UVa Course on
Physics of
Particle Accelerators

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Clarification from last time

Have Phase Stability if

$$-1 < \left(\frac{\mathrm{Tr}\, M}{2} \right) < 1 \rightarrow -1 < 1 - \frac{2\pi^2 \rho_l e V_c}{\lambda E_l} \sin\phi_s < 1$$

$$\frac{2\pi^2 \rho_l e V_c}{\lambda E_l} \sin\phi_s = \frac{\pi f_{RF} e V_c}{f_c m c^2} \cos\phi_s \tan\phi_s = \frac{\pi f_{RF} \Delta\gamma}{f_c} \tan\phi_s$$

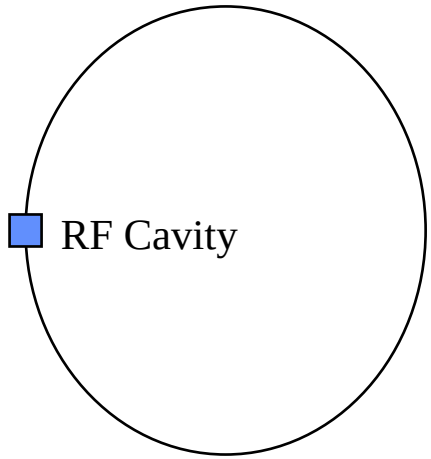
i.e.,

$$\boxed{0 < \nu\pi \tan\phi_s < 2}$$

Synchrotrons

Two basic generalizations needed

- Acceleration of non-relativistic particles
- Difference equation describing per turn dynamics becomes a differential equation with solution involving a new frequency, the synchrotron frequency



Acceleration of non-relativistic particles

For microtron, racetrack microtron and other polytrons, electron speed is at the speed of light. For non-relativistic particles the recirculation time also depends on the longitudinal velocity $v_z = \beta_z c$.

$$t_{recirc} = L / \beta_z c$$

$$\Delta t = \frac{\partial L}{\partial p}\frac{\Delta p}{\beta_z c} + \frac{L}{c}\frac{\partial}{\partial \beta_z}\frac{1}{\beta_z}\Delta\beta_z$$

$$\frac{\Delta t}{t_{recirc}} = \frac{M_{56}}{L}\frac{\Delta p}{p} - \frac{\Delta\beta_z}{\beta_z} = \frac{M_{56}}{L}\frac{\Delta p}{p} - \frac{1}{\gamma^2}\frac{\Delta p}{p}$$

Momentum Compaction

$$\frac{\Delta t}{t_{recirc}} = -\eta_c \frac{\Delta p}{p} \rightarrow \eta_c = \frac{1}{\gamma^2} - \frac{M_{56}}{L}$$

$$2p\Delta pc^2 = 2E\Delta E \rightarrow \frac{\Delta p}{p} = \frac{1}{\beta_z^2}\frac{\Delta E}{E} \rightarrow \frac{\Delta t}{t_{recirc}} = -\frac{\eta_c}{\beta_z^2}\frac{\Delta E}{E}$$

Transition Energy: Energy at which the change in the once around time becomes independent of momentum (energy)

$$\eta_c = 0 \rightarrow \frac{1}{\gamma_t^2} = \frac{M_{56}}{L}$$

Equation for Synchrotron Oscillations

$$\begin{pmatrix} \Delta\phi_{l+1} \\ \Delta E_{l+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -eV_c \sin\phi_s & 1 \end{pmatrix} \begin{pmatrix} 1 & -\dfrac{2\pi L\eta_c}{\lambda\beta_z^2 E_l} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\dfrac{2\pi L\eta_c}{\lambda\beta_z^2 E_l} \\ -eV_c \sin\phi_s & 1 + \dfrac{2\pi L\eta_c}{\lambda\beta_z^2 E_l} eV_c \sin\phi_s \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

Assume momentum slowly changing (adiabatic acceleration)
Phase advance per turn is

$$\cos\Delta\mu = 1 + \frac{\pi L\eta_c}{\lambda\beta_z^2 E_l} eV_c \sin\phi_s \rightarrow \Delta\mu^2 \approx -\frac{2\pi L\eta_c}{\lambda\beta_z^2 E_l} eV_c \sin\phi_s$$

So change in phase per unit time is

$$\frac{\Delta\mu}{T_0} \approx \frac{1}{T_0}\sqrt{-\frac{2\pi L\eta_c}{\lambda\beta_z pc} eV_c \sin\phi_s}$$

yielding synchrotron oscillations with frequency

$$\omega_s = \omega_{rev}\sqrt{-\frac{h\eta_c}{2\pi}\frac{eV_c}{pc}\sin\phi_s}$$

where the *harmonic number* $h = L / \beta_z\lambda$, gives the integer number of *RF* oscillations in one turn

Phase Stable Acceleration

At energies below transition, $\eta_c > 0$. To achieve acceleration with phase stability need $\phi_s < 0$

$$\therefore \omega_s = \omega_{rev}\sqrt{\frac{h\eta_c}{2\pi}\frac{eV_c}{pc}\sin\left(-\phi_s\right)}$$

At energies above transition, $\eta_c < 0$, which corresponds to the case we're used to from electrons. To achieve acceleration with phase stability need $\phi_s > 0$

$$\therefore \omega_s = \omega_{rev}\sqrt{\frac{h\left(-\eta_c\right)}{2\pi}\frac{eV_c}{pc}\sin\phi_s}$$

Large Amplitude Effects

Can no longer linearize the energy error equation.

$$\Delta\phi_{l+1} = \Delta\phi_l - \frac{2\pi L\eta_c}{\lambda\beta_z^2 E_l}\Delta E_l$$

$$\Delta E_{l+1} = \Delta E_l + eV_c\left(\cos\left(\phi_s + \Delta\phi_l\right) - \cos\phi_s\right)$$

$$\frac{d\Delta\phi}{dt} \approx \frac{\Delta\phi_{l+1} - \Delta\phi_l}{T_0} = -\frac{2\pi\eta_c}{\lambda p}\Delta E$$

$$\frac{d\Delta E}{dt} \approx \frac{\Delta E_{l+1} - \Delta E_l}{T_0} = \frac{eV_c\left(\cos\left(\phi_s + \Delta\phi_l\right) - \cos\phi_s\right)}{T_0}$$

$$\frac{d^2\Delta\phi}{dt^2} = -\frac{2\pi\eta_c}{\lambda p T_0}eV_c\left(\cos\left(\phi_s + \Delta\phi\right) - \cos\phi_s\right)$$

Constant of Motion (Longitudinal "Hamiltonian")

$$\frac{d\Delta\phi}{dt}\frac{d^2\Delta\phi}{dt^2} = -\frac{2\pi\eta_c}{\lambda p T_0} eV_c \frac{d\Delta\phi}{dt}\left(\cos\left(\phi_s + \Delta\phi\right) - \cos\phi_s\right)$$

$$\frac{1}{2}\left(\frac{d\Delta\phi}{dt}\right)^2 = -\frac{2\pi\eta_c}{\lambda p T_0} eV_c \left(\sin\left(\phi_s + \Delta\phi\right) - \Delta\phi\cos\phi_s\right) + C$$

$$H\left(\Delta\phi, T_0\Delta E\right) = \frac{1}{2}\frac{2\pi\eta_c}{\lambda p T_0}\left(T_0\Delta E\right)^2 + eV_c\left(\sin\left(\phi_s + \Delta\phi\right) - \Delta\phi\cos\phi_s\right)$$

Equations of Motion

If neglect the slow (adiabatic) variation of p and T_0 with time, the equations of motion approximately Hamiltonian

$$\frac{d\Delta\phi}{dt} = \frac{\partial H}{\partial(T_0 \Delta E)} \qquad \frac{d(T_0 \Delta E)}{dt} = -\frac{\partial H}{\partial \Delta\phi}$$

In particular, the Hamiltonian is a constant of the motion

Kinetic Energy Term

$$T = \frac{1}{2}\frac{2\pi\eta_c T_0}{\lambda p}(\Delta E)^2$$

Potential Energy Term

$$V = eV_c\left(\sin(\phi_s + \Delta\phi) - \Delta\phi\cos\phi_s\right)$$

No Acceleration

$$\phi_s = \pi / 2 \qquad V = eV_c \cos \Delta\phi$$

$$\frac{d^2 \Delta\phi}{dt^2} = -\omega_s^2 \sin \Delta\phi$$

Better known as the real pendulum. Phase space Wiedemann Fig 8.5

With Acceleration

$$\frac{d^2\Delta\phi}{dt^2} = -\frac{\omega_s^2}{\sin\phi_s}\left(\cos\left(\phi_s + \Delta\phi\right) - \cos\phi_s\right)$$

$$\frac{1}{2}\left(\frac{d\Delta\phi}{dt}\right)^2 = -\frac{\omega_s^2}{\sin\phi_s}\left(\sin\left(\phi_s + \Delta\phi\right) - \Delta\phi\cos\phi_s\right) + C$$

Equation for separatrix yields “fish” diagrams in phase space. Fixed points at

$$\cos\left(\phi_s + \Delta\phi\right) = \cos\phi_s \qquad \Delta\phi = 0, -2\phi_s$$

Independent Orbit Recirculators - Motivation

- At final beam energy, $E_f \sim$ several 100 MeV, cost of racetrack microtron is dominated by cost of end magnets

- Cost of end magnets $\propto E_f^3$
  $\Rightarrow$ Standard racetrack microtron (RTM) uneconomical at $E_f \approx 500 - 1000$ MeV

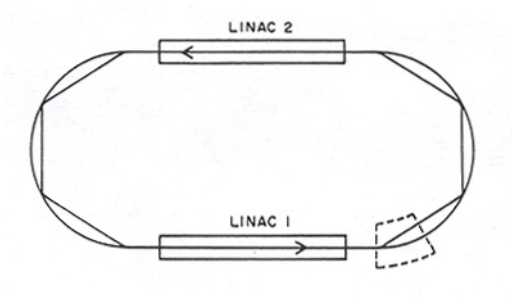
- Bicyclotron and hexatron: one method to overcome the problem but they are similarly limited

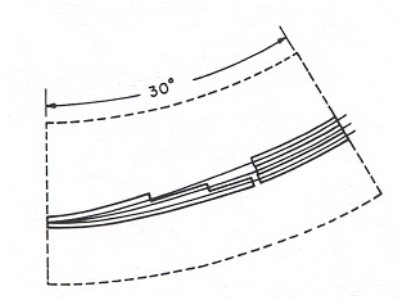
- A distinctly different approach: A recirculation system with independent or separate orbits, *i.e.* orbits which do not share the same uniform field magnets

- Cost $\propto E_f$ (close to the ideal)

The “Mesotron”

- The first of independent orbit recirculating accelerator designs
- Proposed by Bathow et al., (1968) for high duty factor acceleration at very high energies – up to 60 GeV

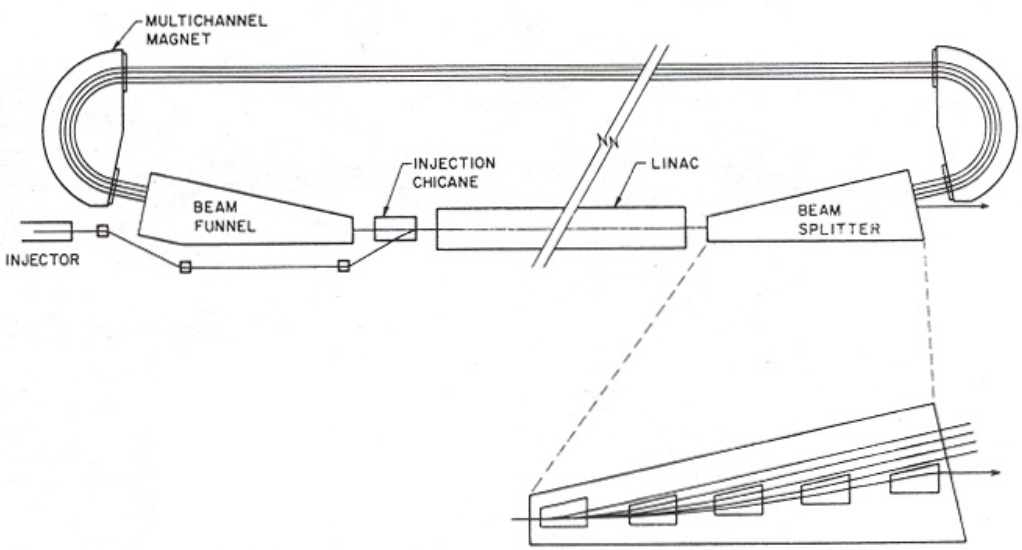




- Although looks similar to a high order polytron, it is distinctly different because of the independent control of every orbit
- At high energies, synchrotron radiation (SR) could present problems and magnetic field values would be restricted to very low values as a consequence.
- At E > 50 GeV, the Mesotron might be cheaper to build than a synchrotron since it has independent DC magnets and can tolerate a much greater energy loss per orbit by SR.

The Stanford–HEPL Superconducting "Recyclotron"

- Main recirculation magnets incorporate four channels (tracks) in which the uniform fields are independently tailored to the momenta of the separate orbits.
  - Use a constant magnet gap with staggered coil windings which produce an appropriately stepped field profile.



No Phase Stability in Independent Orbit Recirculators

- For isochronous ($M_{56} = 0$) transport:

$$\begin{pmatrix} \Delta\phi_{l+1} \\ \Delta E_{l+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -eV_c \sin\phi_s & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

- Usually ϕ_s =0. Higher order effects tend to become important.

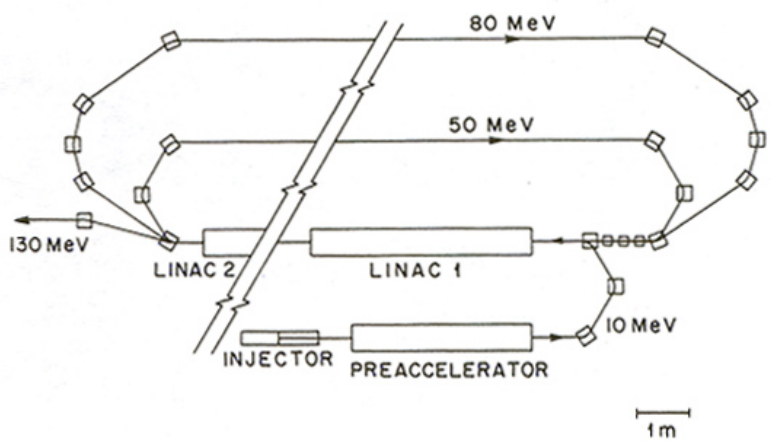
Examples of Isochronous Recirculating Linacs

- The Wuppertal/Darmstadt “Rezyklotron”
- The MIT-Bates Recirculator
- The CEBAF at Jefferson Lab

The Wuppertal/Darmstadt "Rezyklotron"

- The "Rezyklotron" incorporates a superconducting linac at 3 GHz.
- Beam injection energy = 11 MeV, variable extraction energy up to 130 MeV, beam current 20 μA, 100% duty factor. Energy resolution = 2×10^{-4} .
- Two orbits designed with 180^0 isochronous and achromatic bends and two quadrupole doublets and two triplets in the backleg.
- Isochronous beam optics

  Phase oscillations do not occur and energy resolution is determined primarily by second order effects in the linac.



The MIT-BATES Recirculator

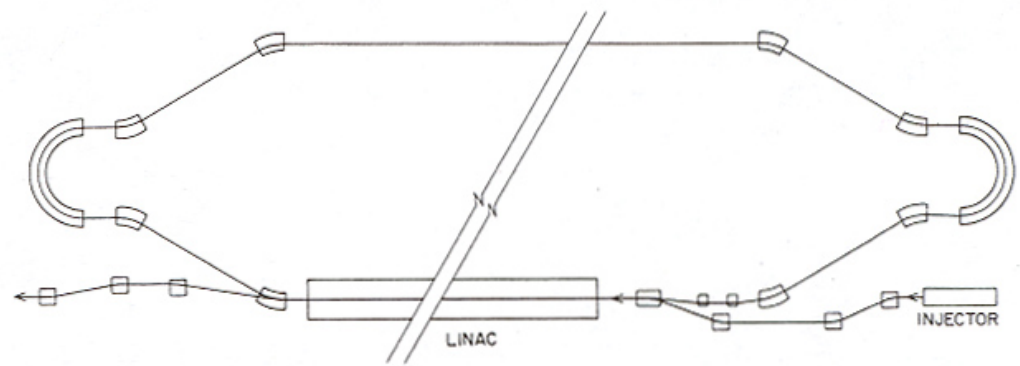
- The MIT-Bates, one-orbit recirculator: An isochronous recirculator
- Severe transient beam loading dictates the isochronous nature of MIT-Bates transport system.

  a) Fluctuations of beam current during each pulse cause variable beam loading The resulting first pass energy variation of ± 0.15%. At a magnet bending radius of about 1m this energy fluctuation would result in bunch length, after recirculation in a non-isochronous orbit, of almost 90° of rf phase!

  b) Total accelerating potential drops by 6% when recirculated beam re-enters the linac and total beam current goes from 8mA to 16 mA. With non-isochronous transport, resulting change in orbit energy would be equivalent to a path length change of many λ_{rf} .
- Both effects were eliminated by an isochronous recirculation design that could accommodate a 6% energy change.
- Flanz *et al*. (1980) successfully designed a recirculator that satisfies all these conditions.

The MIT-BATES Recirculator (cont'd)

- Injection energy = 20 MeV
- Each end of the transport system consists of 5 uniform field dipole magnets which bend by 20°, –20°, 180°, –20° and 20°.
- Edge focusing in these magnets is the only form of focusing in these parts of the orbit.
- Four sextupoles control higher order optical aberrations
- Straight section in the backleg contains 5 quadrupole triplets
- Final energy to date is 750 MeV (?) at an average current of 100 μA (?) (5 mA pulse current) with energy resolution ±0.15% have been achieved.



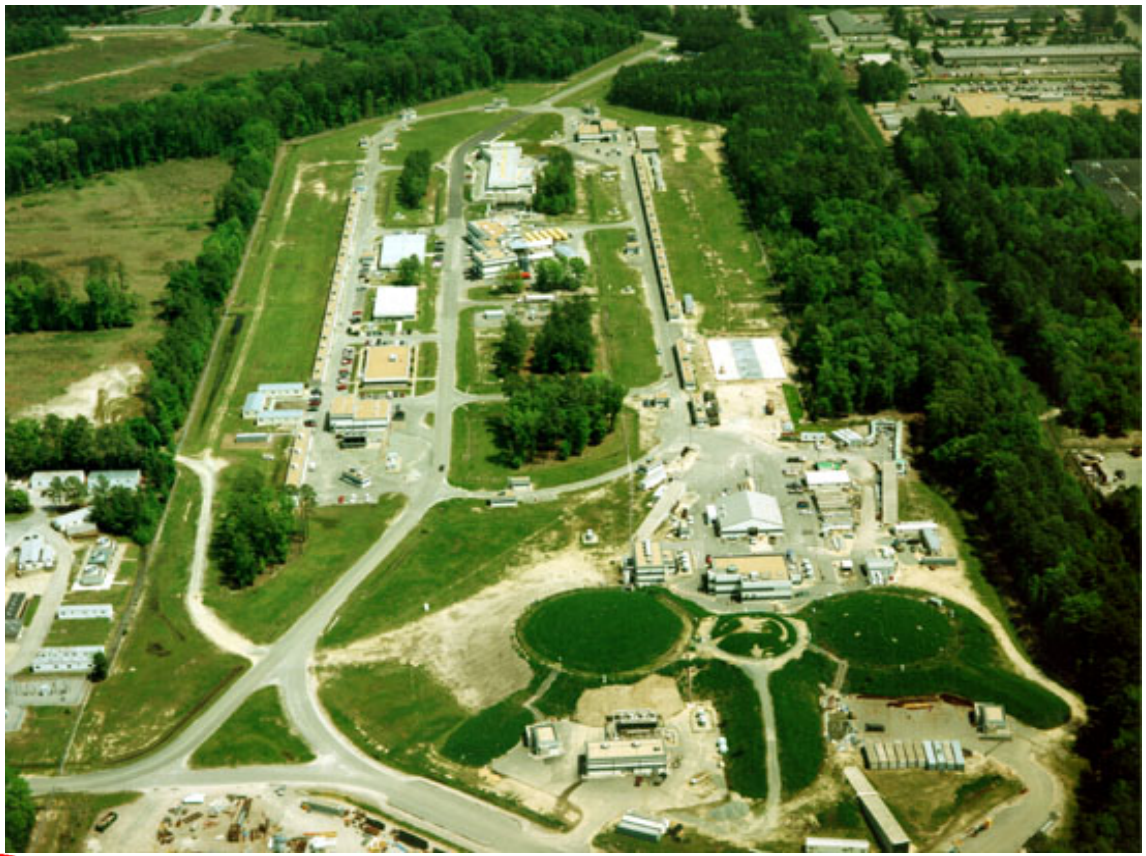
The CEBAF at Jefferson Lab

- The CEBAF accelerator is a 5-pass recirculating srf linac with cw beams of up to 200 μA, geometric emittance < 10^{-9} m, and relative momentum spread of a few 10^{-5}.

- The present full energy is nearly 6 GeV. An upgrade to 12 GeV is planned.

The CEBAF at Jefferson Lab (cont'd)

- Most radical innovations (had not been done before on the scale of CEBAF):
  - choice of srf technology
  - use of multipass beam recirculation
- Until LEP II came into operation, CEBAF was the world's largest implementation of srf technology.



The CEBAF at Jefferson Lab (cont'd)

- SRF Technology

  - srf at 1500 MHz is adopted for CEBAF: result of optimization but ultimately Cornell design had well developed understanding of HOM impedances and Q's and had demonstrated effectiveness of the waveguide-type HOM couplers.

  - Advantage of the design: small energy spread ~ 2.5×10^{-5} and similar relative energy stability are possible

    $\Rightarrow$ tight control of rf phase and amplitude in each cavity is required

  - srf cavities have ~150 Hz bandwidth

    $\Rightarrow$ experience microphonics (mechanical vibrations leading to oscillations in their resonant frequency)

    These oscillations lead to tuning errors of up to 25°.

  - The need to meet tight control requirements led to a defining characteristic of CEBAF rf system: each cavity has its own klystron and low-level rf control system.

The CEBAF at Jefferson Lab (cont'd)

- Recirculation and Beam Optics

  - A straightforward linac would exceed the projects' cost boundaries adopt beam recirculation

  - Relativistic electrons travel at ~c independent of energy. They stay within <1° of rf phase at 1500 MHz of a phase reference point over many kilometers.

  - A recirculating linac sends a beam n times through a linac section 1/n the length of a full-energy linac by means of n transport systems tuned to the energy of the nth path.

  - Each transport system must be unique to accommodate the momentum of the specific beam energy it propagates, but in the accelerating sections bunches of different energy occupy the same spatial locations, and because of c, they stay in phase.

The CEBAF at Jefferson Lab (cont'd)

- Recirculation and Beam Optics (cont'd)

  - Each recirculation path is handled by an independent transport system ⇒ individual beam-line designs can be evolved to manage SR-induced degradation of emittance and energy spread ⇒ Recirculating linacs provide an effective path to very high beam energies while allowing preservation of high beam quality!

  - Decisions were made to
    - Have linac sections in both legs of the racetrack for shorter length.
    - Operate in "linac fashion" (on crest) without phase focusing (unlike RTMs):
      - it makes optimal use of installed accelerating structures and
      - phase focusing is not needed with relativistic beam bunches of subpicosecond duration and appropriate precision rf control.

The CEBAF at Jefferson Lab (cont'd)

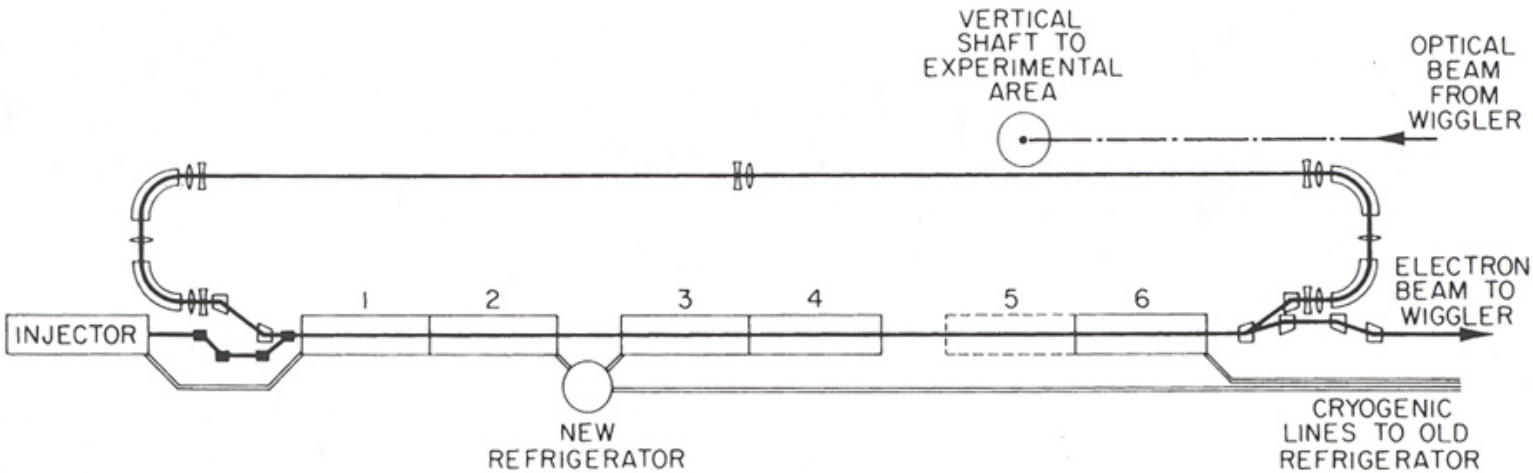
- From these decisions flow several requirements:
  - Linac-to-linac system: achromatic and isochronous (M_{56} <0.2 m) on all passes
  - Pass-to-pass tolerance for phase or path length < 100 μm.
  - Vertical dispersion in the arcs is corrected locally.
- Accelerator Physics
  - Multibunch beam breakup: Threshold current ~ 20 times higher than operating current

Energy Recovery Linacs

- Beam current at CEBAF is limited by the rf power installed and by the beam power on the beam dump, already at 1 MW at 5 GeV and 200 μA.
- Energy recovery is a way to overcome these limits: one can increase the beam current (almost) without increasing the rf power or the beam dump size.
- Basic idea: Bring the beam through the accelerating structures timed in a way so that the second-pass beam is decelerated, i.e. delivering its energy to the cavity fields.
- First demonstration of energy recovery in an rf linac at Stanford University (1986)
- Energy recovery demonstration at world-record current at the Jefferson Lab IR FEL

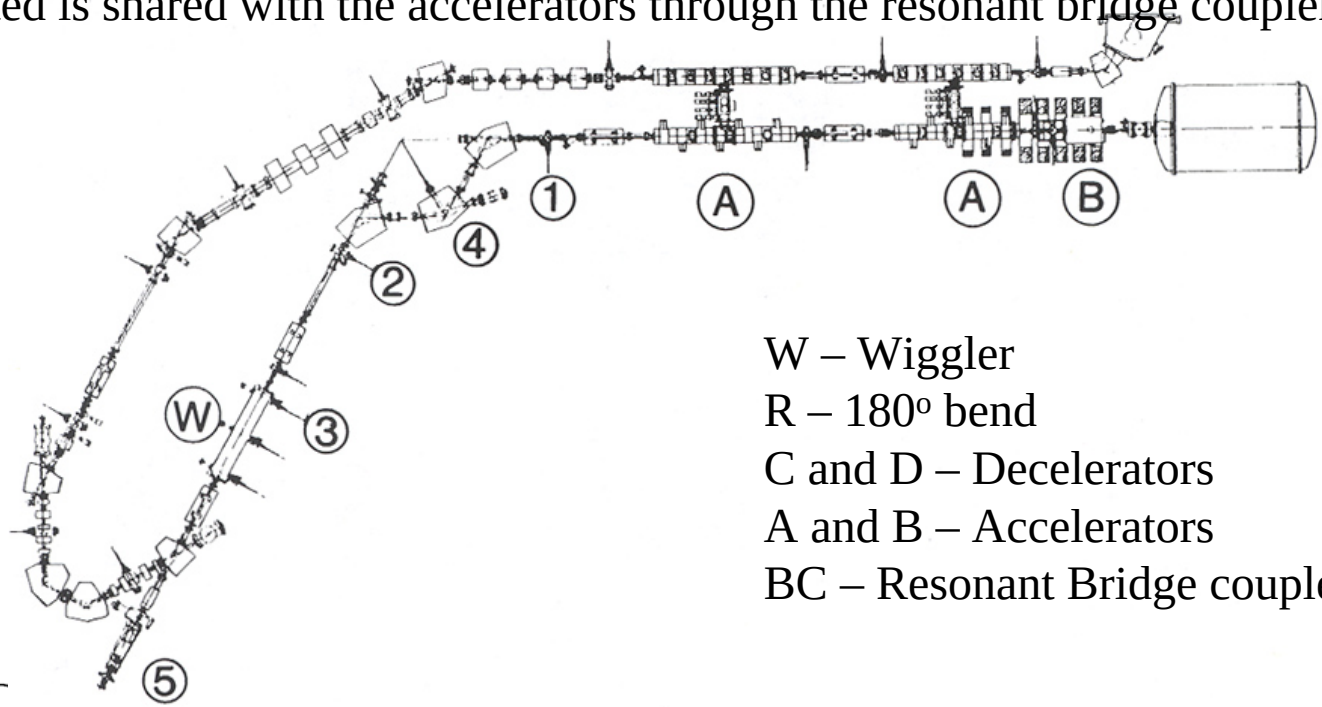
The SCA/FEL Energy Recovery Experiment

- Same-cell energy recovery was first demonstrated in the SCA/FEL in July 1986
- Beam was injected at 5 MeV into a ~50 MeV linac (up to 95 MeV in 2 passes), 150 μA average current (12.5 pC per bunch at 11.8 MHz)
- The previous recirculation system (SCR, 1982) was unsuccessful in preserving the peak current required for lasing and was replaced by a doubly achromatic single-turn recirculation line.
- All energy was recovered. FEL was not in place.



The Los Alamos FEL Energy Recovery Experiment

- Accelerator consists of injector, buncher, and two 10-MeV accelerator sections at 1300 MHz.
- Beam is transported around a 180° bend and through decelerators to a spectrometer.
- Decelerators are coupled to accelerators and klystrons through resonant bridge couplers.
- Electrons lose energy in the decelerators (21 MeV -> 5 MeV), and the rf power generated is shared with the accelerators through the resonant bridge couplers.



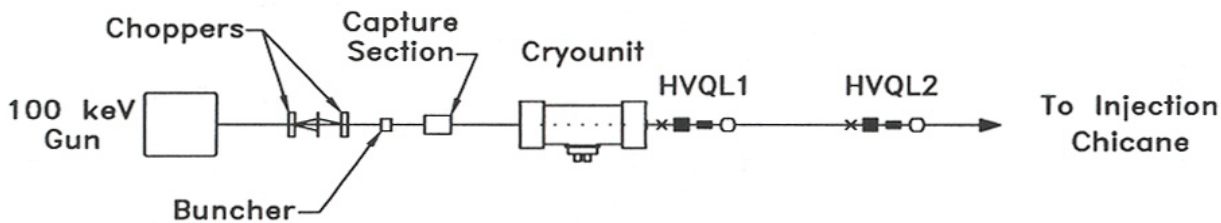
W – Wiggler
R – 180° bend
C and D – Decelerators
A and B – Accelerators
BC – Resonant Bridge couplers

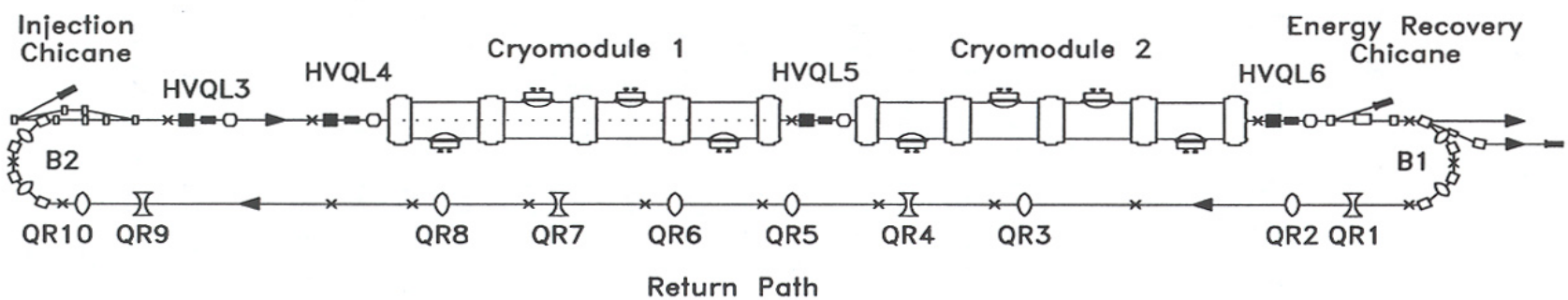
The CEBAF Injector Energy Recovery Experiment

N. R. Sereno, “Experimental Studies of Multipass Beam Breakup and Energy Recovery using the CEBAF Injector Linac,” Ph.D. Thesis, University of Illinois (1994)

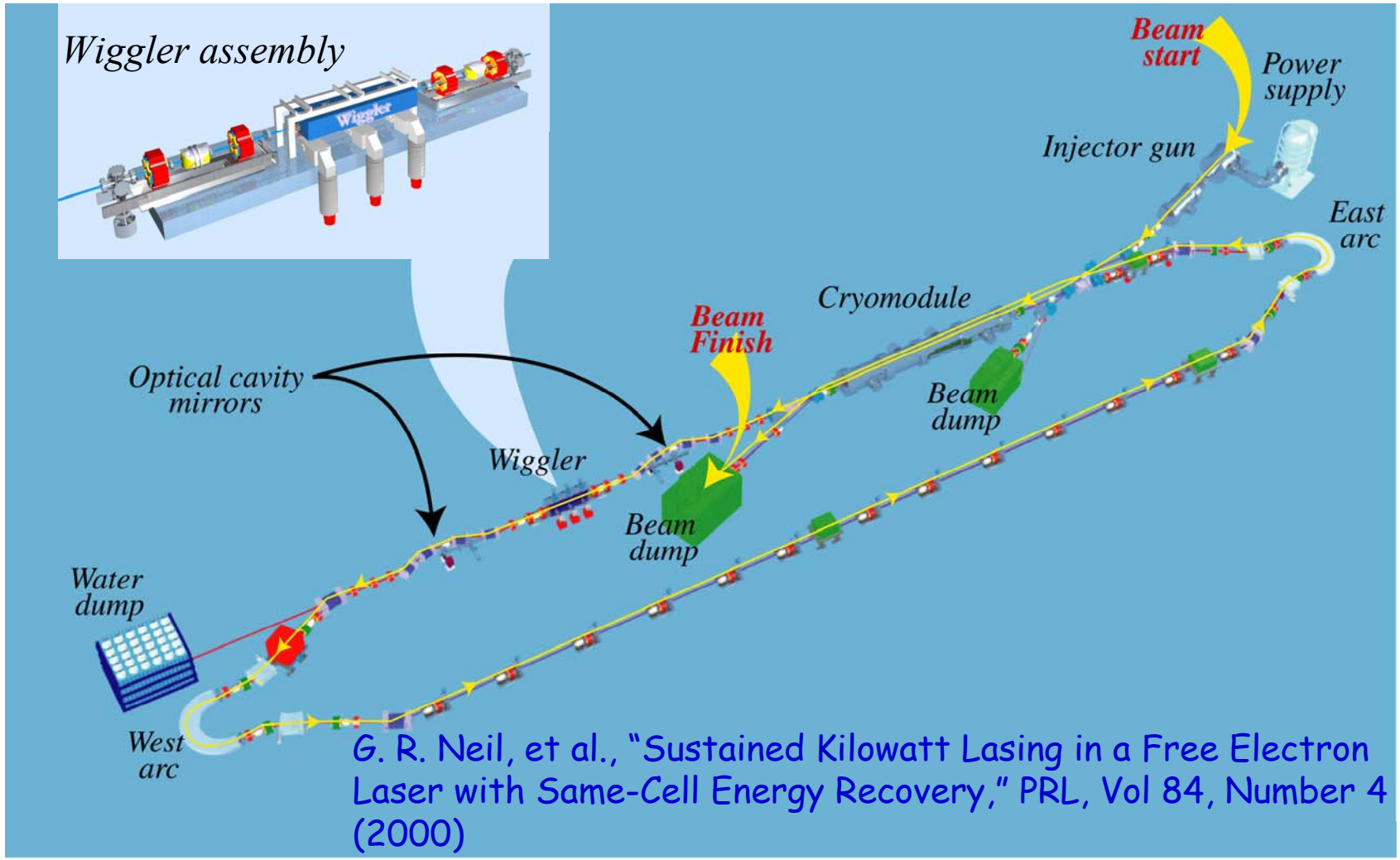
64 – 215 uA in accelerating mode

up to 30 uA in energy recovery mode





The JLab 2.13 kW IRFEL and Energy Recovery Demonstration



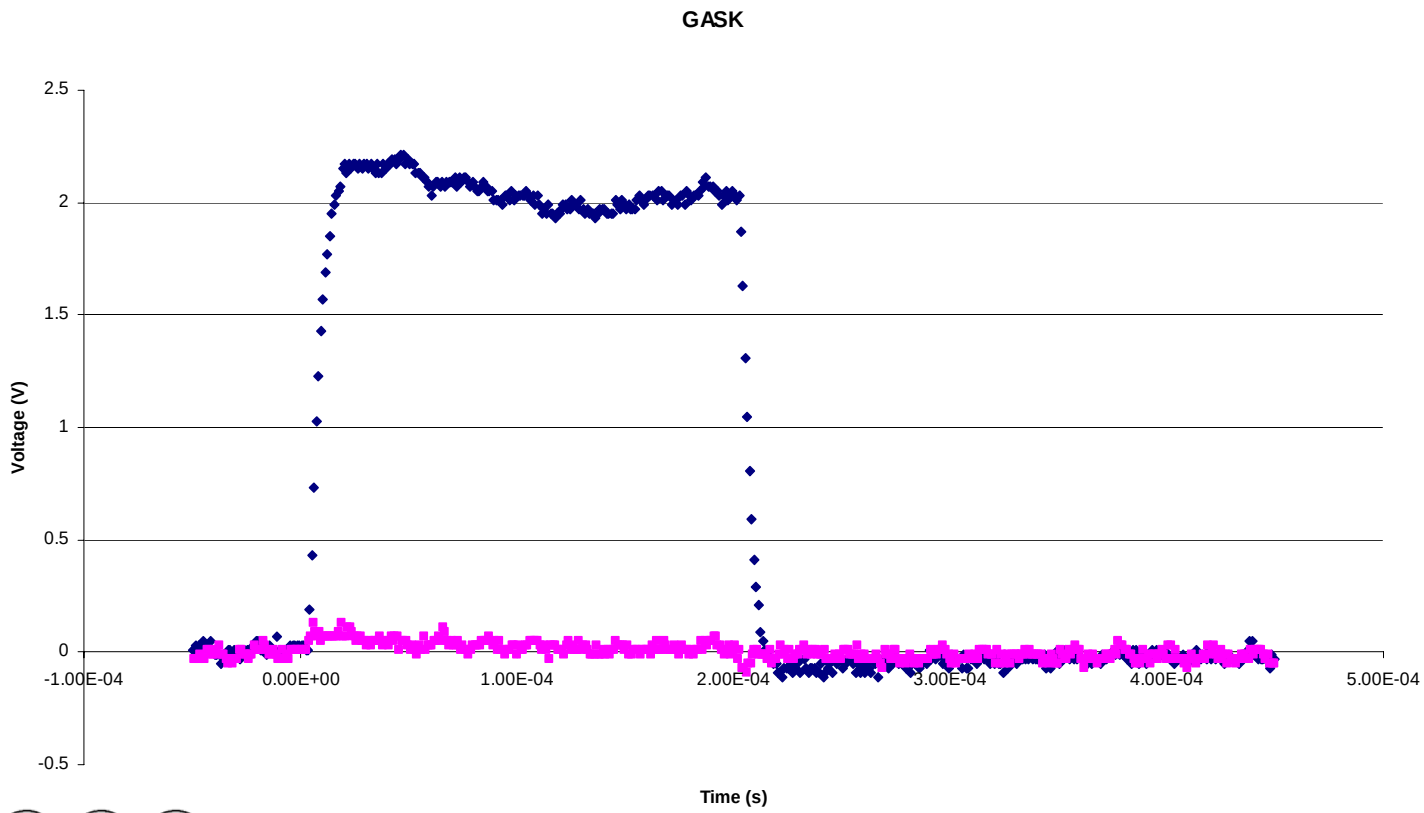
G. R. Neil, et al., "Sustained Kilowatt Lasing in a Free Electron Laser with Same-Cell Energy Recovery," PRL, Vol 84, Number 4 (2000)

IR FEL Parameters

Parameter		Nominal	Achieved
Beam energy at wiggler		42 MeV	20-48 MeV
Beam current		5 mA	5 mA
Single bunch charge		60 pC	60-135 pC
Bunch repetition rate		74.85 MHz	18.7-74.85 MHz
Normalized emittance		13 mm-mrad	5-10 mm-mrad
RMS bunch length at wiggler		0.4 psec	0.4 psec
Peak current		60 A	60 A
FEL extraction efficiency		$\frac{1}{2}$%	>1%
dp/p	rms before FEL full after FEL	$\frac{1}{2}$% 5%	$\frac{1}{4}$% 6-8%
CW FEL Power		~1 kW	2.13 kW

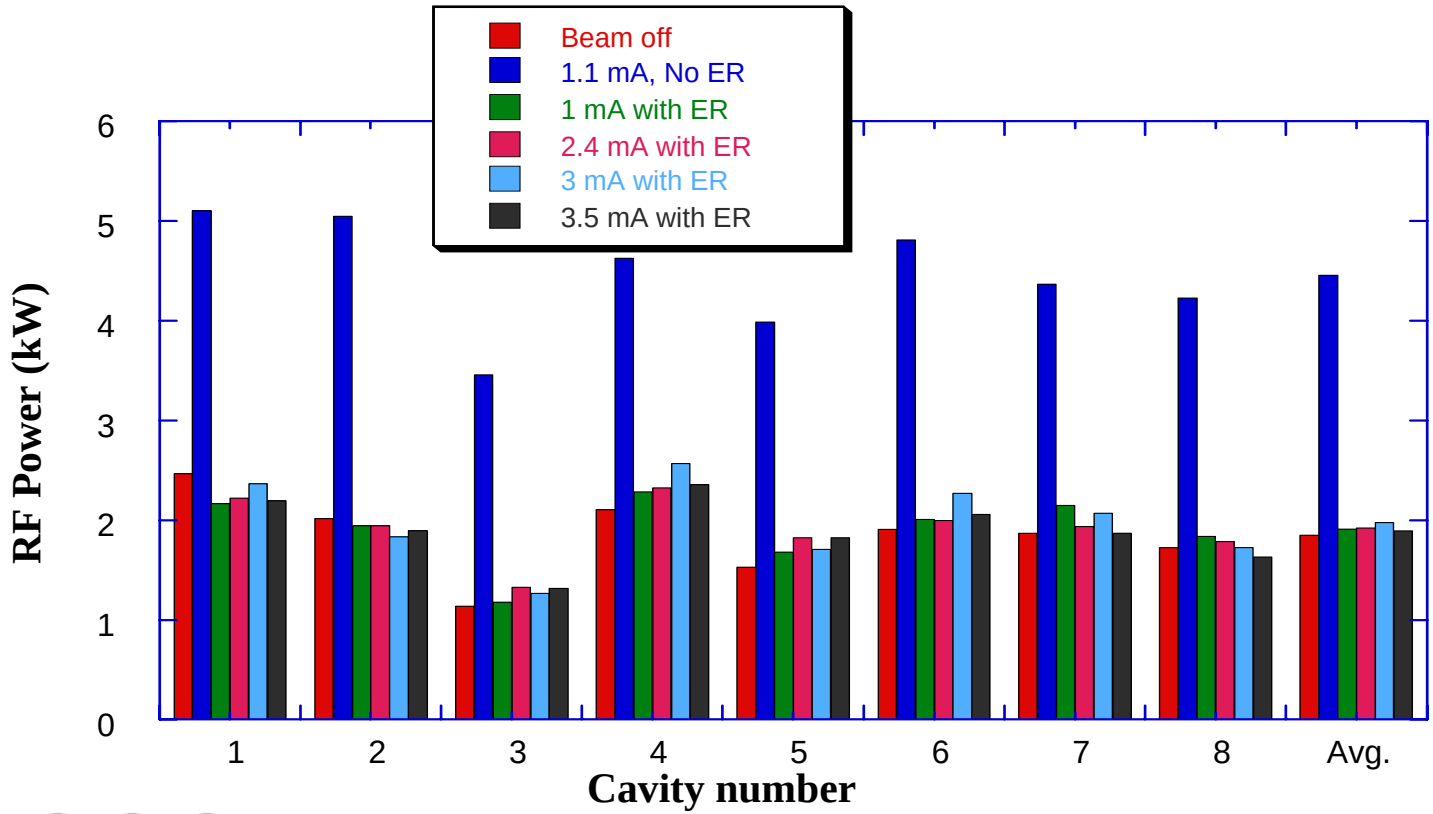
Energy Recovery Works

Gradient modulator drive signal in a linac cavity measured without energy recovery (signal level around 2 V) and with energy recovery (signal level around 0).

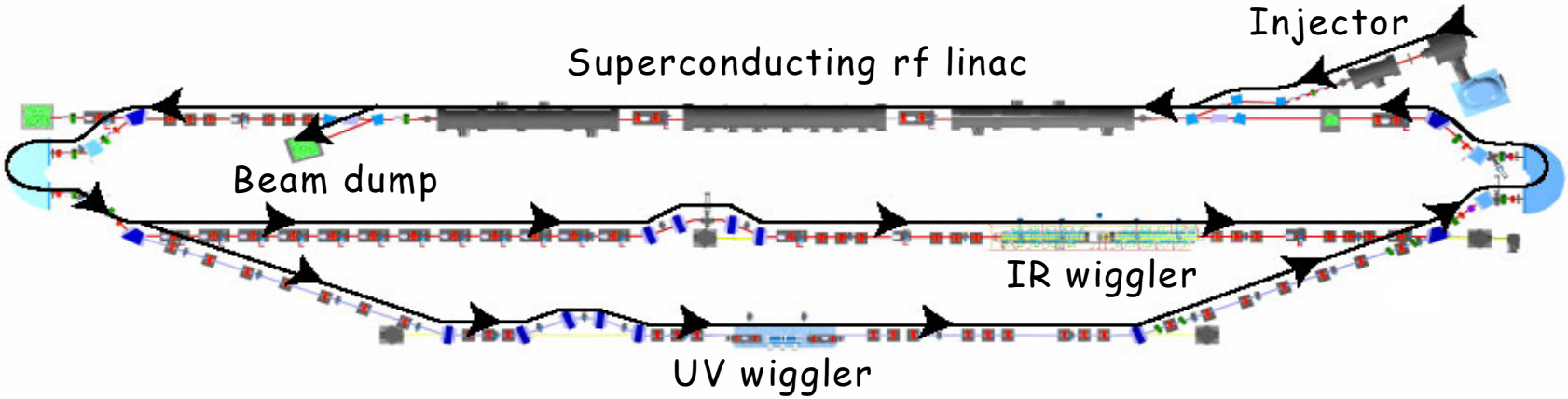


Energy Recovery Works (cont'd)

With energy recovery the required linac rf power is ~ 16 kW, nearly independent of beam current. It rises to ~ 36 kW with no recovery at 1.1 mA.



JLab 10kW IR FEL and 1 kW UV FEL



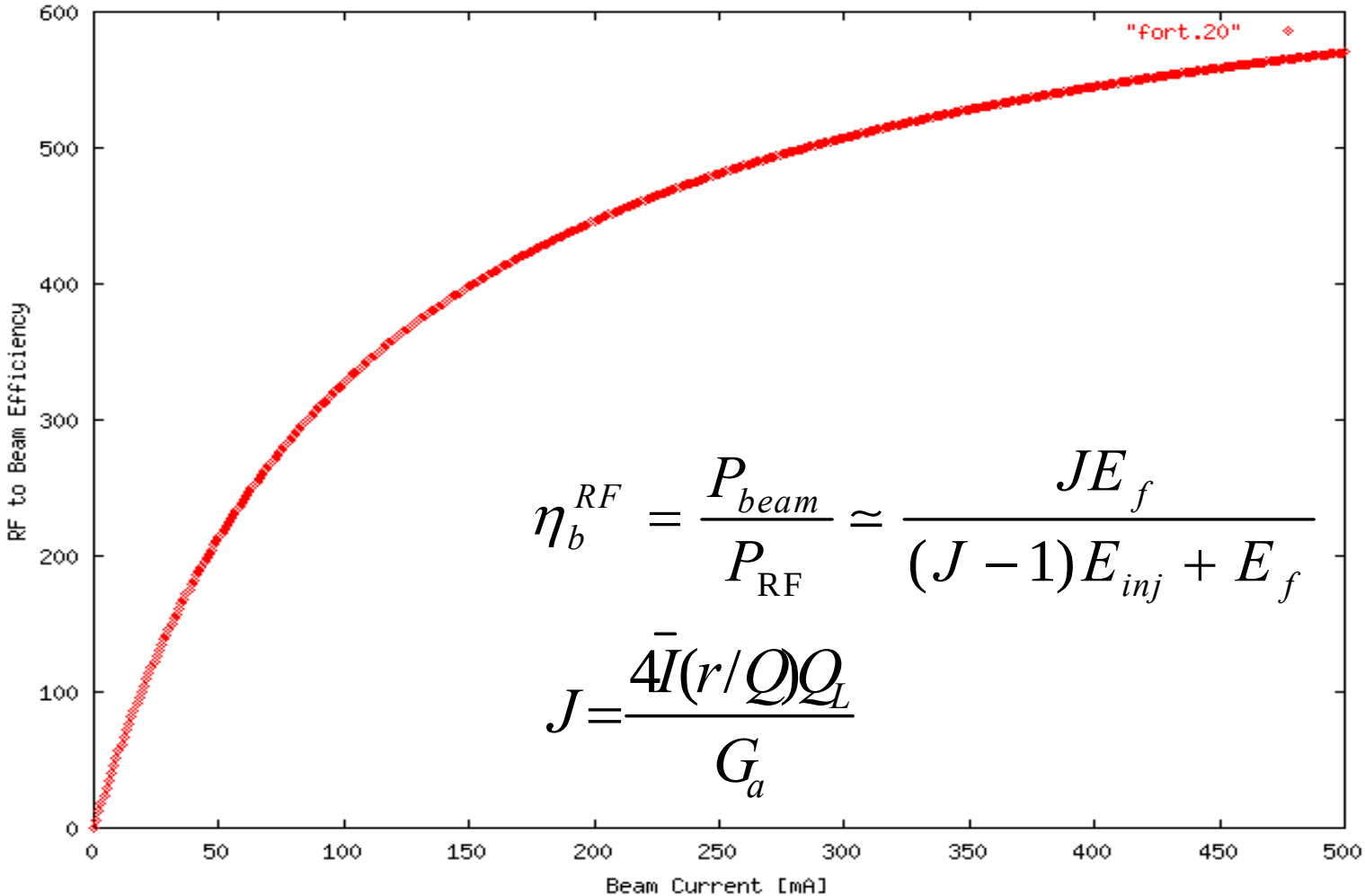
Achieved 8.5 kW CW IR power on June 24, 2004!
Energy recovered up to 5mA at 145 MeV, up to 9mA at 88 MeV

System Parameters for Upgrade (IR&UV)

	Demo	IR Upgrade	UV Upgrade	Achieved
Energy (MeV)	35-48	80-210	200	20-48
I_{ave} (mA)	5	10	5	5
Beam Power (kW)	200	2000	1000	240
Charge/bunch (pC)	60	135	135	135
Rep. Rate (MHz)	18.75-75	4.7-75	2.3-75	18.75-75
Bunch Length* (psec)	0.4	0.2	0.2	0.4(60 pC)
Peak Current (A)	60	270	270	>60 A
σ_E/E	0.5%	0.5%	0.125%	<0.25%
e_N (mm-mrad)	<13	<30	<11	5-10
FEL ext. efficiency	0.5%	1%	0.25%	>0.75%
FEL power (kW)	1	>10	>1	2.1
Induced energy spread (full)	5%	10%	5%	6-8%

* rms value

Rf to Beam Efficiency



$$\eta_b^{RF} = \frac{P_{beam}}{P_{\mathrm{RF}}} \simeq \frac{JE_f}{(J-1)E_{inj} + E_f}$$

$$J = \frac{4\bar{I}(r/Q)Q_L}{G_a}$$

Benefits of Energy Recovery

- Required rf power becomes nearly independent of beam current.
- Increases overall system efficiency.
- Reduces electron beam power to be disposed of at beam dumps (by ratio of E_{fin}/E_{inj}).
- More importantly, reduces induced radioactivity (shielding problem) if beam is dumped below the neutron production threshold.