Multi-pass, multi-bunch instability in Recirculating Linacs

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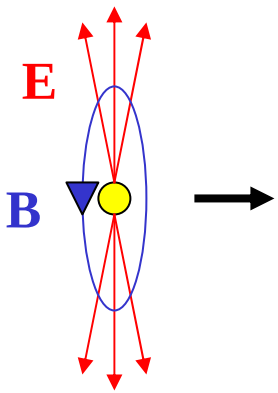
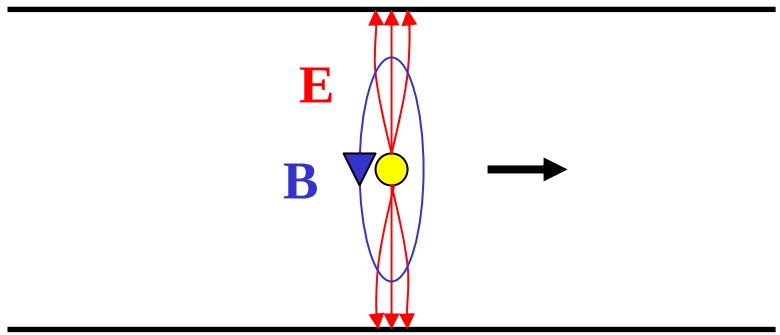
Fields of a relativistic charge

Free Space



Infinite Smooth Pipe



In free space, the force between ultra-relativistic particles vanishes

$$\vec{E}_1 = \frac{q_1 \vec{R}}{R^3} \frac{(1-\beta^2)}{(1-\beta^2 \sin^2\theta)^{3/2}}$$

$$\vec{B}_1 = \frac{\vec{V} \times \vec{E}_1}{c}$$

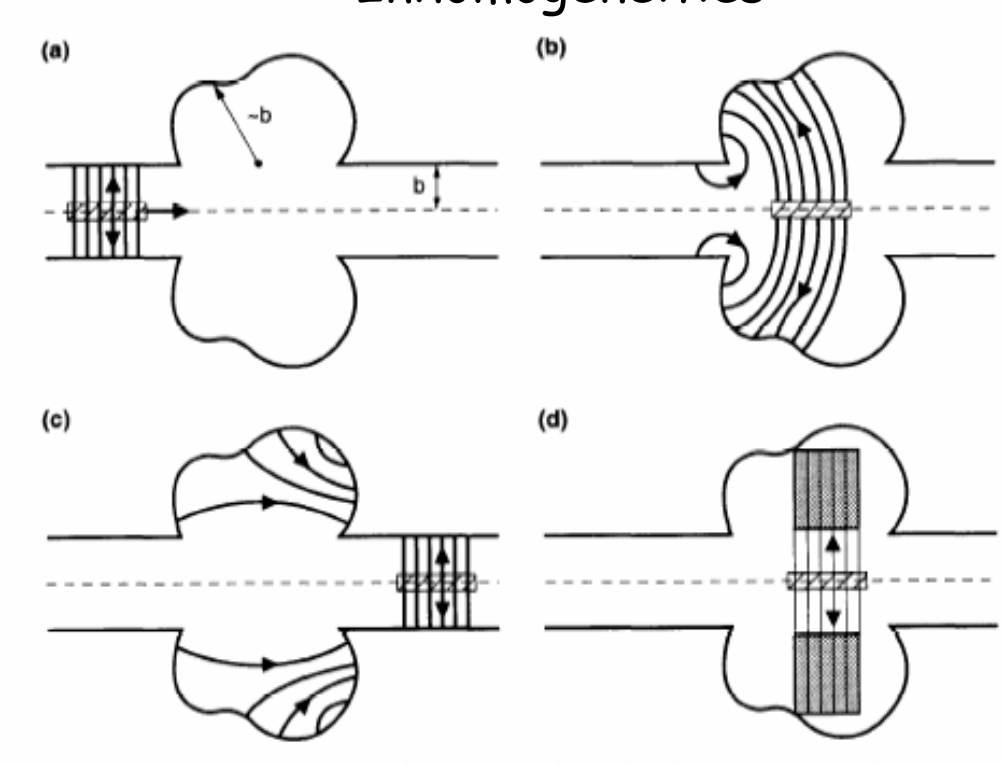
$$\vec{F} = q_2 \vec{E}_1 + \frac{q_2}{c} \vec{V} \times \vec{B}_1$$

$$F_x(\theta = 0) = \frac{q_1 q_2}{R^2}(1-\beta^2) = \frac{1}{\gamma^2}\frac{q_1 q_2}{R^2}$$

$$F_y(\theta = \pi/2) = \frac{q_1 q_2}{R^2}(1-\beta^2)^{1/2} = \frac{1}{\gamma}\frac{q_1 q_2}{R^2}$$

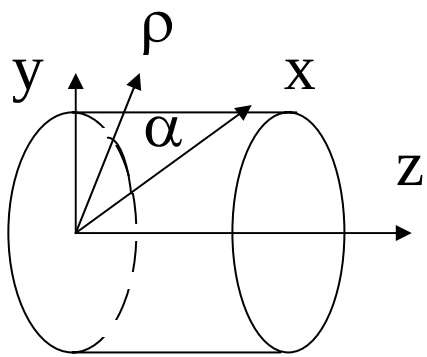
Fields of a relativistic charge

Inhomogeneities



Fields induced by ultra-relativistic particles in inhomogeneities can produce longitudinal and transverse forces affecting particles of the test bunch and following bunches. These forces depend on the charge of bunches, displacement of bunches, and geometry of inhomogeneities.

Modes of a pillbox cavity (Jackson)



Fields in a cavity are described by the wave equation and satisfy boundary conditions

$$\left(\nabla^2 + \mu\varepsilon\frac{\omega^2}{c^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{B}\end{Bmatrix} = 0 \qquad \begin{matrix}\vec{n}\times\vec{E} = 0\\ \vec{n}\cdot\vec{B} = 0\end{matrix}$$

General solution of the wave equation for TM modes can be written as

$$E_z \propto J_m\left(u_{mn}\frac{r}{R}\right)\cos\left(\frac{p\pi z}{d}\right)\cos(m\alpha) \qquad B_z = 0$$

$$E_r \propto -J'_m\left(u_{mn}\frac{r}{R}\right)\sin\left(\frac{p\pi z}{d}\right)\cos(m\alpha) \qquad B_r \propto J_m\left(u_{mn}\frac{r}{R}\right)\cos\left(\frac{p\pi z}{d}\right)\sin(m\alpha)$$

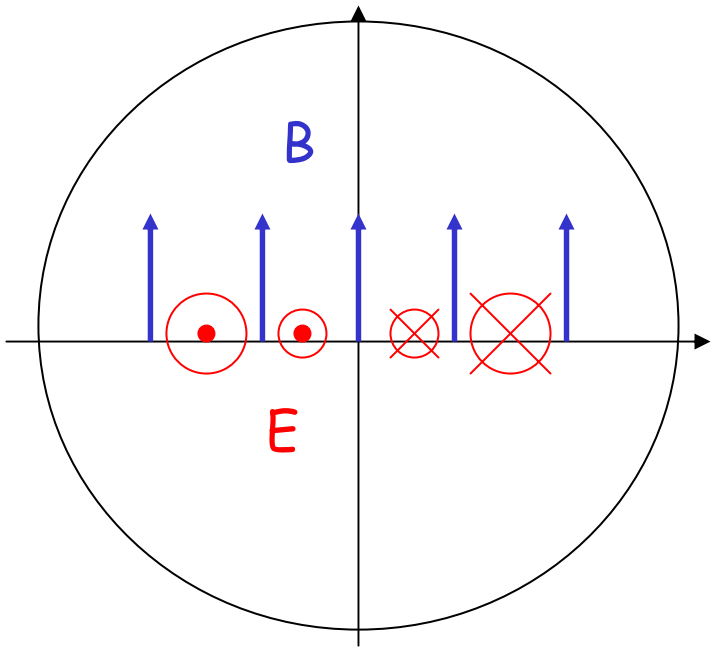
$$E_\alpha \propto J_m\left(u_{mn}\frac{r}{R}\right)\sin\left(\frac{p\pi z}{d}\right)\sin(m\alpha) \qquad B_\alpha \propto J'_m\left(u_{mn}\frac{r}{R}\right)\cos\left(\frac{p\pi z}{d}\right)\cos(m\alpha)$$

Dipole mode (lowest frequency)

$$E_z \propto J_1\left(u_{11}\frac{r}{R}\right)\cos(\alpha) \propto r\cos(\alpha), \quad r << R$$

$$B_\alpha \propto J_1'\left(u_{11}\frac{r}{R}\right)\cos(\alpha) \propto \left(J_0\left(u_{11}\frac{r}{R}\right) + J_2\left(u_{11}\frac{r}{R}\right)\right)\cos(\alpha)$$

$$\approx \cos(\alpha), \quad r << R$$



Power dissipation in a dipole mode

Let's define dipole mode voltage at some r=a as

$$\tilde{V}_a = \left| \int E_z(r=a,z) e^{i\omega z/c} dz \right| \cos(\varphi) = V_a \cos(\varphi)$$

Let's assume that the cavity "resistance" is R. The expression

$$P = V_a^2 / R$$

has right units but depends on a. The expression

$$P = V_a^2 / (a^2 R)$$

does not depend on a but has wrong units. The expression

$$\boxed{P = \frac{V_a^2 c^2}{a^2 R \omega^2}}$$

is just right. But what is R?

Power dissipation in a dipole mode

A quality factor of a mode is defined as:

$$\boxed{Q = \omega \frac{Stored\ Energy}{Power\ Loss} = \omega \frac{U}{P_c}}$$

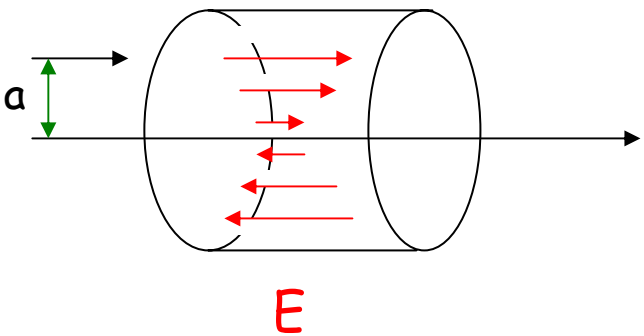
Without sources: $P = -\frac{dU}{dt} = \frac{\omega}{Q} U \quad => \quad U = U_0 e^{-\frac{\omega}{Q} t}$

Definition of R: $\boxed{R = (R/Q)Q}$, (R/Q) is a geometrical factor

$$\boxed{\left(\frac{R}{Q}\right) = \frac{c^2}{a^2 \omega^2} \frac{V_a^2}{\omega U}}$$

(R/Q) is calculated by simulation codes according to the last formula

Interaction of particles with a dipole mode



In an empty cavity, a bunch induces voltage δV_a proportional to the bunch charge q and bunch displacement x.

If there was induced voltage V_a, the stored energy in the cavity is proportional to

$$U \propto (V_a + \delta V_a)^2 = V_a^2 + 2V_a \delta V_a + \delta V_a^2$$

In the first order, $\boxed{\delta U \propto 2V_a \delta V_a \propto qV_a x}$

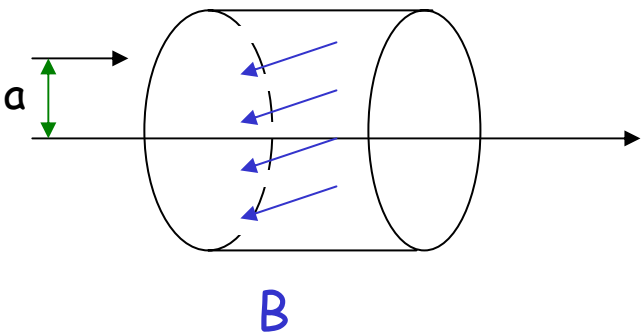
If the self field can be neglected, particle energy gain is equal to

$$\delta E_k = q\frac{\tilde{V}_a}{a}x = q\frac{V_a \cos(\varphi)}{a}x = -\delta U$$

$$\Rightarrow \delta U = -q\frac{V_a \cos(\varphi)}{a}x$$

which is in good agreement with our initial guess

Deflection of particles by a dipole mode



$$\frac{d\vec{p}_\perp}{dt} = \frac{e}{c}\vec{v}\times\vec{B}$$

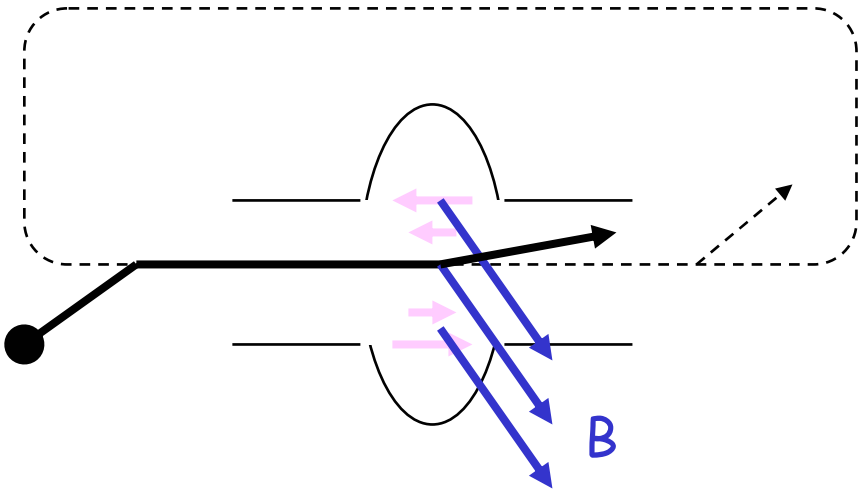
$$\delta p_x = -\frac{e}{c}\int B_y(z)e^{\frac{i\omega z}{c}}dz$$

Definition: $$V_\perp = \frac{\delta p_x c}{e} \quad => \quad V_\perp = -\int B_y(z)e^{\frac{i\omega z}{c}}dz$$

From Maxwell's equation $$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \quad => B_y = -\frac{icE_z(r=a)}{a\omega}$$

$$V_\perp = \frac{ic}{a\omega}\int E(a)e^{\frac{i\omega z}{c}}dz = -\frac{cV_a\sin(\varphi)}{a\omega}$$

Let's put everything together

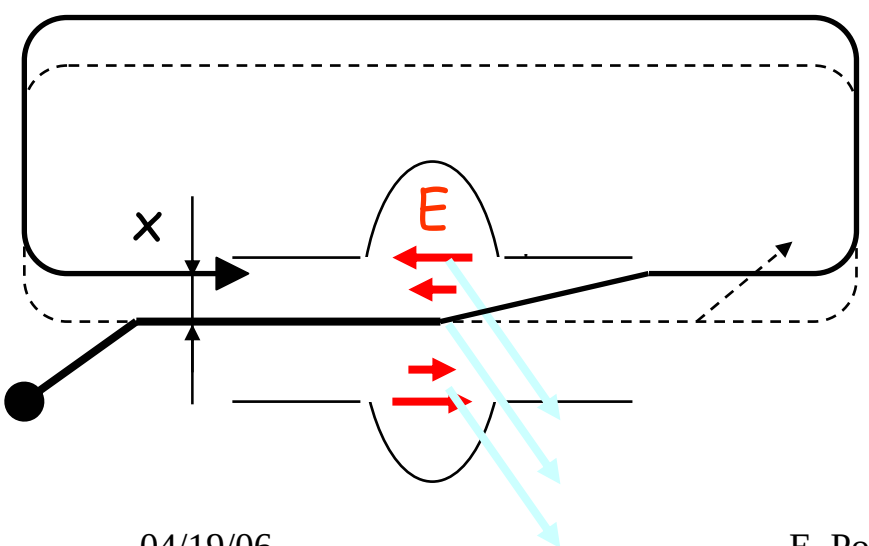


$$x_1' = \frac{V_\perp}{V_b} = \frac{-\frac{cV_a}{\omega a}\sin(\varphi)}{V_b}, \quad V_b = \frac{pc}{e}$$

$$x_2 = m_{11}x_1 + m_{12}x_1'$$

$$\Delta U_1 = -qV_a \cos(\varphi)\frac{x_1}{a}$$

$$\Delta U_2 = -qV_a \cos(\varphi + \omega T_r)\frac{x_2}{a}$$



The equilibrium is given by

$$\dot{U}_{cavity} = \dot{U}_{beam} - P$$
$$= \langle \delta U_1 + \delta U_2 \rangle_\varphi \cdot f_b - P = 0$$

Instability threshold

The threshold corresponds to equilibrium between deposited and dissipated power. At the equilibrium, the stored HOM energy does not change (dU/dt=0)

$$\frac{dU}{dt} = -\frac{V_a^2}{a^2}\left(I_b \frac{m_{12}}{V_b}\frac{c}{\omega}\frac{\sin(\omega T_r)}{2} + \frac{1}{(\omega/c)^2\left(\frac{R}{Q}\right)Q} \right) = 0$$

$$\boxed{I_{th} = -\frac{2V_b}{(\omega/c)\left(\frac{R}{Q}\right)Q\, m_{12}\sin(\omega T_r)}}$$

The formula yields two regions: $m12\sin(\omega Tr)<0$ – low-threshold region, $m12\sin(\omega Tr)>0$ – "pseudo"-stable.

This is first order approximation approach: the beam can be unstable even if $m12\sin(\omega Tr)>0$.

Mode voltage evolution above and below I_{th}

$$\frac{V_a^2}{a^2} = \omega \left(\frac{\omega}{c}\right)^2 \left(\frac{R}{Q}\right) U$$

$$\boxed{\frac{dU}{U} = -dt \frac{\omega}{Q} \frac{I_{th} - I}{I_{th}}}$$

$$\boxed{U = U_0 \exp\left(-t \frac{\omega}{Q} \frac{I_{th} - I_b}{I_{th}}\right)} \qquad \boxed{V = V_0 \exp\left(-t \frac{\omega}{2Q} \frac{I_{th} - I_b}{I_{th}}\right)}$$

The system HOM+beam can be described by the effective quality factor:

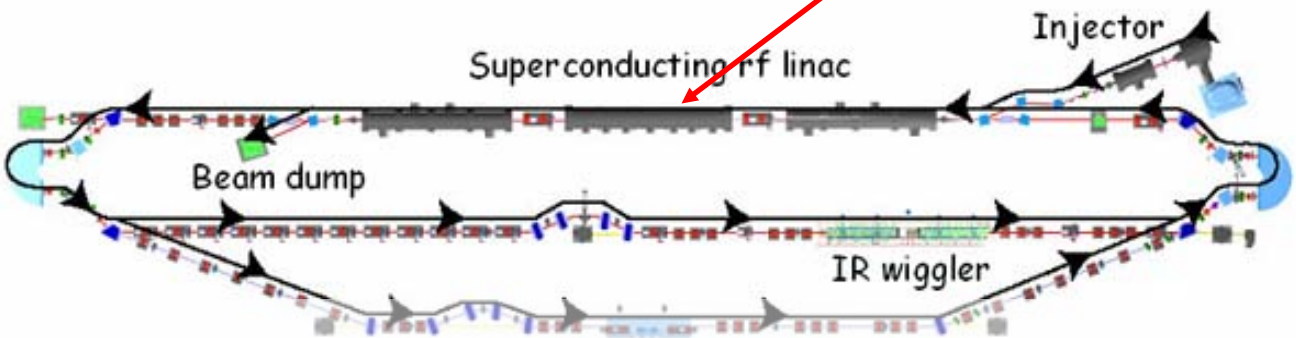
$$\boxed{Q_{eff} = Q \frac{I_{th}}{I_{th} - I}} \quad \Leftrightarrow \quad \boxed{\tau_{eff} = \tau_0 \frac{I_{th}}{I_{th} - I}}$$

Experimental observation of the multi-pass, multi-bunch instability
frequently called multi-pass Beam Breakup (BBU)

JLab FEL Upgrade

Energy(MeV)	80-200
Charge per bunch (pC)	135
Bunch rep.rate (MHz)	4-75
Average current (mA)	10
Laser power (kW)	10

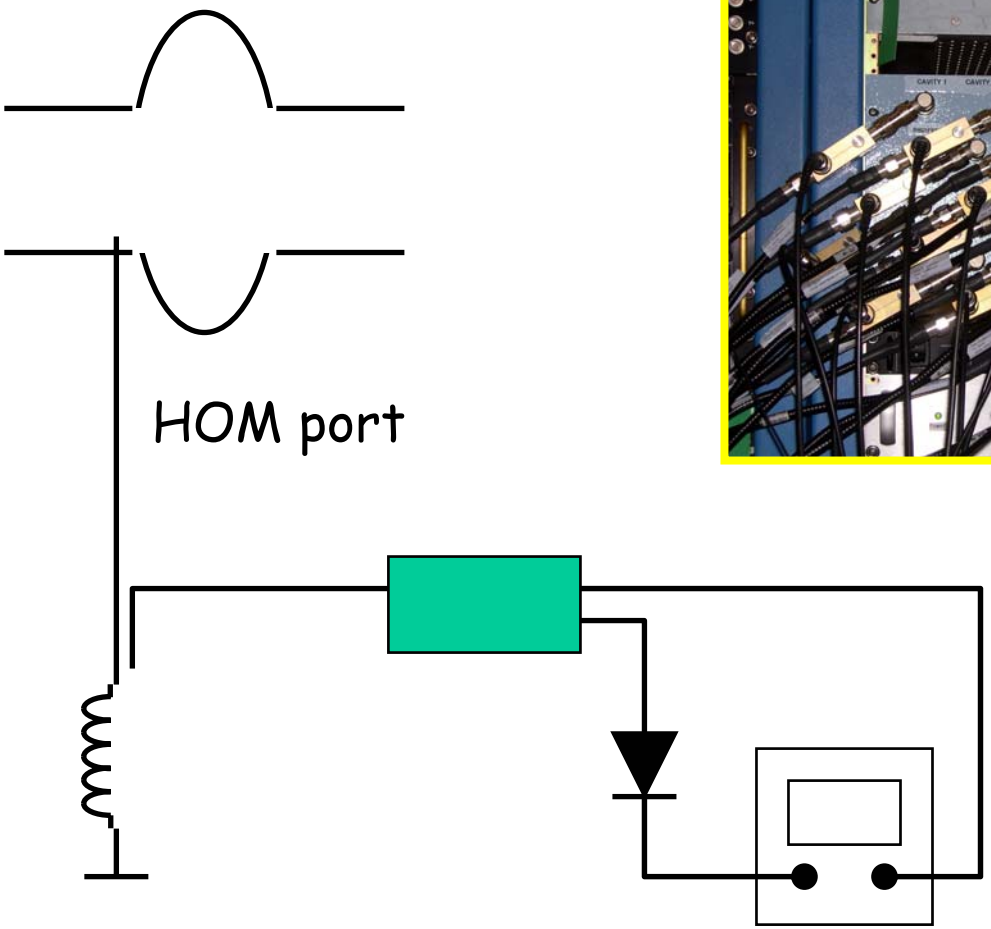
Cavities of Zone 3 have higher accel. gradient than Zone 2,4. The Q of dipole HOMs is also higher. HOMs of Zone 3 impose BBU limit.

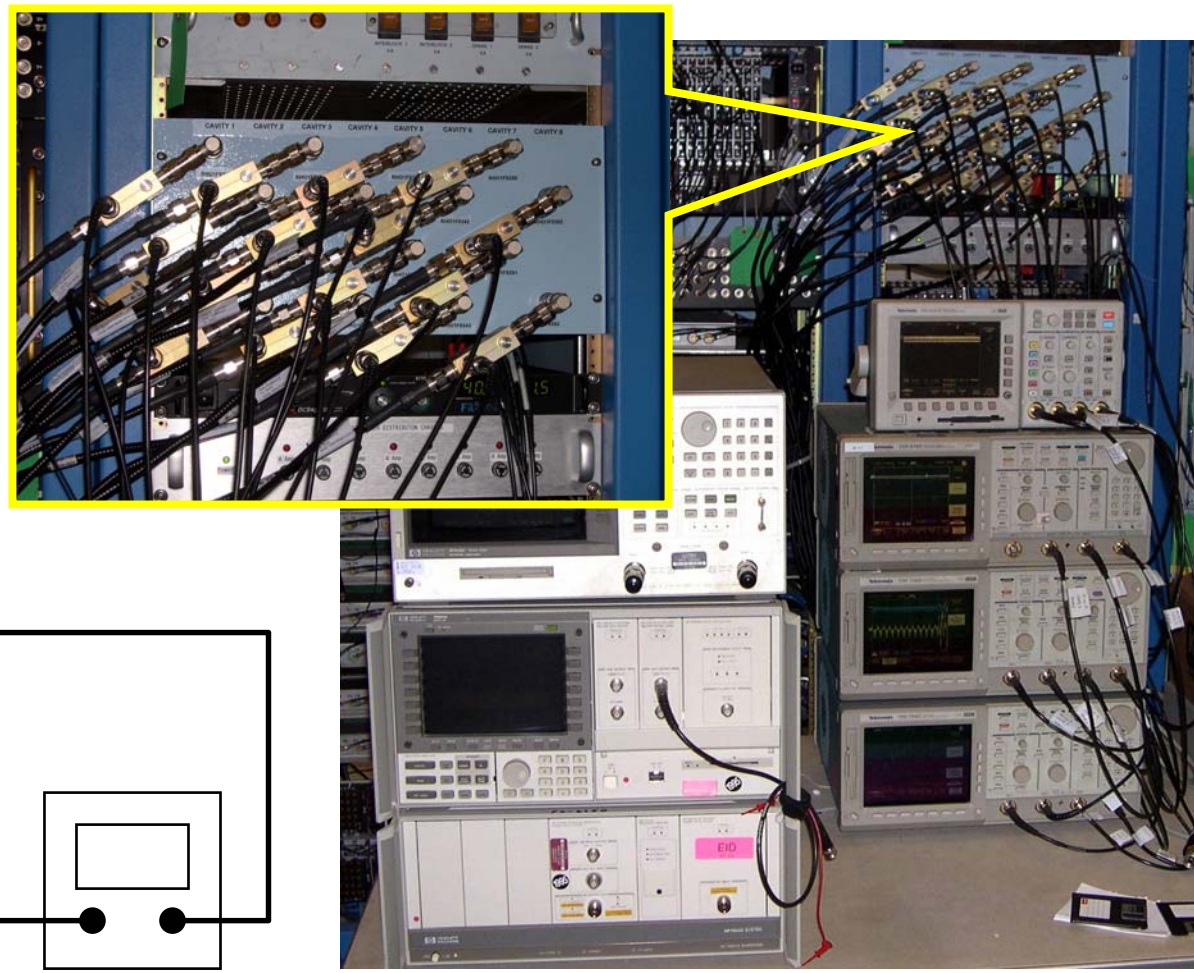


UV wiggler

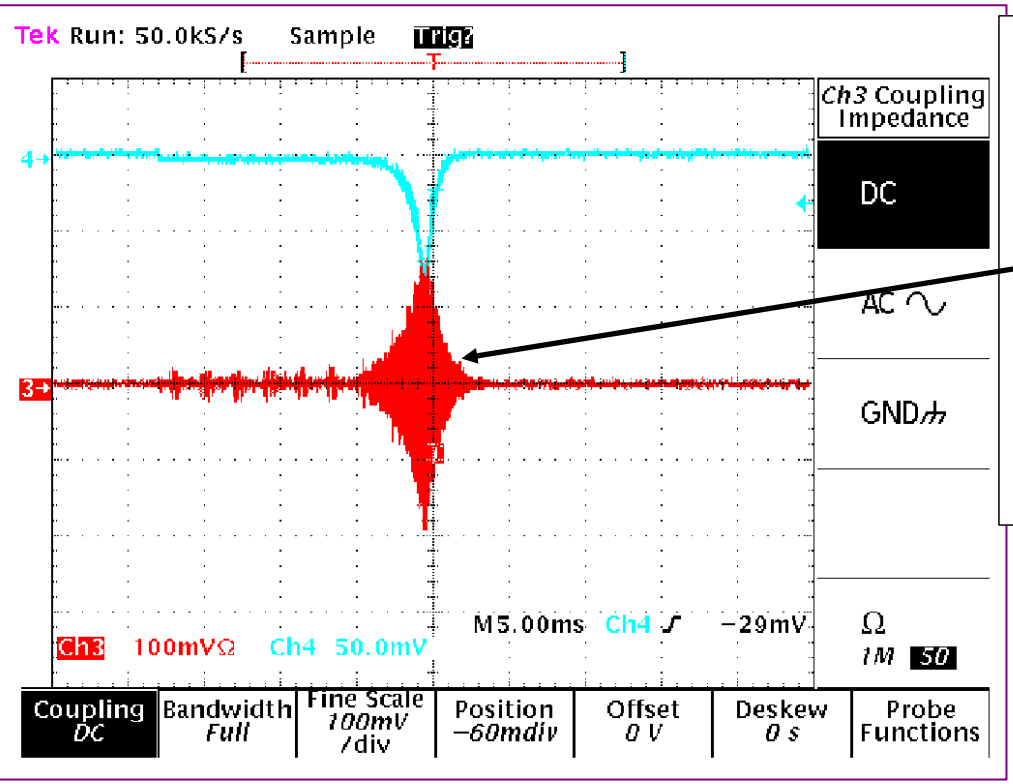
Direct observation of the BBU threshold

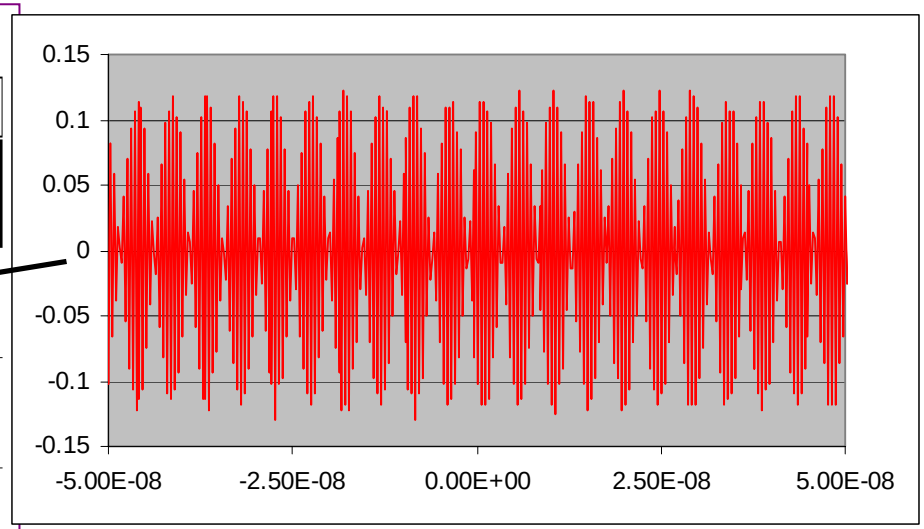
Schottky diodes where used to measure HOM power from the HOM ports.





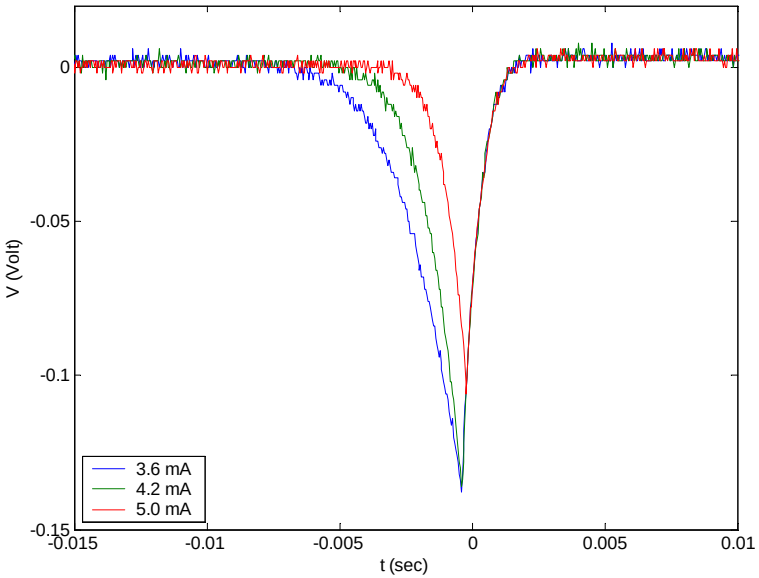
Direct observation of the Instability



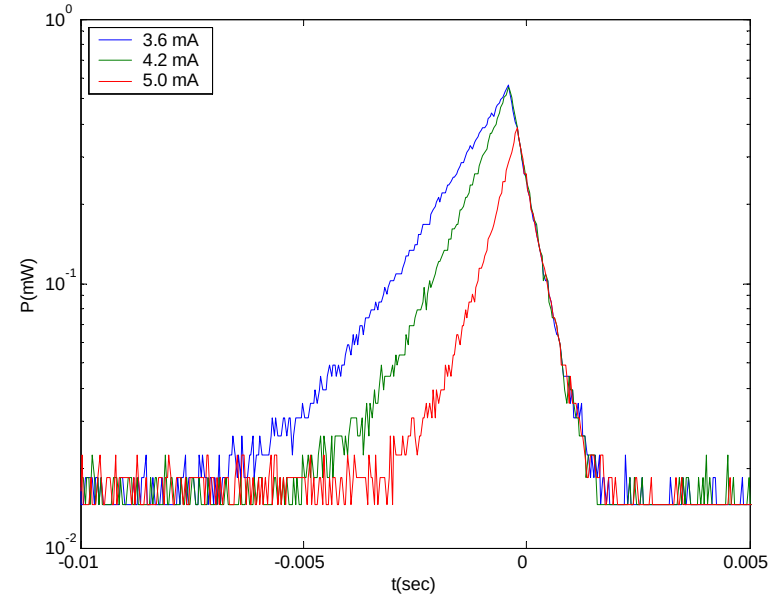


Cav 7, F_{hom}=2106 MHz, Ith=2.7 mA

HOM voltage growth rate measurements

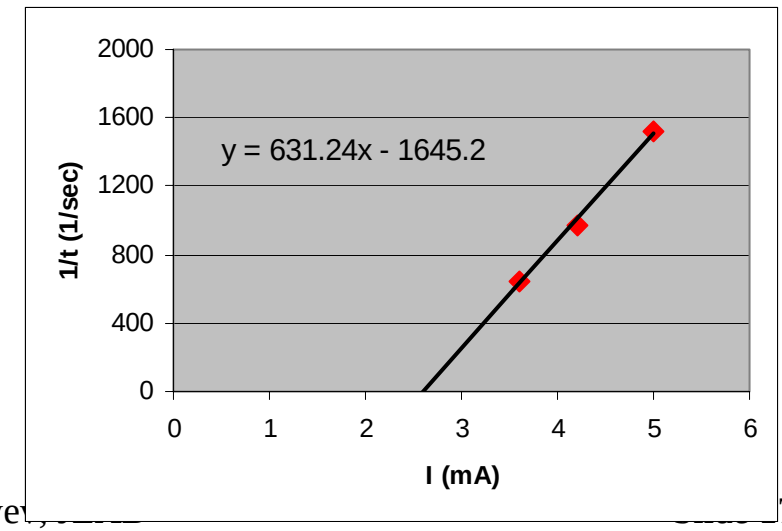


invert
+
adjust
+
log



$$\tau_{eff} = \tau_0 \frac{I_{th}}{I_{th} - I}$$

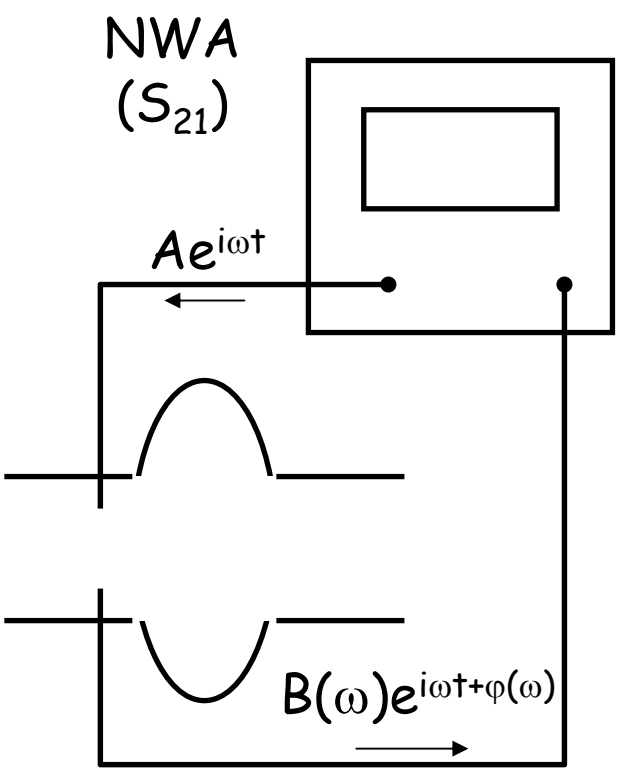
Cav 7, F_{hom}=2106 MHz,
Ith=2.6 mA



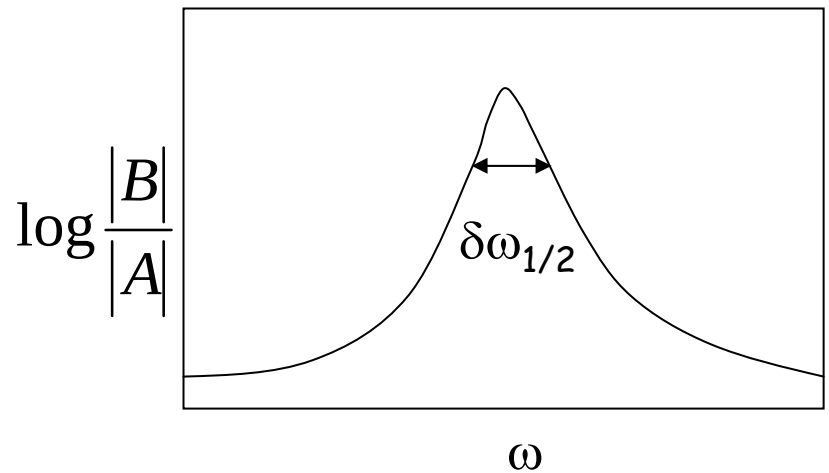
Transfer function measurements
Measurement of the threshold current below the threshold

Transfer Function (TF) measurements

Transfer function technique is a measurement of system's response to a harmonic excitation as a function of the signal frequency. This method can be used to predict system's response to external excitation and determine stability limits.



Cavity modes show typical resonant response to harmonic excitation

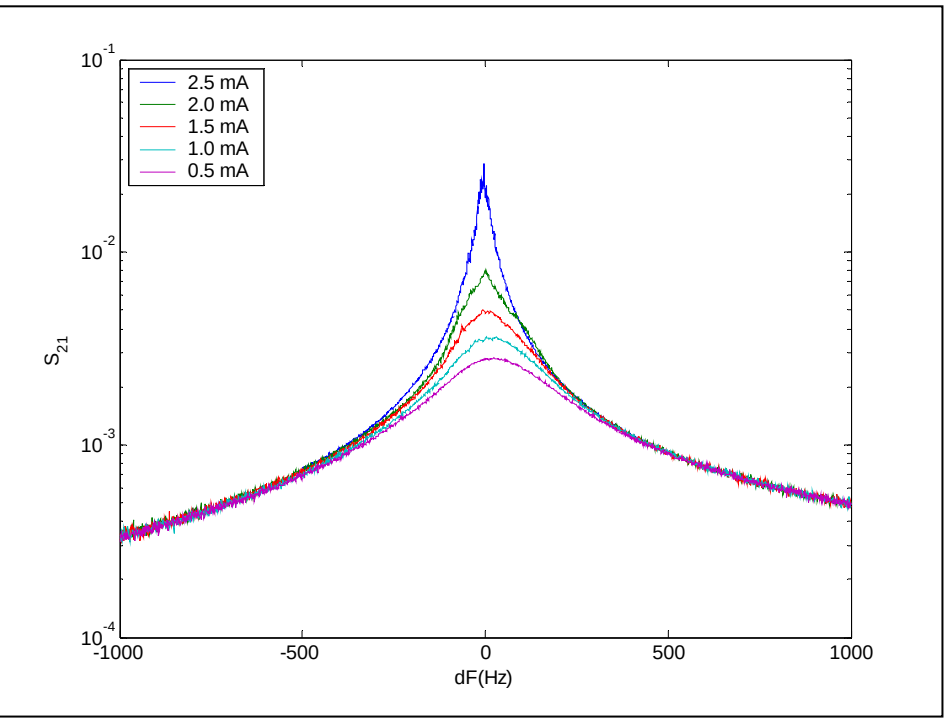


$$Q = \omega_c / \delta\omega_{1/2}$$

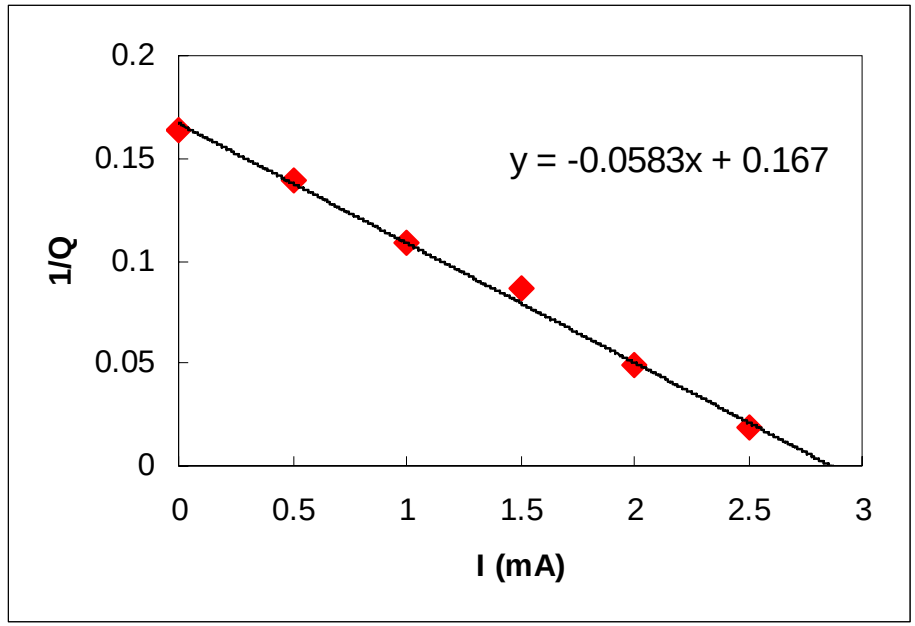
TF measurements at the FEL

Cav 7, F_{hom}=2106 MHz

$$Q_{eff} = Q\frac{I_{th}}{I_{th} - I} \quad => \quad \frac{1}{Q_{eff}} = \frac{1}{Q}\frac{I_{th} - I}{I_{th}}$$



If $I=I_{th}$, $1/Q_{eff}=0$

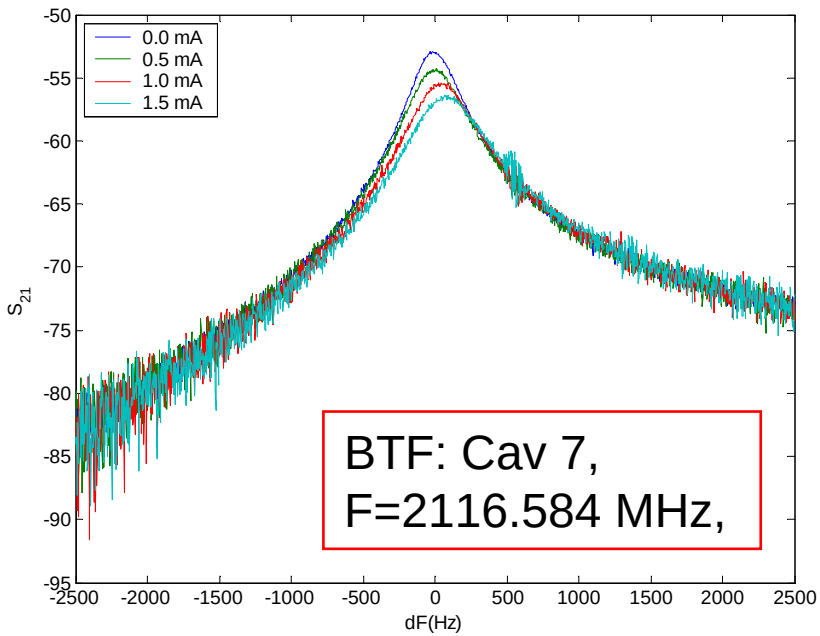


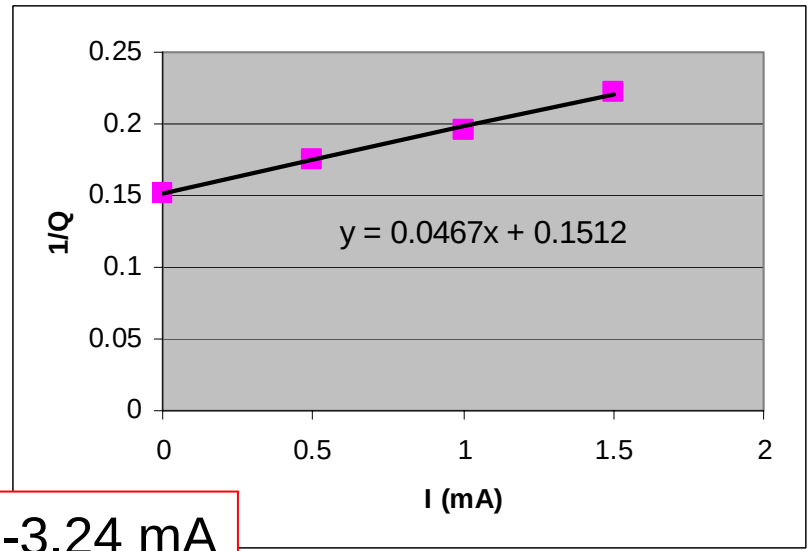
Projected threshold current is 2.9 mA

TF measurements for $m_{12}\sin(\omega T_r)>0$

$$Q_{eff} = Q\frac{I_{th}}{I_{th}-I} = Q\frac{-|I_{th}|}{-|I_{th}|-I} = Q\frac{|I_{th}|}{|I_{th}|+I}$$

$$\frac{1}{Q_{eff}} = \frac{1}{Q}\frac{|I_{th}|+I}{|I_{th}|}$$





I_{th}=-3.24 mA

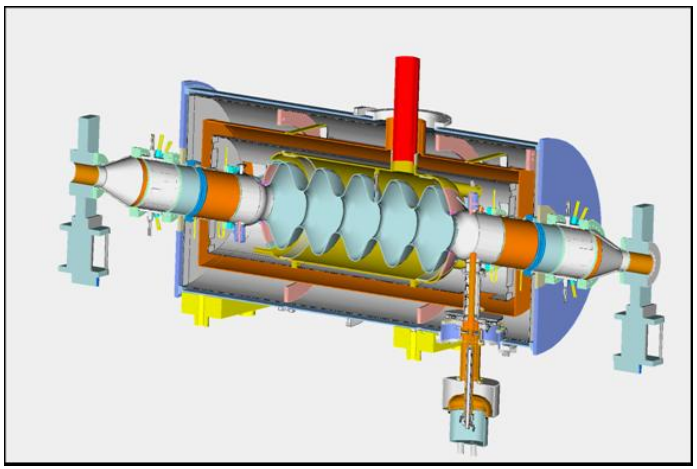
For $m_{12}\sin(\omega T_r)>0$, instability still can happen at very high currents (~10A). (J. Bisognano, G. Krafft, S. Laubach (1987), B. Yunn, 1991 Hoffstaetter, Bazarov (2004))

Suppression of multi-pass instability

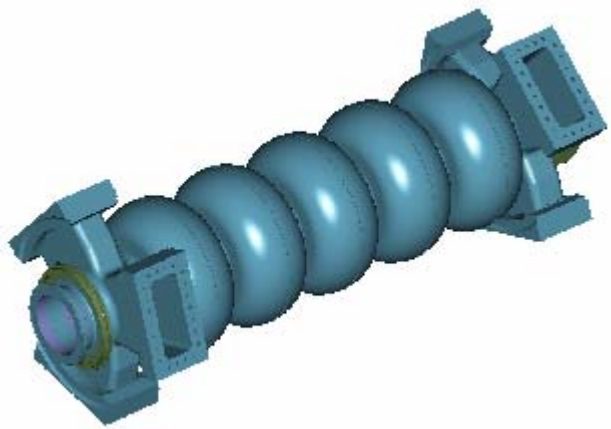
Design of cavities with low-Q HOMs

- Design of multi-cell cavities with low-Q, low-R/Q HOMs seems to be the most reliable way to increase the BBU threshold in large-scale ERLs.The work is under way at BNL, JLAB, Cornell U…
- To provide adequate damping, HOM power must be effectively evacuated from a cavity and damped outside

Brookhaven National Lab

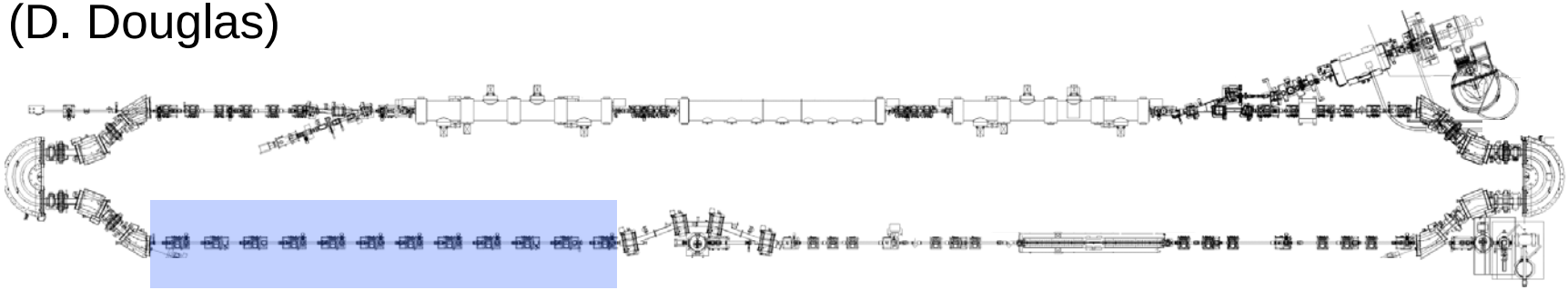


JLAB

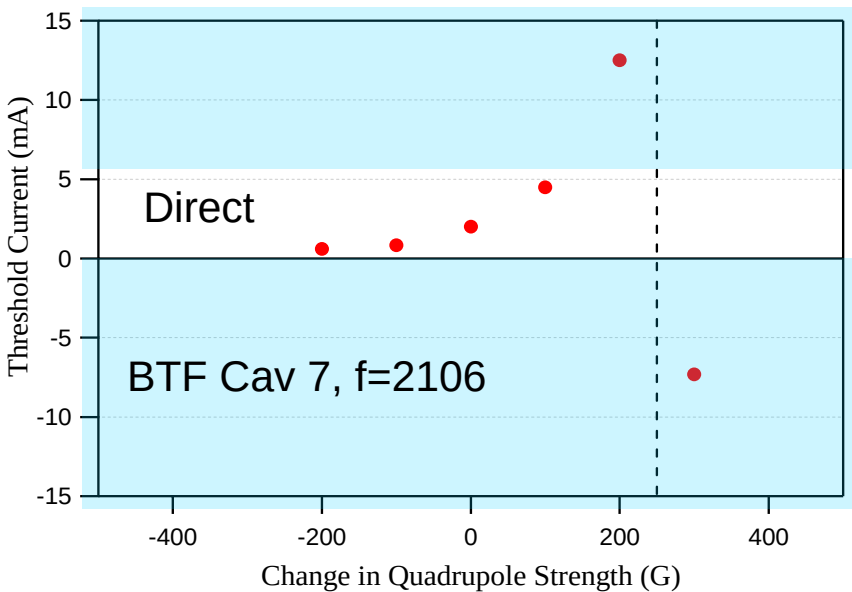


BBU suppression at the FEL: Point-to-Point focusing (Phase Trombone)

(D. Douglas)



Quadrupoles of the blue region were adjusted to maximize I_{th}



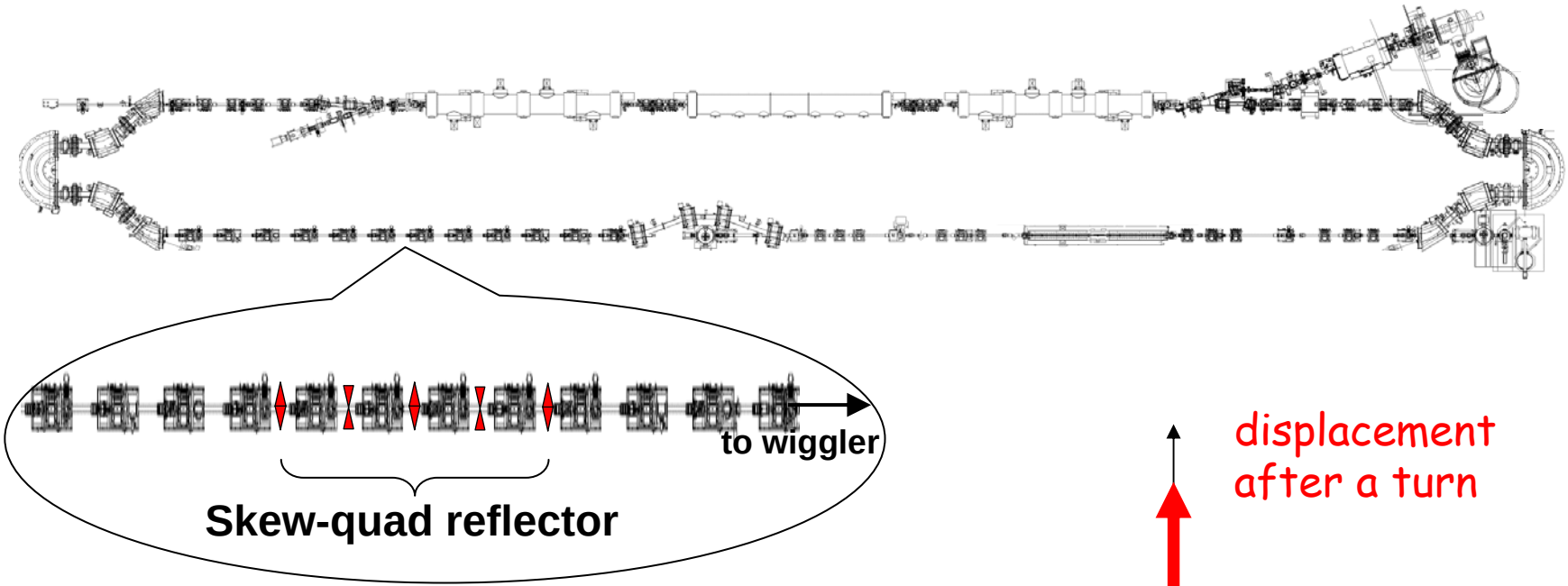
$$I_{th} = -\frac{2V_b}{(\omega/c)\left(\frac{R}{Q}\right) Q\, m_{34} \sin(\omega T_r)}$$

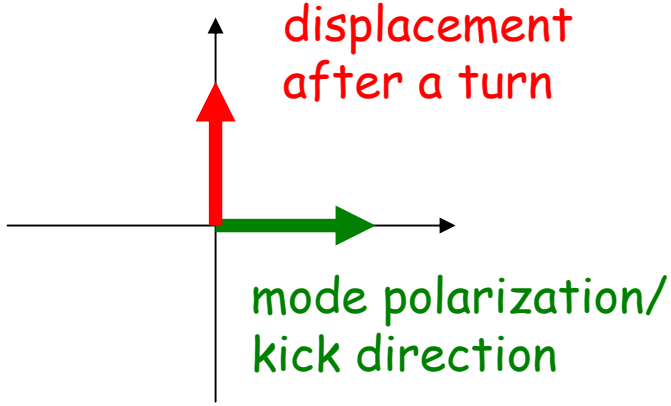
$$m_{34} \propto \sin(\psi) = \sin(n\pi + \delta) \approx (-1)^n \delta$$

Realistically, the threshold increase factor in the FEL is limited to 2 – 3.

BBU suppression at the FEL: Reflector

Idea: R. Rand, T. Smith (1980). Implemented: D. Douglas + FEL team (2004)

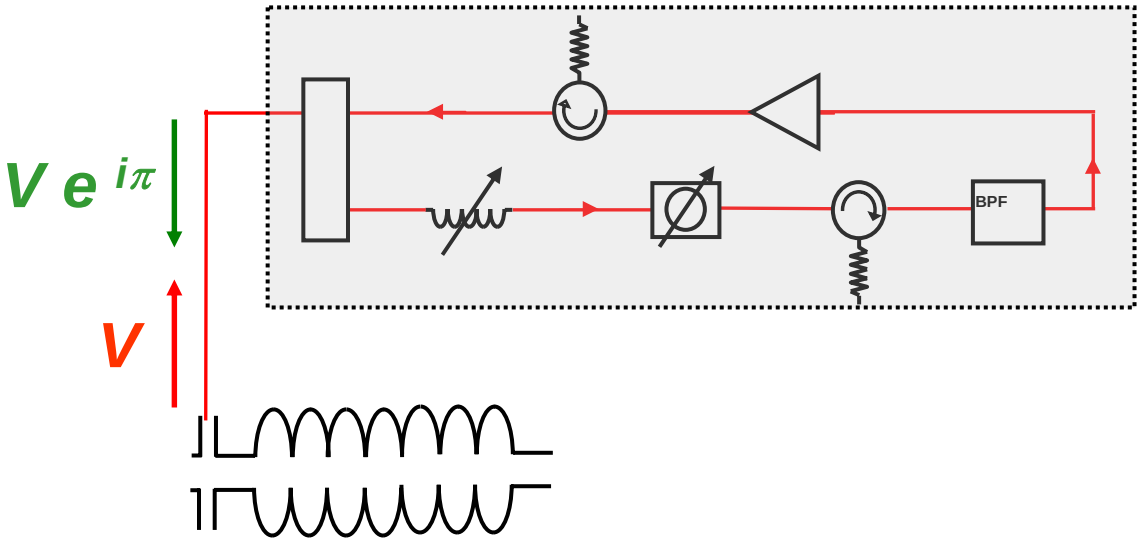


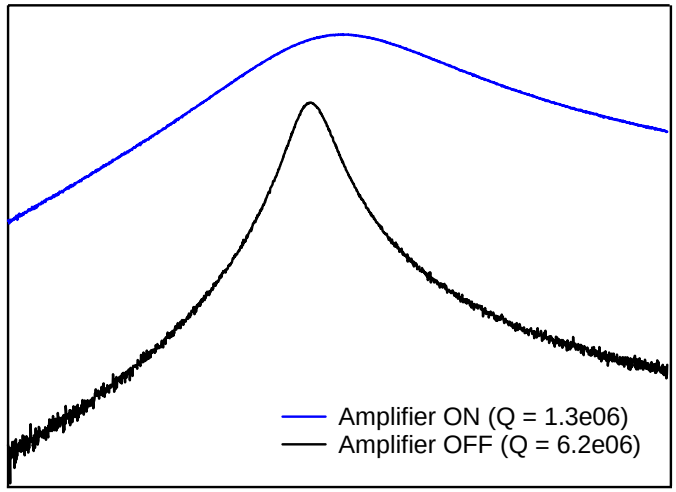


$$\begin{bmatrix} 0 \\ 0 \\ m_{32}x' \\ m_{24}x' \end{bmatrix} = \begin{bmatrix} 0 & 0 & m_{13} & m_{14} \\ 0 & 0 & m_{23} & m_{24} \\ m_{31} & m_{32} & 0 & 0 \\ m_{41} & m_{42} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ x' \\ 0 \\ 0 \end{bmatrix}$$

Threshold was increased by a factor of 5

BBU suppression at the FEL: Cavity Feedback





HOM resonance curve with and without feedback (no beam)

Suppression factors up to 20 were achieved for a short period of time.
Suppression factors of 5 to 10 were achieved reliably.

Other collective effects

- Coulomb force ("Space Charge") at lower energies
- Short range fields arising due to irregularities and beam pipe resistance
- Coherent synchrotron radiation
- Intra-beam particle scattering

…

References

- Cavities and modes:
  - Jackson's Electrodynamics
  - H. Padamsee, J. Knobloch, T. Hays, "RF Superconductivity for accelerators"
- Collective effects and instabilities:
  - Alex Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators"