (for RHIC) and E_e=10 GeV.

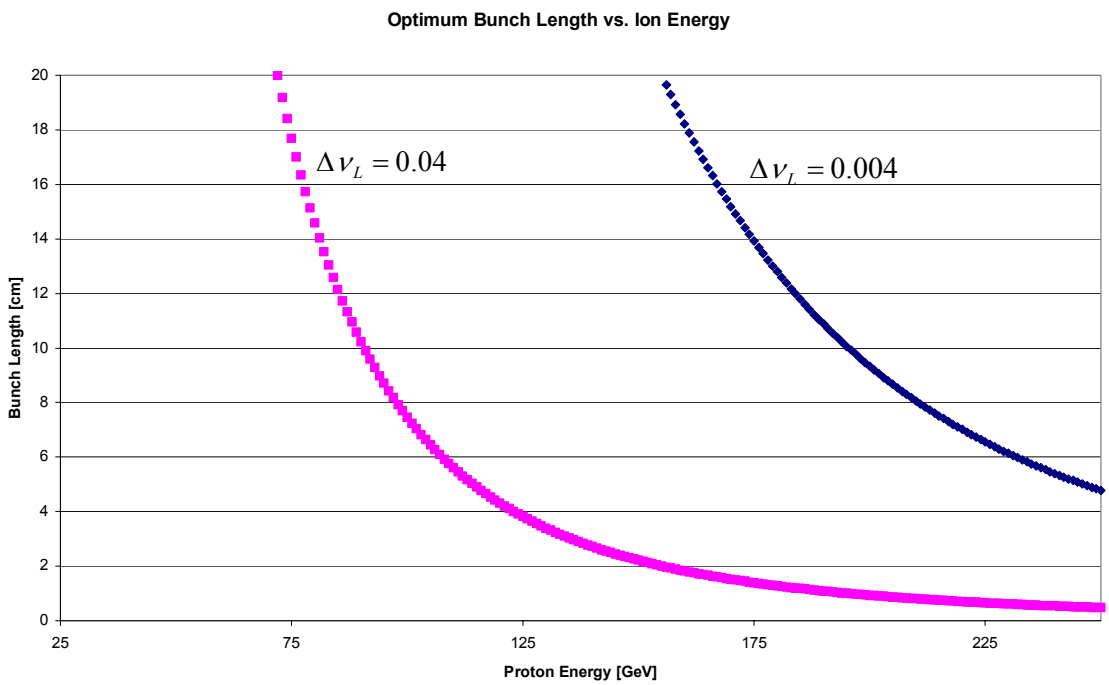

Figure 1: Proton bunch length as function of proton beam energy for RHIC parameters. The two curves correspond to two values of the Laslett tuneshift. In both cases the beam-beam parameter is assumed equal to 0.2, as expected in the presence of a circulator ring.

Note that for energies below about 75 GeV, the optimum bunch length is longer that ~20 cm, which may make operations difficult. From optics considerations at the IP, the minimum beta function at the IP can not be much smaller than ~1 cm, which sets a lower limit for the ion bunch length. Bunch length of 1 cm corresponds to proton beam energy of about 200 GeV.

We now write the expression for the maximum luminosity, eq. (3), using eq. (8) for the bunch length:

$$L_{\max} = \frac{N_e f_c}{r_e C} \gamma_e (\Delta\nu_L) \sqrt{2\pi}\gamma_i^2 \frac{E_i}{E_e} \tag{9}$$

Figure 2 is a plot of maximum luminosity vs. proton beam energy, for 10^{11} electrons per bunch and two values of the Laslett tuneshift as before. Equation (9) is used for the energy range between 75 and 200 GeV, where the optimum bunch length assumes values between 20 cm and 1 cm. For energies above 200 GeV, the maximum luminosity is limited by the beam-beam tuneshift and, for RHIC parameters it is equal to 4 x 10^{34} cm^{-2} sec^{-1}, independent of proton energy. For energies below ~75 GeV, the maximum luminosity is limited by the Laslett tuneshift and is given by:

$$L = N_e f_c(\Delta\nu_L)\gamma_i^3 \frac{\sqrt{2\pi}}{r_i C} \quad . \tag{10}$$

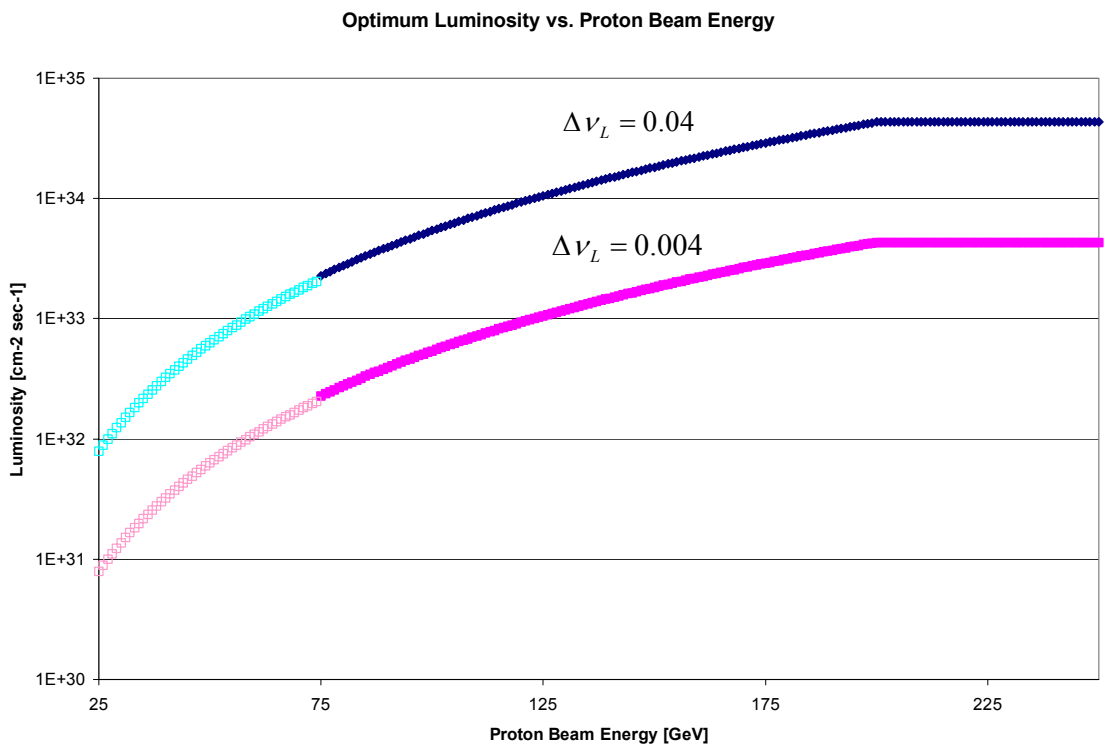

Figure 2: Maximum luminosity vs. proton beam energy for RHIC collider parameters.