Coherent Synchrotron Radiation in Particle Accelerators

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To address the questions:

- What is coherent synchrotron radiation (CSR)?
- Why is it a concern in particle accelerators?
- How we investigate the problem?
- What is the present status of CSR research?

Outline

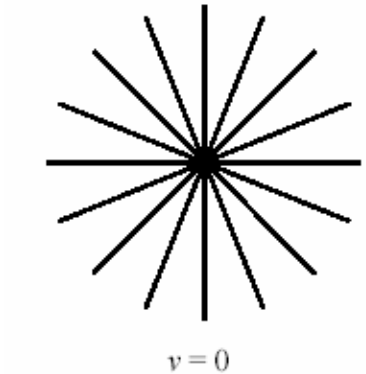
- Synchrotron Radiation by a Single Electron
- Coherent Synchrotron Radiation (CSR) by an Electron Bunch
- Collective CSR Interaction for High Brightness Beams
- CSR Effect in Particle Accelerators
- Investigation of the CSR Effects

1. Synchrotron Radiation by a Single Electron

Electromagnetic Field of a Moving Charge

Coulomb Field for a Point Charge at Rest

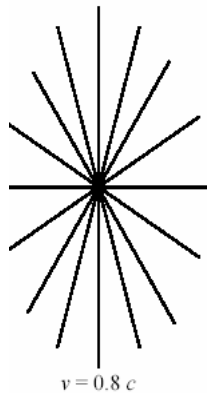
$$\vec{E} = \frac{q}{r^3}\vec{r}$$



v=0

Coulomb Field for a Point Charge with Uniform Motion

$$\vec{E} = \frac{q\vec{r}}{r^3}\frac{1}{\gamma^2(1-\beta^2\sin^2\psi)^{3/2}}$$



v=0.8c

Radiation by an Accelerated Point Charge

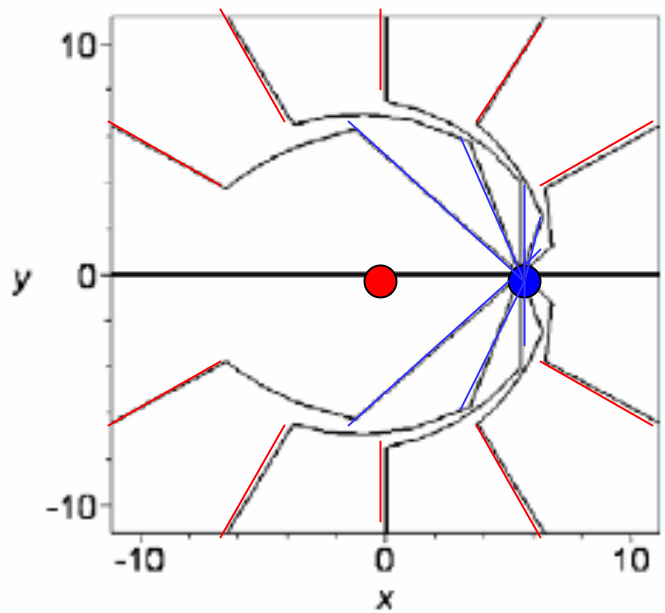
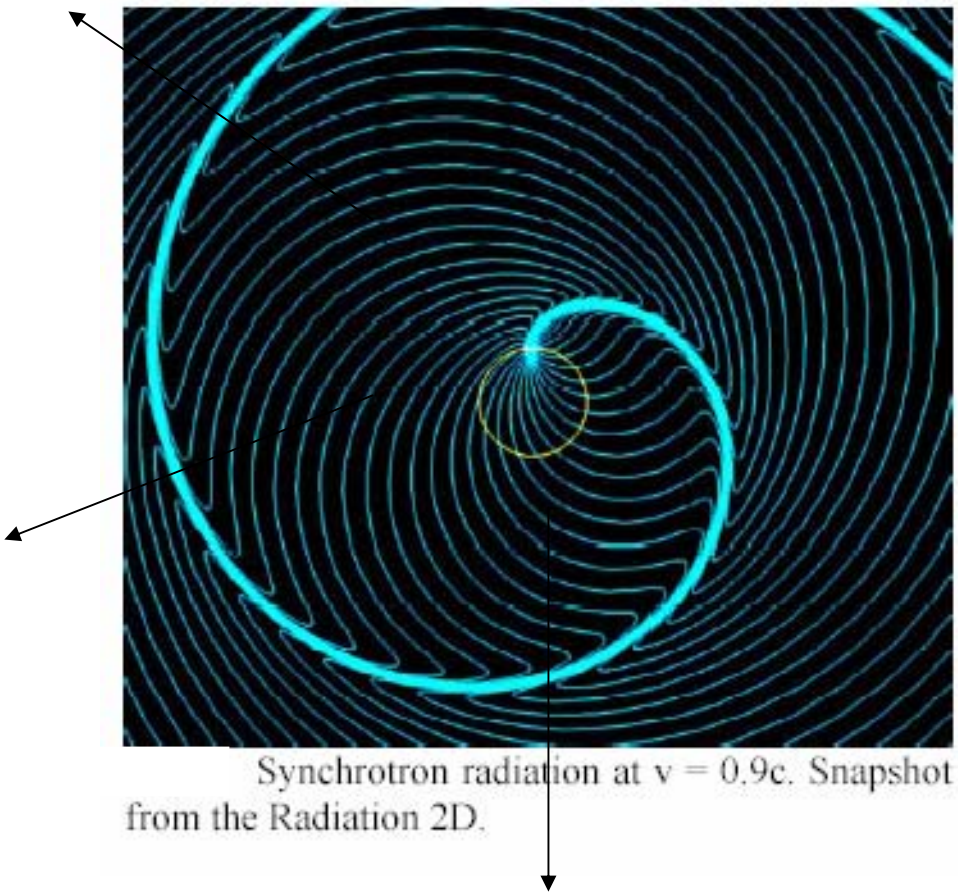


FIG. 6: The electric field lines of a briefly accelerated charge.

Black field line connecting Coulomb fields: radiation fields due to acceleration

Synchrotron Radiation

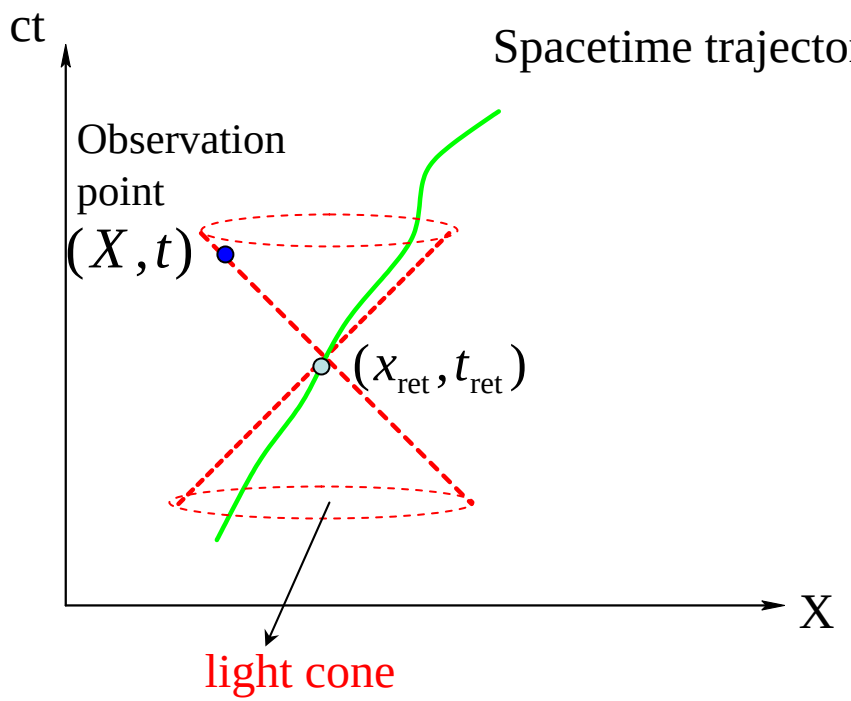


Synchrotron radiation at v = 0.9c. Snapshot from the Radiation 2D.

- **Electromagnetic radiation emitted by charged particles moving with centripetal acceleration**

- **First discovered by GE in 1947 in a type of accelerator known as synchrotron, as an unwanted by-product of storage ring accelerator.**

Single Particle Lienard-Wiechert Field



Spacetime trajectory of source electron

$$\vec{R} = \vec{X} - \vec{x}_{\text{ret}}, \qquad \vec{n} = \frac{\vec{R}}{R},$$

$$\vec{\beta} = \frac{\vec{v}_{\text{ret}}}{c}, \qquad \dot{\vec{\beta}} = \frac{\dot{\vec{v}}_{\text{ret}}}{c}$$

Retardation Condition :

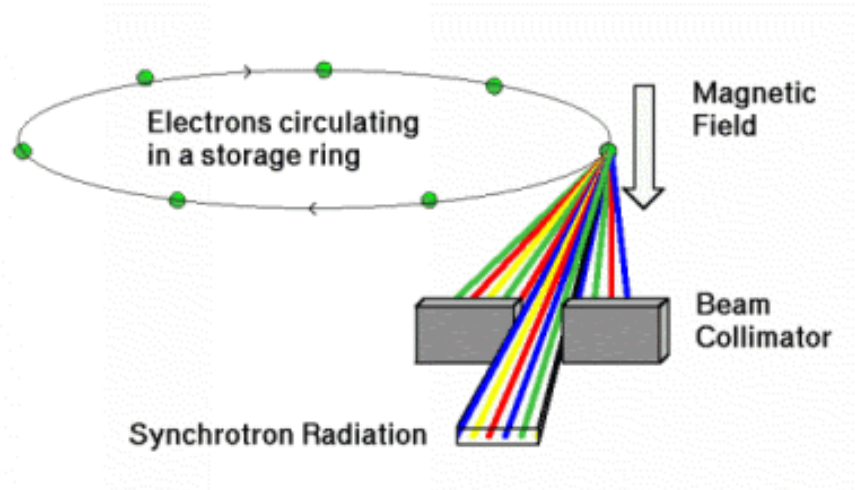
$$c(t - t_{\text{ret}}) = | X - x_{\text{ret}} |$$

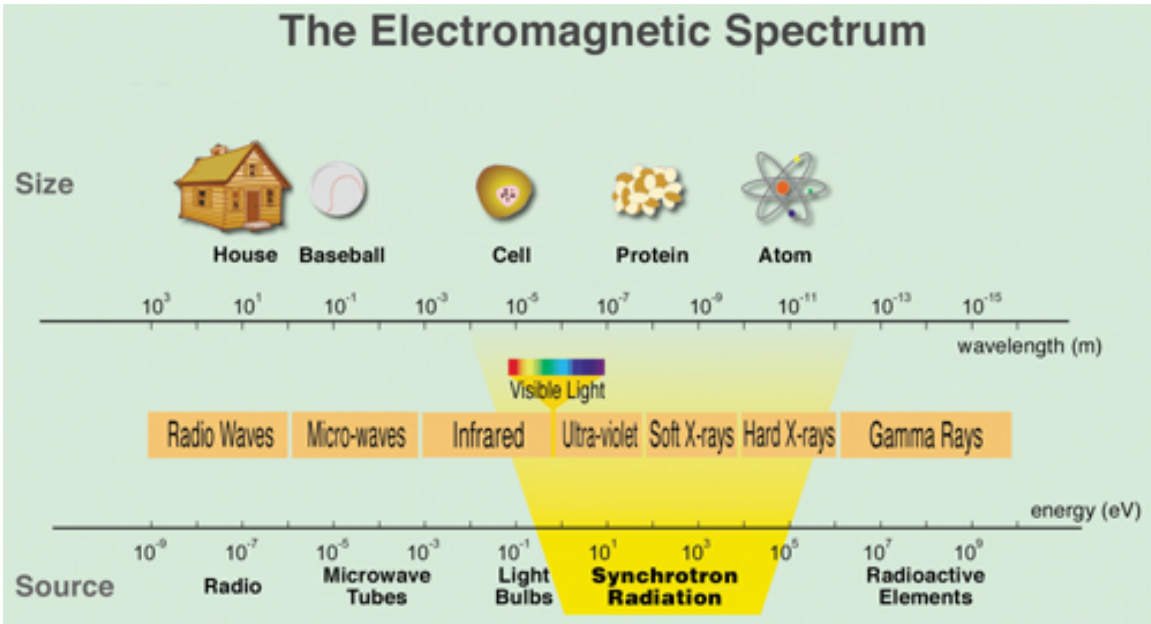
Lienard-Wiechert Field:

$$\begin{cases} \vec{E} = e\left[\dfrac{\vec{n}-\vec{\beta}}{\gamma^2(1-\vec{\beta}\cdot\vec{n})^3 R^2}\right]_{\text{ret}} + \dfrac{e}{c}\left[\dfrac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{\beta}\cdot\vec{n})^3 R}\right]_{\text{ret}} \\ \vec{B} = \vec{n}\times\vec{E} \end{cases}$$

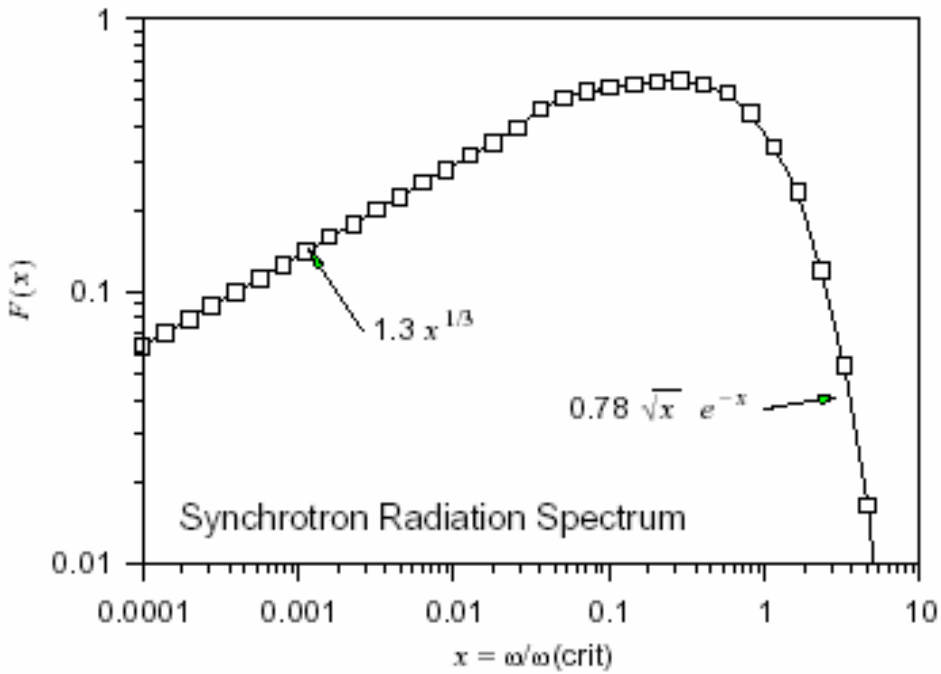
Features of Synchrotron Radiation

- extremely intense
- wide energy specturm
- highly polarized
- emitted in very short pulses (nanosec)





Spectrum of Synchrotron Radiation



The synchrotron radiation spectral function $F(x)$.

Cutoff at critical frequency: $$\omega_c = 3\left(\frac{E}{mc^2}\right)^3 \frac{c}{\rho}$$

2. Coherent Synchrotron Radiation (CSR) by an Electron Bunch

Coherence of Radiation by Many Electrons

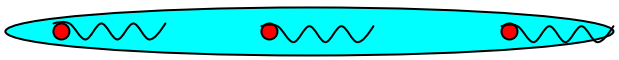
- Typically in accelerators, 10^{10} electrons travel in a tight bunch at v~c.

- The single particle radiation power is $P_0 = |E_0|^2$

- What is the radiation power spectrum for N electrons?

•For radiation wavelength << bunch length,

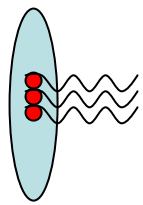
$$\left|E_i E_j\right| = 0, \qquad P_{\text{tot}} = \left|\sum_{i=1}^{N} E_i\right|^2 \approx N\left|E_0\right|^2 = \boxed{N} P_0 \qquad (\lambda << \sigma_z)$$



$\sigma_s > \lambda$: incoherent radiation

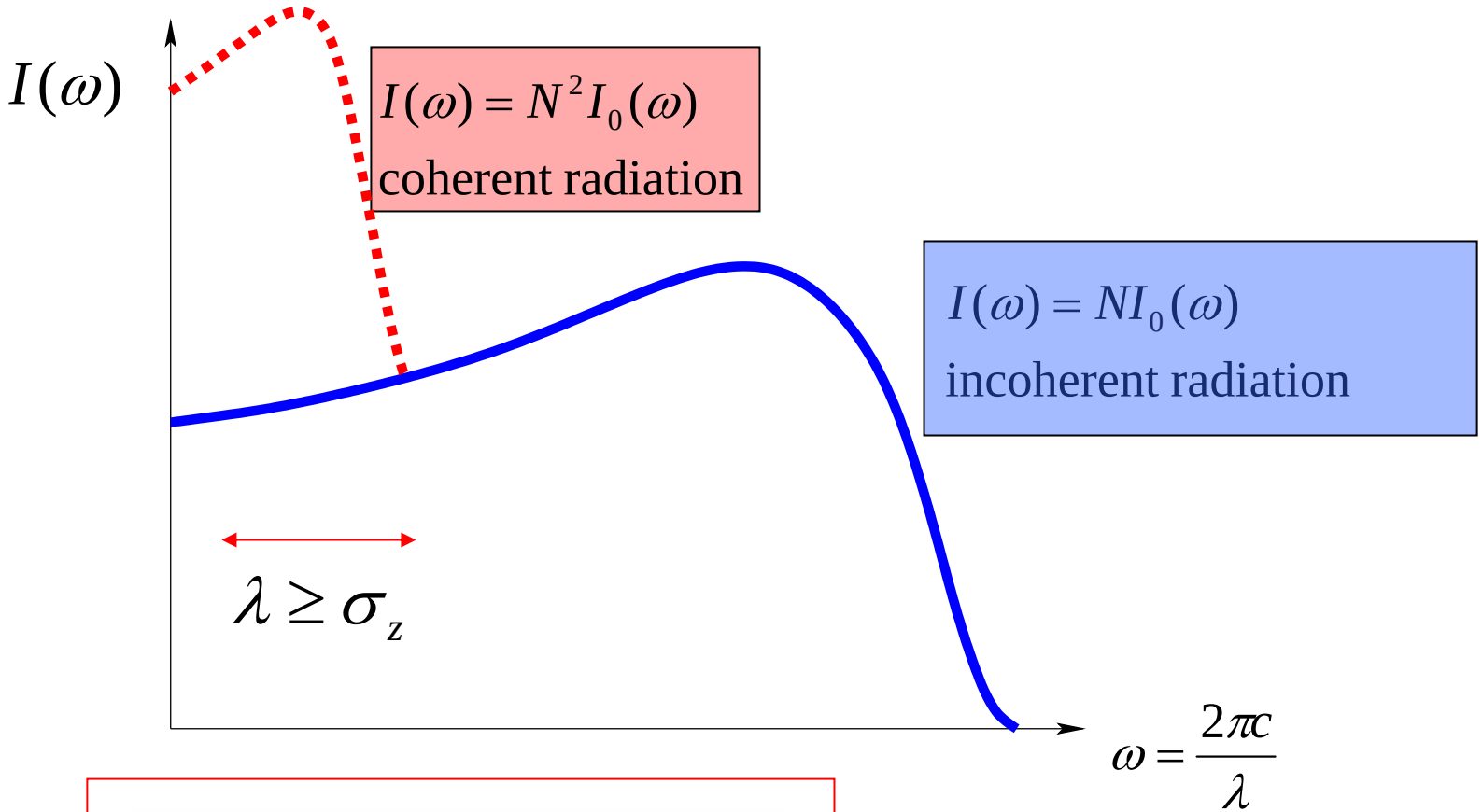
•For radiation wavelength > bunch length,
the radiation from single electron adds coherently,

$$P_{\text{tot}} = \left|\sum_{i=1}^{N} E_i\right|^2 \approx \left|NE_0\right|^2 = \boxed{N^2}\ P_0 \qquad (\lambda \approx \sigma_z)$$



$\sigma_s \leq \lambda$: coherent radiation

Radiation Power Spectrum for N electrons



$$\left.\frac{dP}{d\omega}\right|_{\text{bunch}} = \frac{dP}{d\omega} N \left(1 + N |\hat{f}(\omega)|^2\right)$$

Recent Observation of CSR

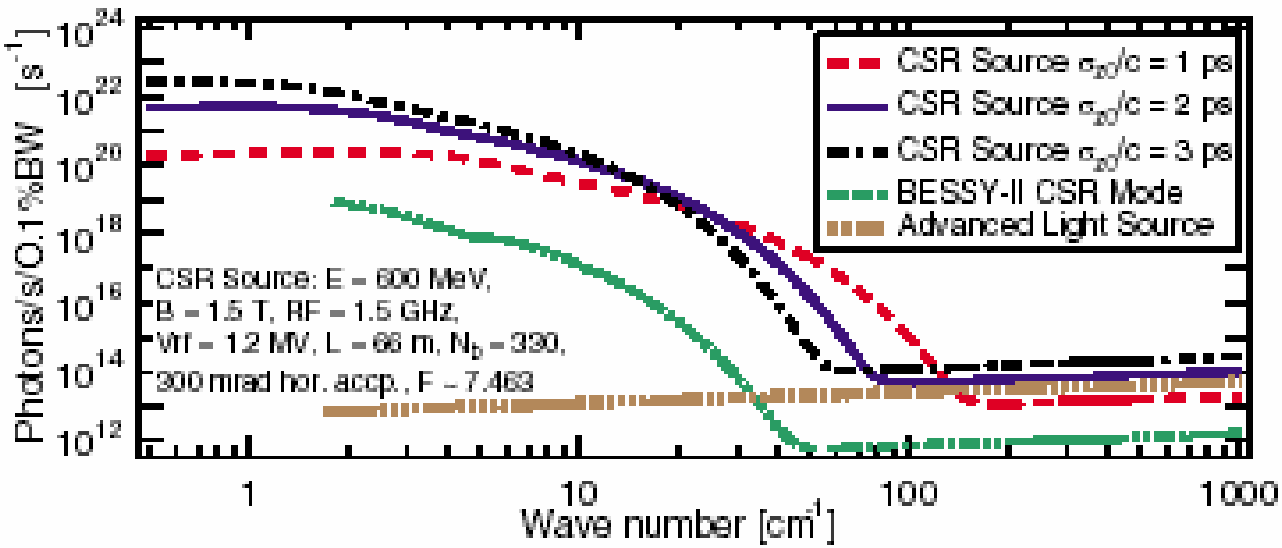
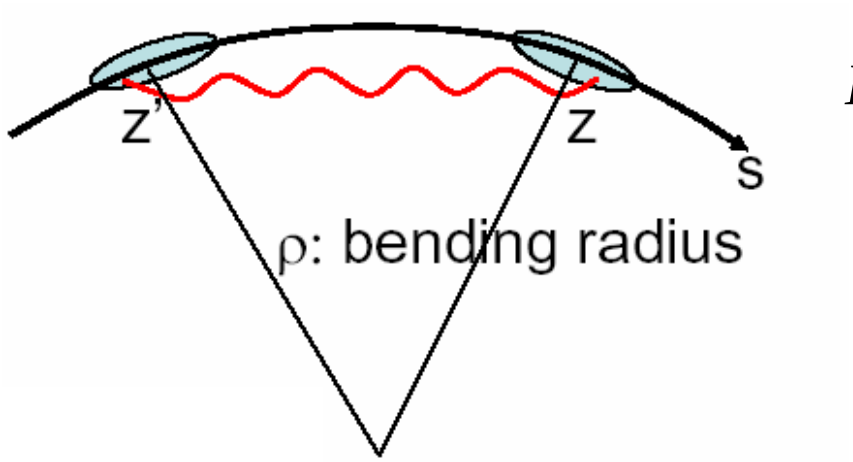


FIG. 4 (color online). Example of source optimized for the CSR production using the criteria described in this Letter. The photon flux is compared with the cases of a conventional SR source (ALS) and of BESSY II CSR mode with 400 bunches, 19.2 μA/bunch, $\sigma_{z0}/c = 1.8$ ps, and 60 mrad horizontal acceptance.

3. Collective CSR Interaction for High Brightness Beams

Where does the extra radiation power come from?

→ Kinetic energy loss of the electrons due to bunch self-interaction on curved orbit



$$\vec{E}(r,t) = \int \vec{E}_0(\vec{r},t;\vec{r}',t')n(\vec{r}',t')d\vec{r}'$$

Power loss by one electron:

$$P = e\vec{E}\cdot\vec{v} \propto Ne$$

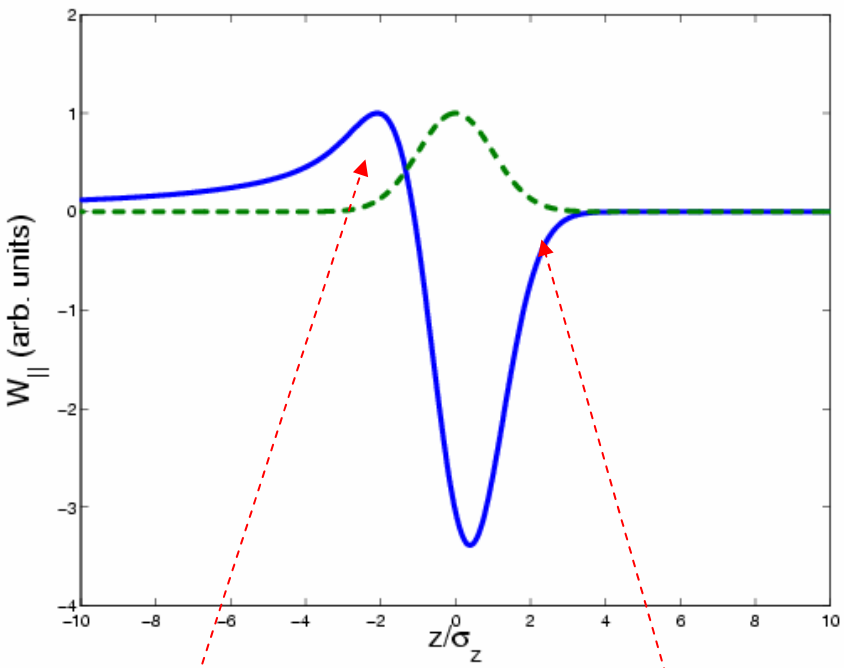
Power loss by the bunch:

$$P_{\text{tot}}(t) = \int P(\vec{r},\vec{v},t)\, f(\vec{r},\vec{v},t)d\vec{r}d\vec{v} \propto N^2e^2$$

Radiation emitted from a tail particle at retarded time overtakes the bunch to interact with a head particle

$$t' = t - \frac{|r-r'|}{c}$$

CSR Collective Force on the Bunch



Bunch head:
gain energy

Bunch tail:
lose energy

Overtaking Length

$$L_0 = (24\sigma_z \rho^2)^{1/3}$$

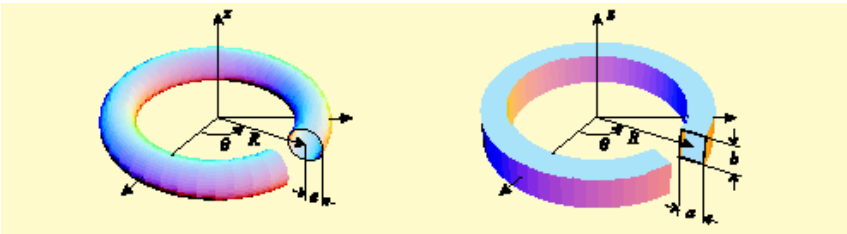
For example,

$$\sigma_z = 1\,\text{mm},\ \ \rho = 10\text{m},\ \ L = 1.34\text{m}$$

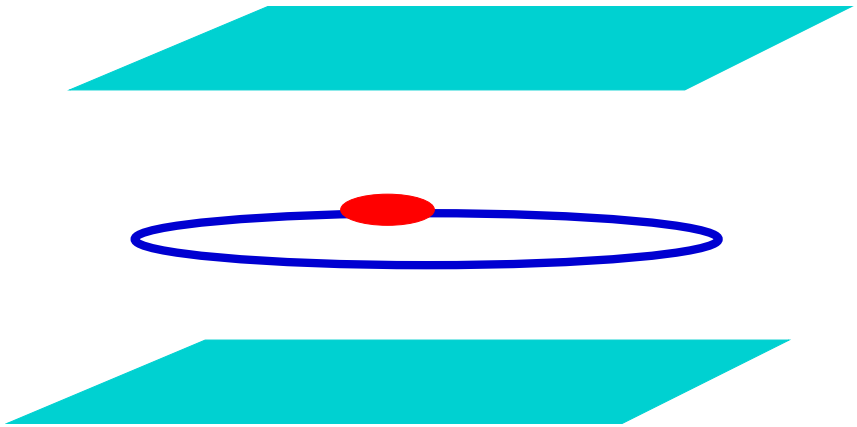
$$\frac{d(\Delta E)}{ds} = -\frac{e}{4\pi}\int_{-\infty}^{z} \underbrace{\frac{2Z_0}{3\rho^{2/3}(z-z')^{1/3}}}_{\text{CSR "wake", stronger at smaller scale}} \frac{dI}{dz'}dz'$$

Shielding by Vacuum Chamber

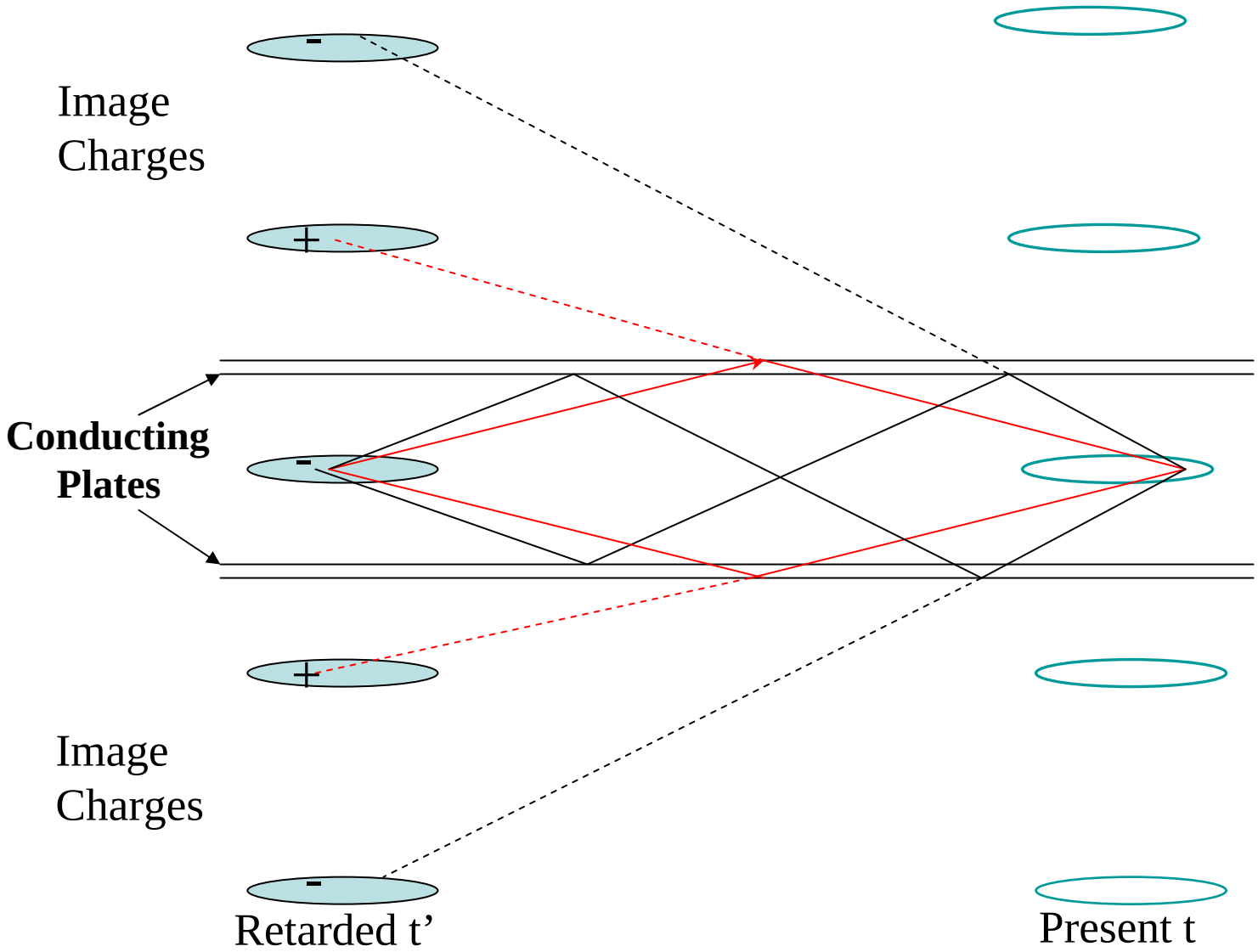
•Electron beams in accelerators are surrounded by vacuum chamber



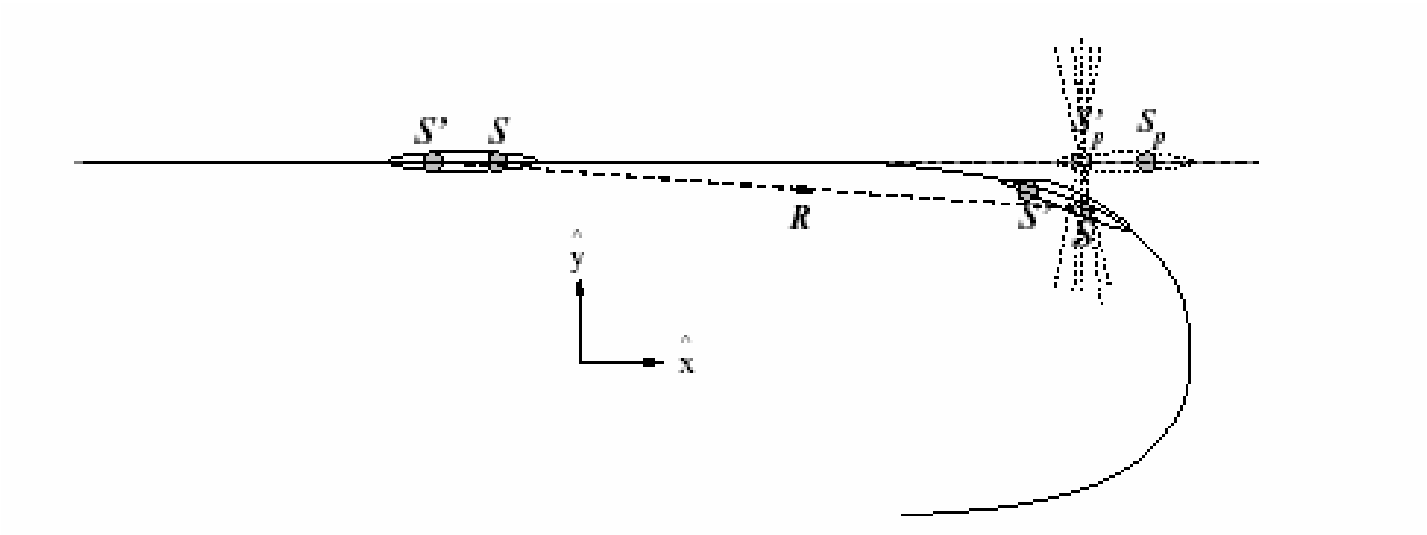
•Simple model: parallel plate shielding



Bunch Self-Interaction in Vacuum Chamber



Transient Interaction at Entrance of a Bend



As the bunch enters into the bend, in addition to the radiation fields, the bunch also sees Coulomb field emitted from the bunch's pseudo-position ——— as if the particles continue to travel with their Velocity at retarded time.

4. CSR Effect in Particle Accelerators

Accelerator Facilities with High Brightness Electron Beams

A. Light Sources

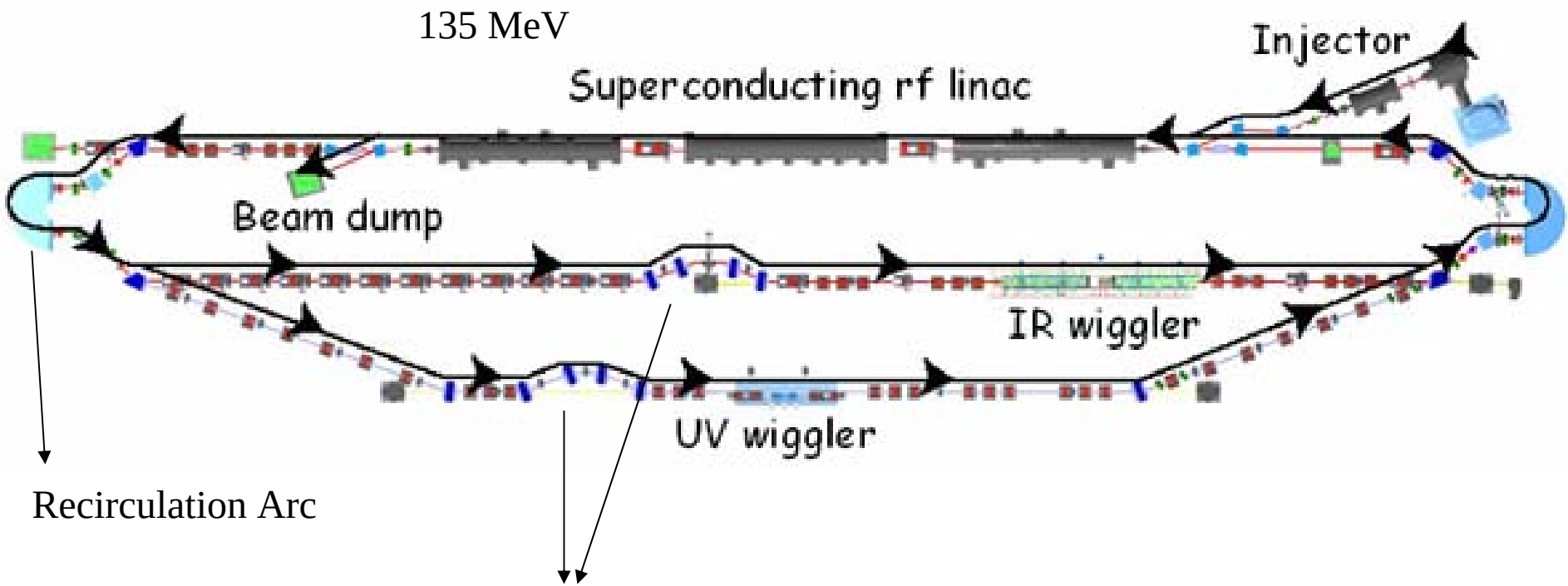
- Storage Ring Light Souces
- Linac Based Free Electron Laser

B. Colliders

- Storage Ring Colliders
- Linear Colliders

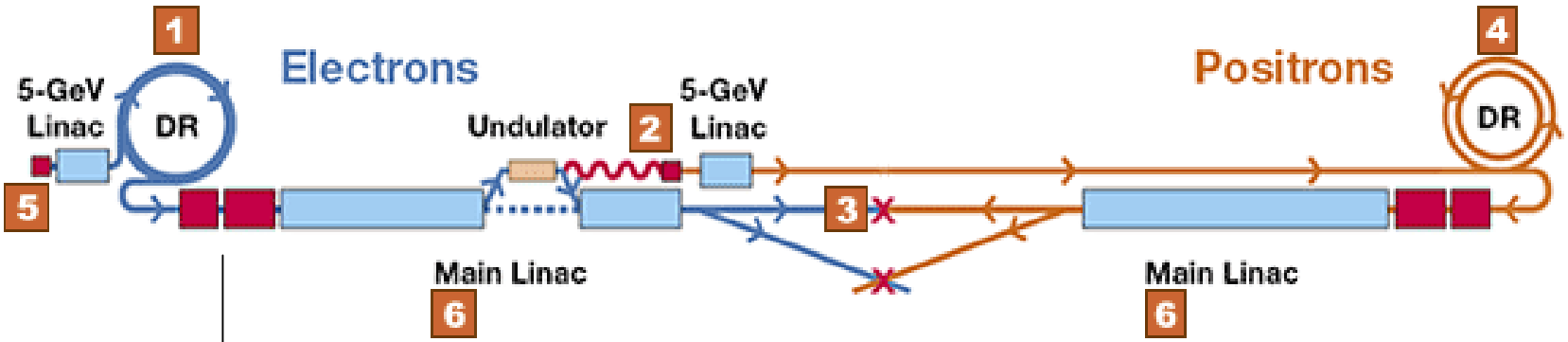
Example of Free Electron Laser:

Eenergy Recovery Linac based FEL (Jefferson Lab)

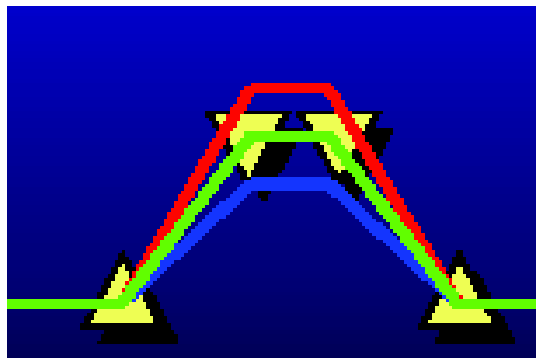


Example of Linear Collider:

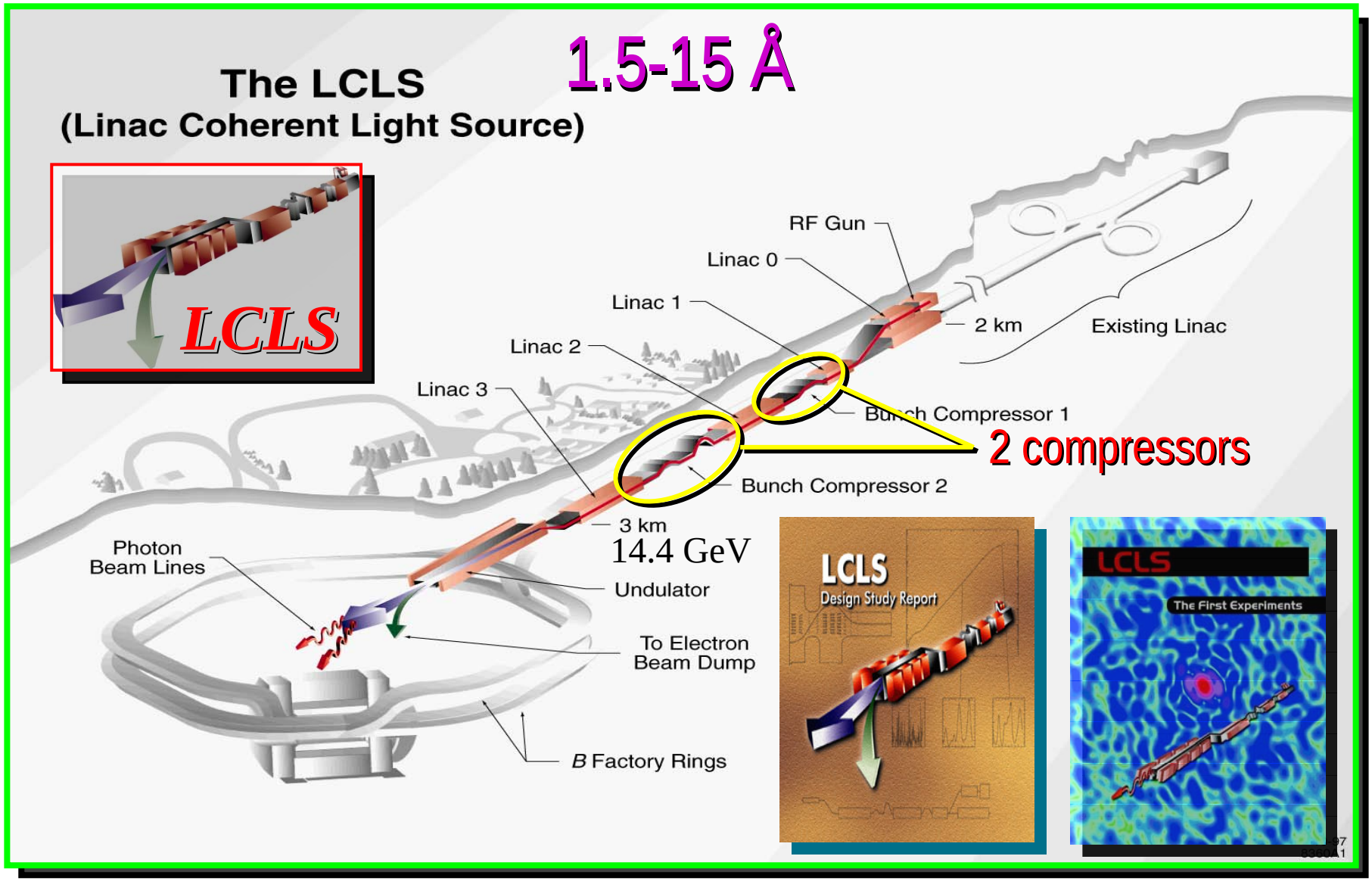
The International Linear Collider (ILC)



Magnetic Bunch Compressor:

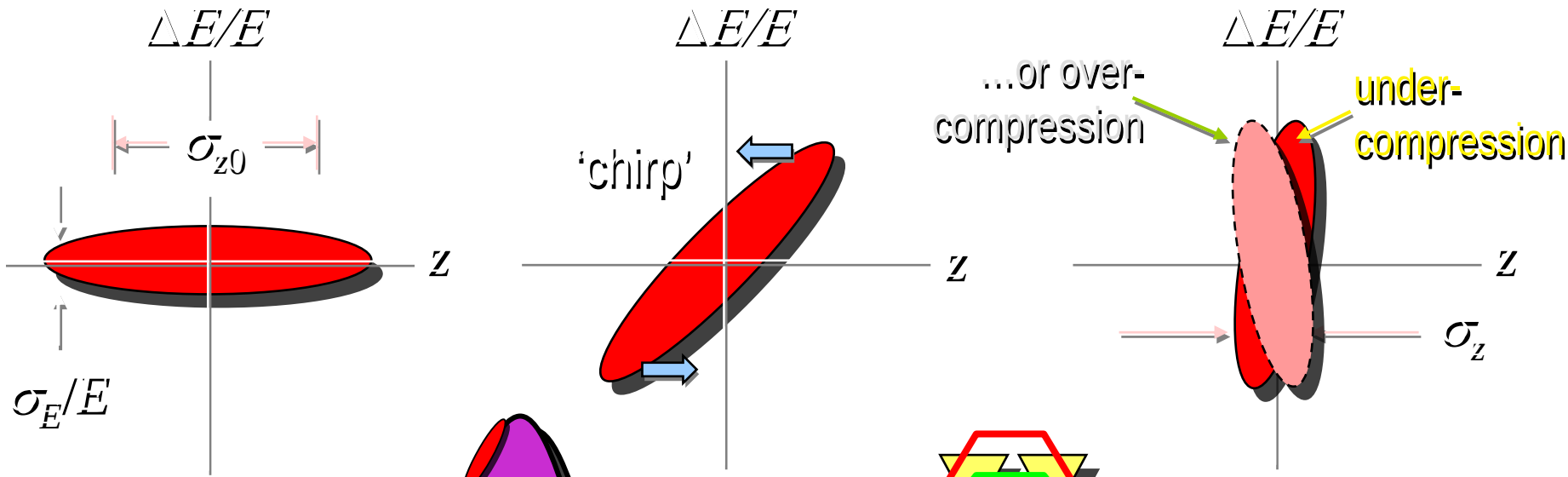


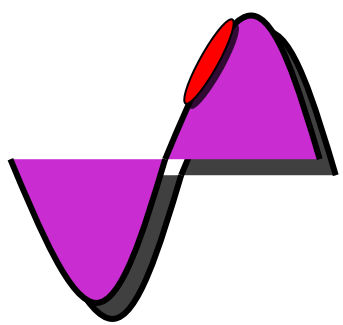
LCLS at *SLAC*



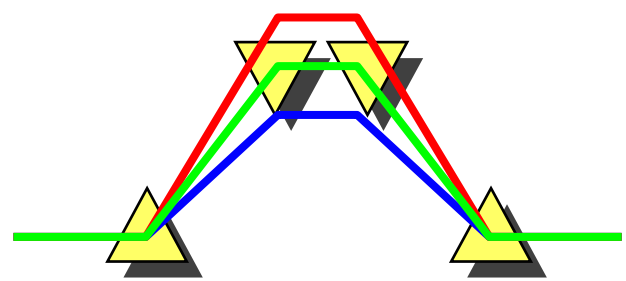
X-FEL based on last 1-km of existing *SLAC* linac

Magnetic Bunch Compression





$$V = V_0 \sin(\omega\tau)$$



$$\Delta z = R_{56}\Delta E/E$$

RF Accelerating Voltage

Path Length-Energy Dependent Beamline

Requirement of Accelerator Design

- Colliders: event rate $\propto$ luminosity

High luminosity $\longleftrightarrow$ Large particle number in small beam phase space area

or, high brightness electron beam

- Light Sources: high brilliance of photon flux

$\updownarrow$

high brightness electron beam

Pros and Cons of the CSR Effect

- **Pros (for storage ring light source)**
  Can enhance radiation power by a factor of N for radiation wavelength ~ bunch length

- **Cons (for FEL, circular and linear colliders)**

Collective CSR Interaction on Curved Orbit

↓

Energy Spread of Particles in the Bunch

↓

Particle with different energy being bent differently by the same external magnetic field

↓

Degradation of Electron Beam Brightness

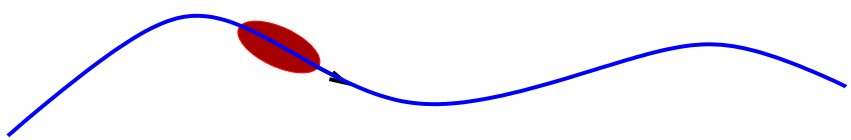
5. Investigation of the CSR Effect

- Theory
- Numerical Simulation
- Experiment

Theoretical Approach

- A bunch of electrons guided by an external EM field to follow the design orbit

s: pathlength along the orbit



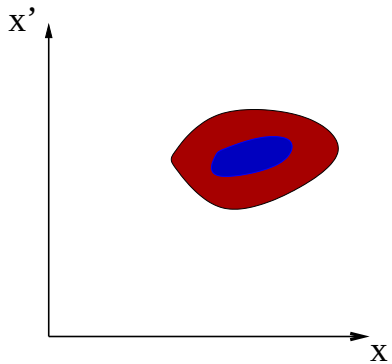
- Due to external and collective interaction, the phase space point (x,x') of a particle varies along the design orbit

$$\begin{cases} \dfrac{dx}{ds} = x' \\ \dfrac{dx'}{ds} = F_{\mathrm{col}}[\, f(s_{\mathrm{ret}})] \end{cases}$$

$$\text{Phase space trajectory}: \; x(s), x'(s;)$$

Theoretical Approach (con'd)

- Phase space distribution at s:

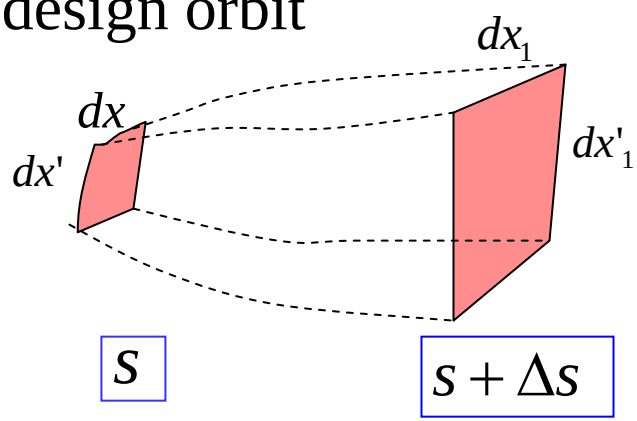


$$f(x, x', s)$$

- # of particles within infinitesimal phase space volume:

$$dN = f(x, x', s)dxdx'$$

- Flow of infinitesimal phase space volume along the design orbit



Theoretical Approach (con'd)

- Conservation of Particle Number

$$dN = f(x, x', s)dxdx' = f(x_1, x_1', s + \Delta s)dx_1 dx_1'$$

$\longrightarrow$ Vlasov Equation

Collective CSR Force

$$\frac{\partial f}{\partial s} + \frac{\partial f}{\partial x} x' + \frac{\partial f}{\partial x'} F_{\text{col}}[f] = 0$$

Hamiltonian system: $dxdx' = dx_1 dx_1'$

- Perturbation $f = f_0 + f_1$ $(|f_1| << |f_0|)$

Given $F_{\text{col}}[f]$, $f(x, x', s)$ is solved from the Vlasov Eq. using perturbative approximation.

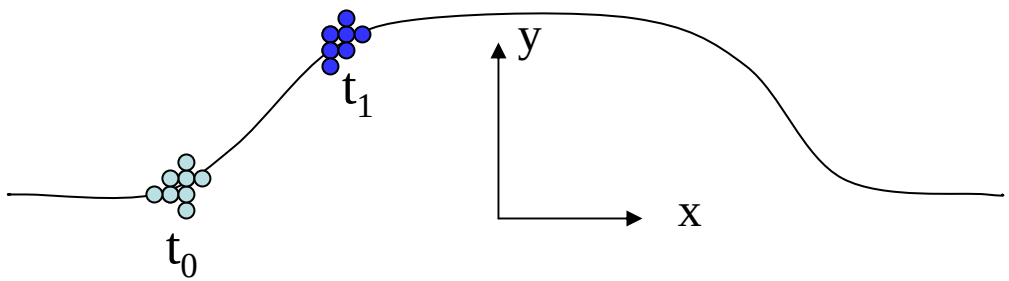
Numerical Simulation

Should be

- Faithful representation of phase space distribution, including fine details
- Accurate calculation of collective forces, including retardation
- Self-consistent
- Not slow

A. Macroparticle Model

- **Transverse and Longitudinal bunch dynamics**



- **Bunch is simulated by a set of macroparticles**

——— Macros: 2D round Gaussian disc

——— Single macro density distribution:

$$n_m(\vec{r}-\vec{r}_0(t)) = \frac{1}{2\pi\sigma_m^2}\,\mathrm{Exp}\left[-\frac{(x\text{-}x_0(t))^2+(y-y_0(t))^2}{2\sigma_m^2}\right]$$

Macro centroid vector

Macro size

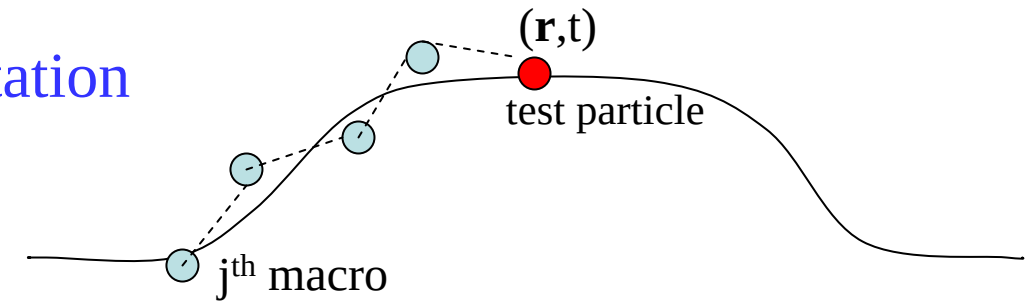
——— Bunch charge distribution: $\rho(\vec{r},t) = q_m \sum n_m(r - r_0^{(j)}(t))$

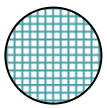
- CSR Force Computation $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$

$$\vec{E}(\vec{r},t) = \sum_j \vec{E}^{(j)}(\vec{r},t), \qquad \vec{E}^{(j)}(\vec{r},t) = q_m \int \frac{d^2\vec{r}\,'}{|\vec{r}-\vec{r}\,'|} F[\vec{r}\,' - \vec{r}_0^{(j)}(t'), \vec{v}_0^{(j)}(t')]$$

for retarded time $t' = t - |\vec{r} - \vec{r}\,'|/c$

- Interpretation





If we devide jth macro into grid, each grid emits photon (which reaches **r** at t) at different **r**’,t’.

Pros and Cons of Macroparticle Model:

- straight-forward calculation of CSR force including retardation
- self-consistent

but

- slow
- noisy

B. Semi-Lagrangian Simulation

(low noise, can reveal fine details of phase space)

Dynamical Equation

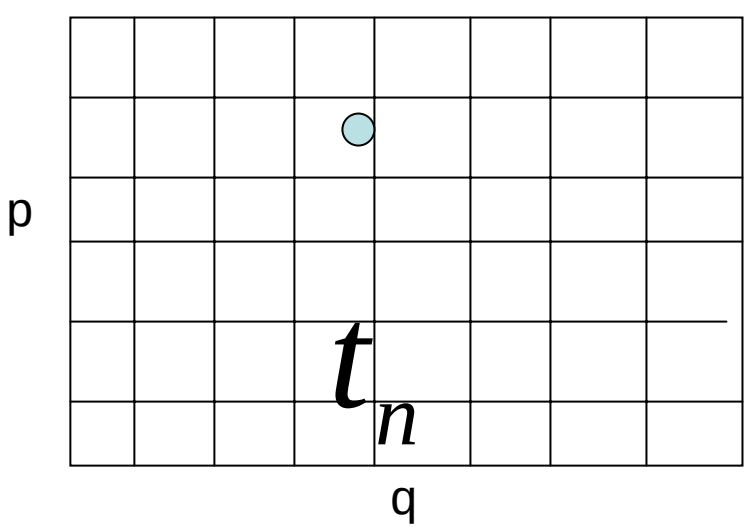
$$\begin{cases} \dfrac{dq}{dt} = F(p,q,t;f) \\ \dfrac{dp}{dt} = G(p,q,t;f) \end{cases} \longrightarrow$$

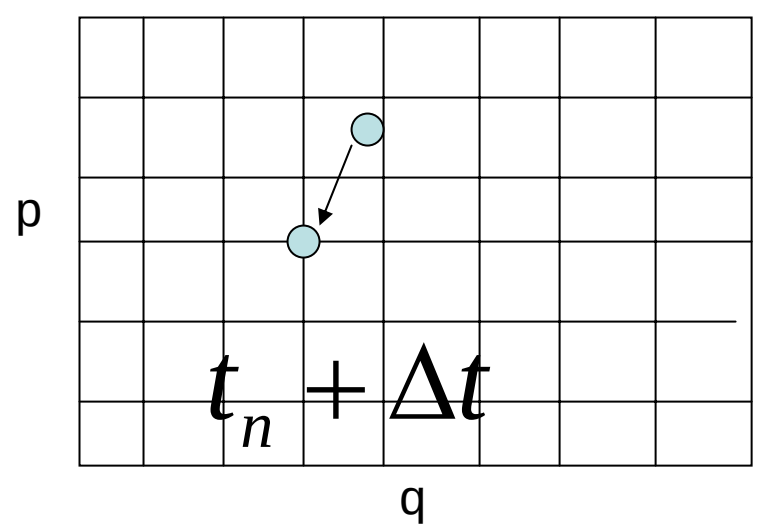
Difference Equation

$$\begin{cases} q(t_{n-1}) = q(t_n) - F(p,q,t_n;f)\Delta t \\ p(t_{n-1}) = p(t_n) - G(p,q,t_n;f)\Delta t \end{cases}$$

Knowing $f(p_i, q_j, t_n)$ on the mesh

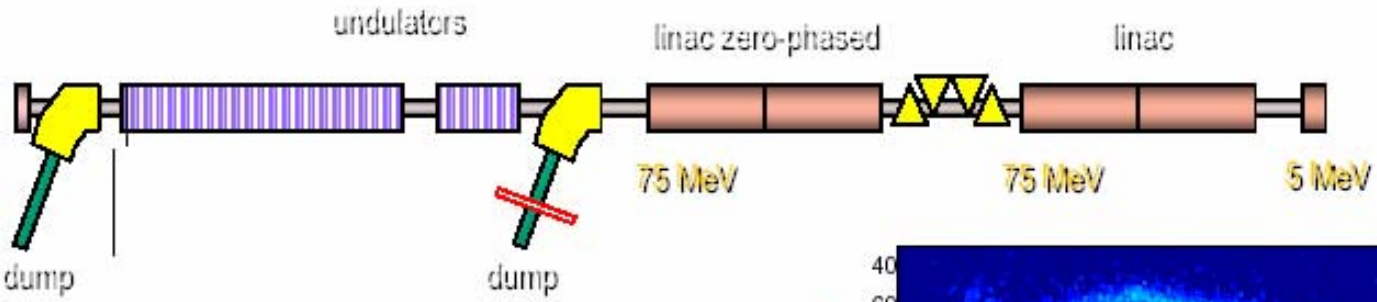
Find $f(p_i, q_j, t_n + \Delta t)$ on the mesh

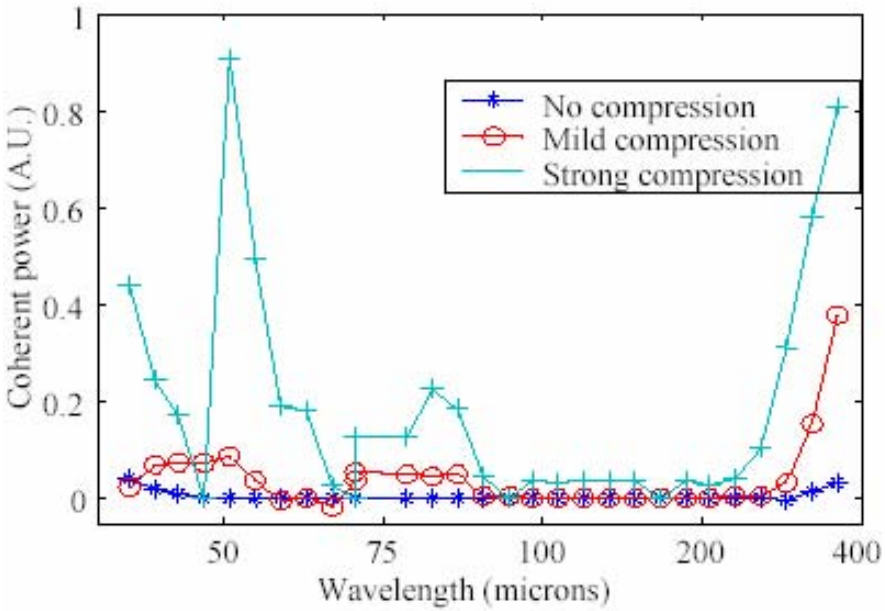




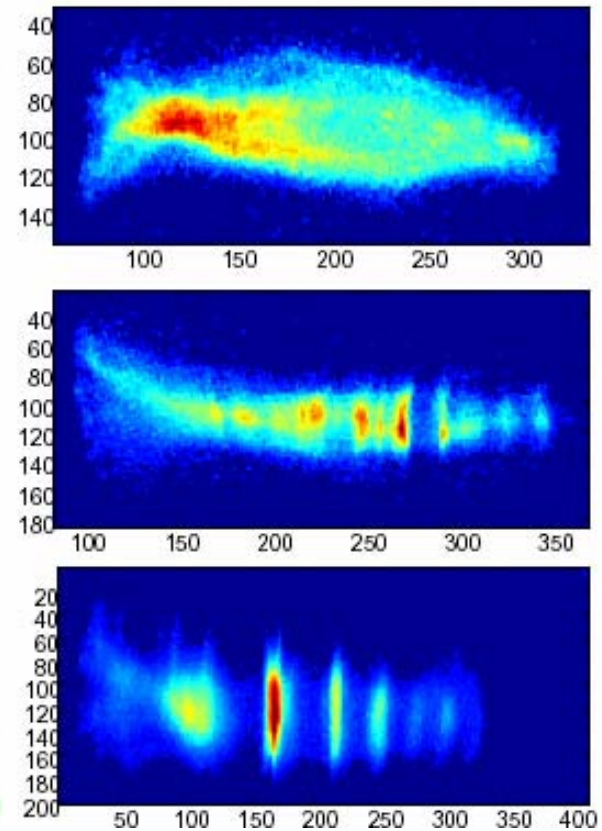
Experimental Observation

Magnetic bunch compression





Stronger compression



V. Graves, et al PAC'01, T. Shaftan et al PAC'03)

Microbunching Instability

Small Initial energy perturbation

↓ bend geometry

density modulation

↓ CSR in bend

Stronger energy modulation

↓ bend geometry

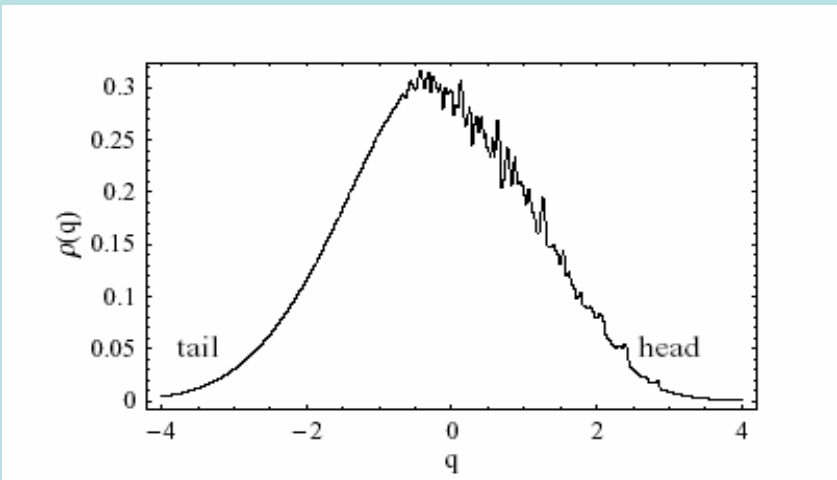
Stronger density modulation

Remark: intrinsic energy spread provide natural stabalizing mechanism

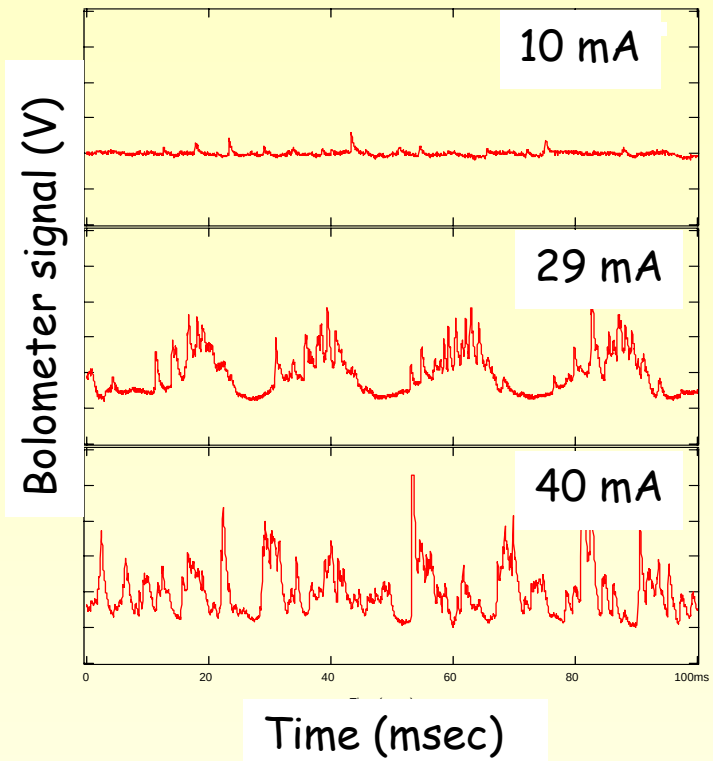
→ Threshold of instability

Experimental Observation (con'd)

CSR can drive a microbunching instability in the electron bunch, resulting in a periodic bursts of terahertz synchrotron radiation, resulting in a noisy source.



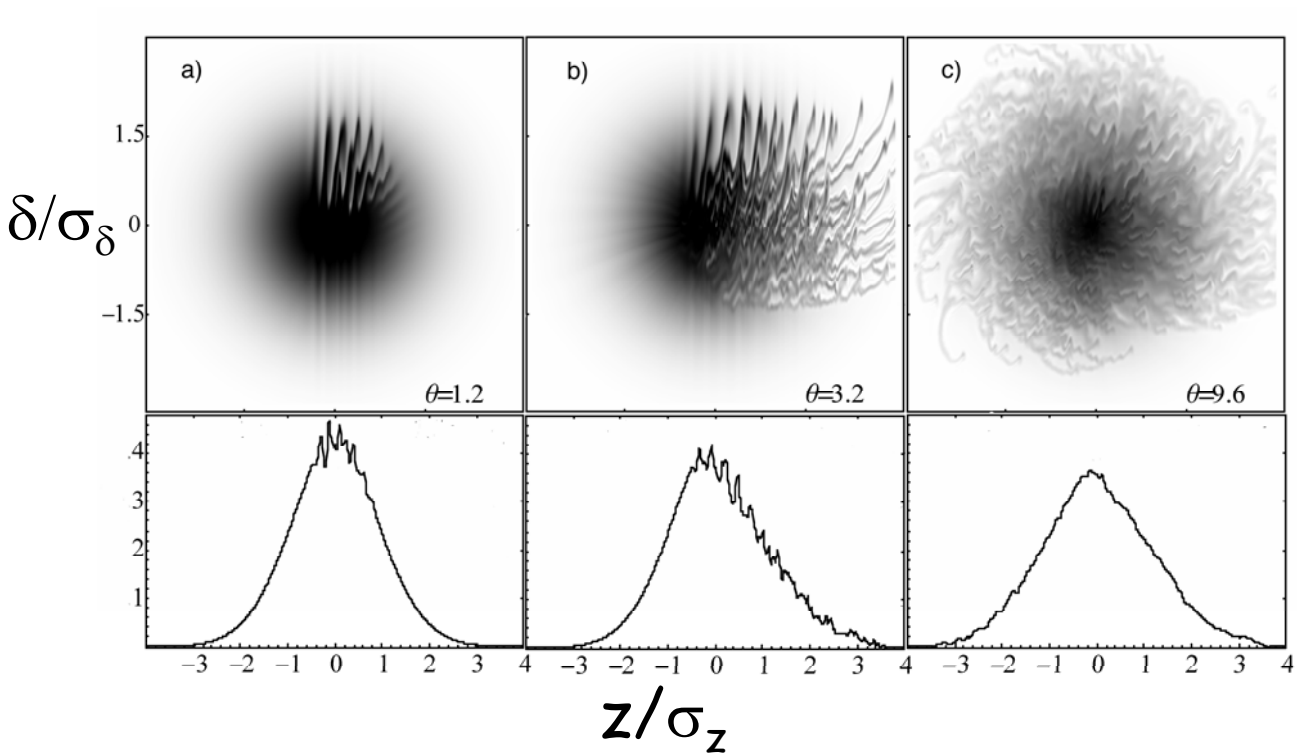
Simulated instability showing bunch shape



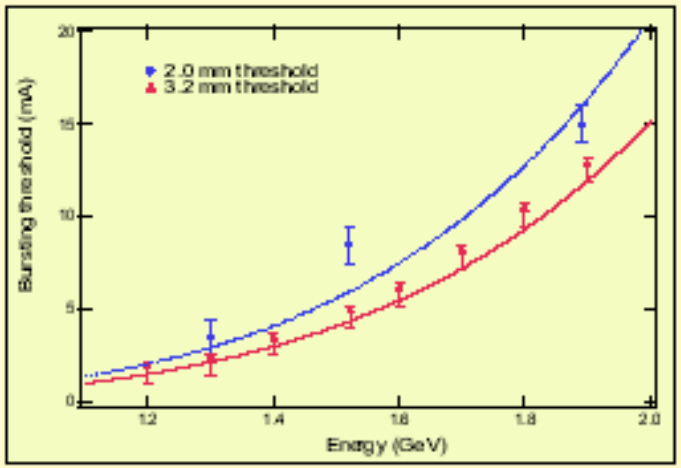
Bursts of far-IR CSR observed on a bolometer. Threshold depends on beam energy, bunch length, energy spread, and wavelength.

Simulated Results

Small perturbations to the bunch density can be amplified by the interaction with the radiation. Instability occurs if growth rate is faster than decoherence from bunch energy spread.



Experiment Compared with Theory



Bursting threshold as a function of electron beam energy at 3.2 and 2 mm wavelength.

Very good agreement with theory for threshold of the instability.

Conclusion

- Coherent synchrotron radiation is recognized as an important factor to impact beam dynamics for high brightness beams transporting in magnetic bends.

- The retardation and sensitivity of this effect pose serious challenge to theory, simulation and experiments.

- Big progress has been made on each aspects. However, more detailed and precise understanding and simulation of CSR are needed in order to have predictive power for machine design.