USPAS Course on
Recirculated and Energy Recovered Linacs

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Lecture 14:
Emittance and energy spread growth due to synchrotron radiation

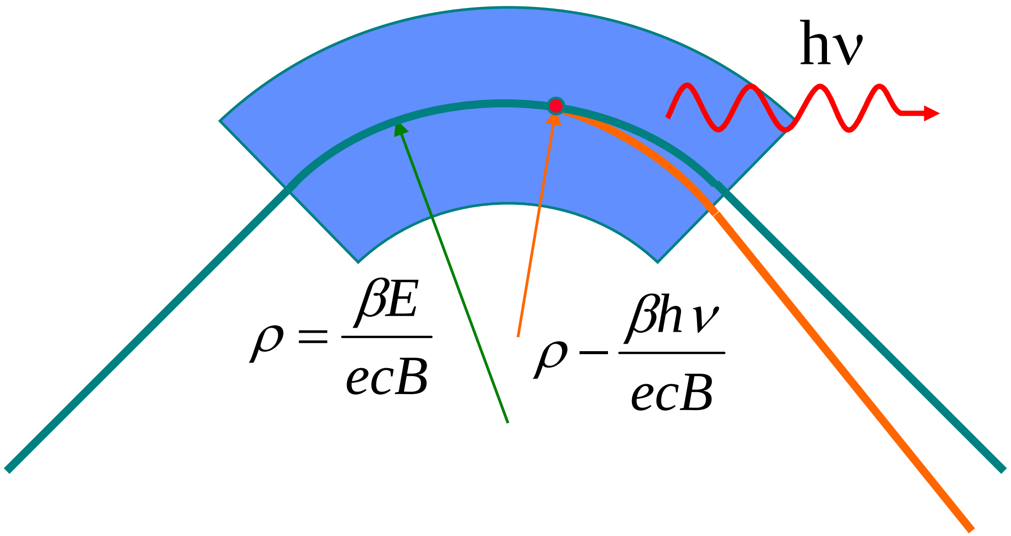
Quantum excitation

'Quantum excitation' in accelerator physics refers to diffusion of phase space (momentum) of e^- due to recoil from emitted photons.

Because radiated power scales as $\propto \gamma^4$ and critical photon energy (divides synchrotron radiation spectral power into two equal halves) as $\propto \gamma^3$, the effect becomes important at high energies (typically ≥ 3 GeV).

Here we consider spontaneous synchrotron radiation ($\lambda << \sigma_z$, so that the radiation power scales linearly with the number of electrons). When radiation wavelength becomes comparable with the bunch length (or density modulation size), radiated power becomes quadratic with peak current. This coherent synchrotron radiation (CSR) effects can be important at all energies when bunch length becomes is sufficiently short.

Recoil due to photon emission



Photon emission takes place in forward direction within a very small cone (~ $1/\gamma$ opening angle). Therefore, to 1st order, photon removes momentum in the direction of propagation of electron, leaving position and divergence of the electron intact at the point of emission.

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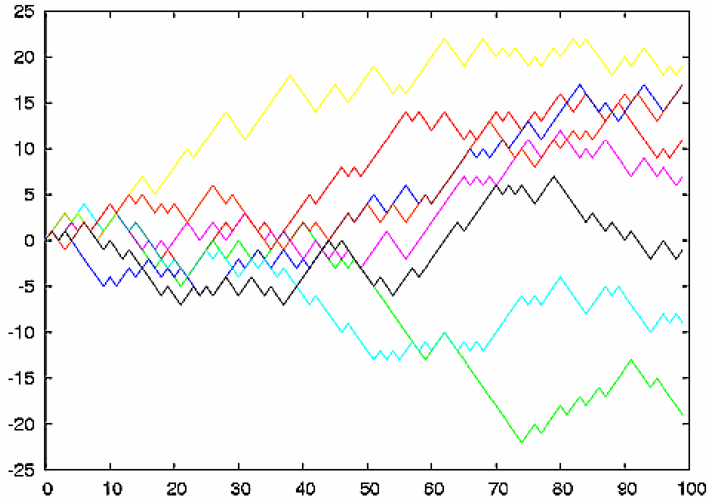
Energy spread

Synchrotron radiation is a stochastic process. Probability distribution of the number of photons emitted by a single electron is described by Poisson distribution, and by Gaussian distribution in the approximation of large number of photons.

If emitting (on average) N_{ph} photons with energy E_{ph}, random walk growth of energy spread from its mean is

$$\sigma_E^2 = N_{ph} E_{ph}^2$$

random walk



If photons are emitted with spectral distribution $N_{ph}(E_{ph})$, then one has to integrate:

$$\sigma_E^2 = \int E_{ph}^2 N(E_{ph}) dE_{ph}$$

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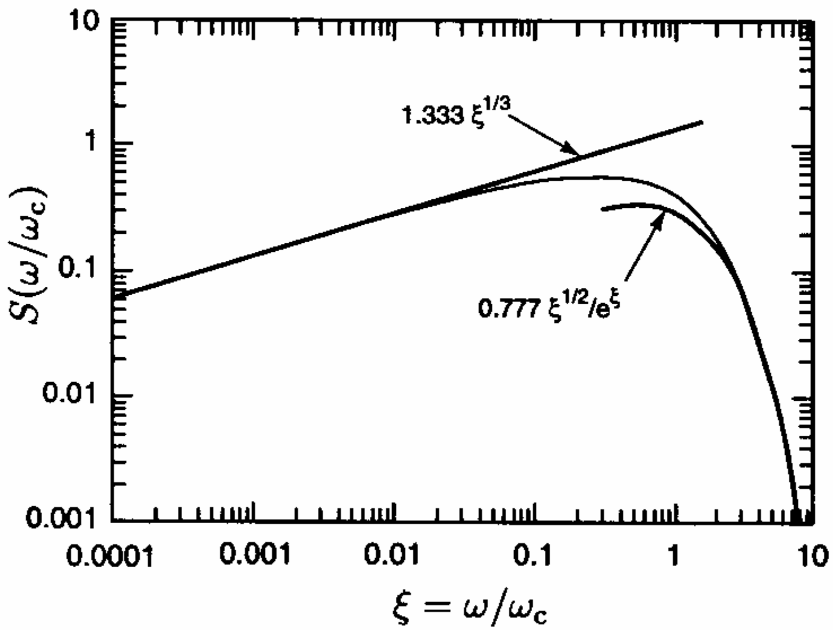
Spectrum of synchrotron radiation from bends

Photon emission (primarily) takes place in deflecting magnetic field (dipole bend magnets, undulators and wigglers). Spectrum of synchrotron radiation from bends is well known (per unit deflecting angle):

$$\frac{dN_{ph}}{d\psi} = \frac{4\alpha}{9}\gamma\frac{\Delta\omega}{\omega}\frac{I}{e}S\left(\frac{\omega}{\omega_c}\right)$$

$$\omega_c \equiv \frac{3}{2}c\gamma^3/\rho$$



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Energy spread from bends

$$\sigma_E^2 = \frac{55}{32\sqrt{3}\pi} C_\gamma \hbar c (mc^2)^4 \gamma^7 \int \frac{ds}{\rho}$$

Sands' radiation constant for e⁻: $C_\gamma = \frac{4\pi r_c}{3(mc^2)^3} = 8.86 \cdot 10^{-5} \frac{\text{m}}{\text{GeV}^3}$

For constant bending radius ρ and total bend angle Θ (Θ = 2π for a ring) energy spread becomes:

$$\boxed{\frac{\sigma_E^2}{E^2} = 2.6 \cdot 10^{-10} E^5 (\text{GeV}^5) \frac{1}{\rho^2 (\text{m}^2)} \frac{\Theta}{2\pi}}$$

Radiated energy loss:

$$E_\gamma = C_\gamma \frac{E^4}{\rho} \frac{\Theta}{2\pi} \qquad E_\gamma (\text{MeV}) = 0.0886 \frac{E^4 (\text{GeV}^4)}{\rho (\text{m})} \frac{\Theta}{2\pi}$$

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Energy spread from planar undulator

$$\sigma_E^2 = \int E_{ph}^2 N(E_{ph}) dE_{ph} \approx N_{ph}\varepsilon_\gamma^2$$

Photon in fundamental $\varepsilon_\gamma = \dfrac{hc}{\lambda_p}\dfrac{2\gamma^2}{(1+\frac{1}{2}K^2)}$ $\quad \varepsilon_\gamma(\text{eV}) = 950\dfrac{E^2(\text{GeV}^2)}{\lambda_p(\text{cm})(1+\frac{1}{2}K^2)}$

Radiated energy / e⁻ $E_\gamma = \dfrac{4\pi^2 r_c E^2 K^2 L_u}{3\lambda_p^2 mc^2}$ $\quad E_\gamma(\text{eV}) = 725\dfrac{E^2(\text{GeV}^2)K^2}{\lambda_p^2(\text{cm}^2)}L_u(\text{m})$

Naively, one can estimate $N_{ph} \approx \dfrac{E_\gamma}{\varepsilon_\gamma} = 0.763\dfrac{K^2(1+\frac{1}{2}K^2)}{\lambda_p(\text{cm})}L_u(\text{m})$

$$\frac{\sigma_E^2}{E^2} \approx 7\cdot 10^{-13}\frac{E^2(\text{GeV}^2)K^2}{\lambda_p^3(\text{cm}^3)(1+\frac{1}{2}K^2)}L_u(\text{m})$$

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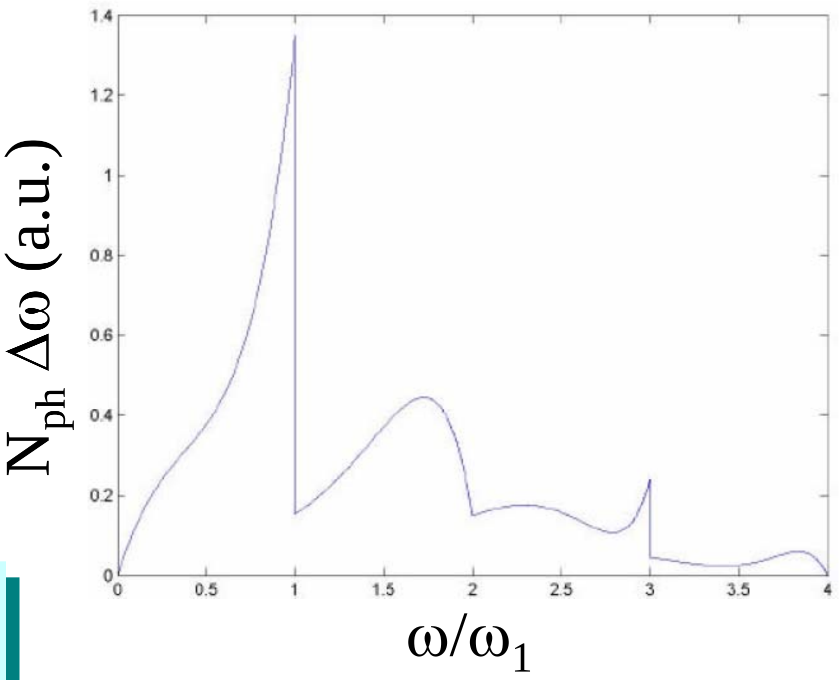
Energy spread from planar undulator (contd.)

In reality, undulator spectrum is more complicated with harmonic content for K ≥ 1 and Doppler red shift for off-axis emission.

More rigorous treatment gives

$$\frac{\sigma_E^2}{E^2} \approx 4.8 \cdot 10^{-13} \frac{E^2(\mathrm{GeV}^2) K^2 F(K)}{\lambda_p^3(\mathrm{cm}^3)} L_u(\mathrm{m})$$

with $F(K) \approx 1.2K + (1 + 1.33K + 0.4K^2)^{-1}$



Emittance growth

Consider motion:

$$x = x_\beta + \eta_x \frac{\Delta E}{E} \qquad x' = x'_\beta + \eta'_x \frac{\Delta E}{E}$$

where $x_\beta = a_x \sqrt{\beta_x(s)} e^{i\psi_x(s)}$

As discussed earlier, emission of a photon leads to:

$$\delta x = 0 = \delta x_\beta + \eta_x \frac{E_{ph}}{E} \qquad \delta x_\beta = -\eta_x \frac{E_{ph}}{E}$$

$$\delta x' = 0 = \delta x'_\beta + \eta'_x \frac{E_{ph}}{E} \qquad \delta x'_\beta = -\eta'_x \frac{E_{ph}}{E}$$

changing the phase space ellipse $a_x^2 = \gamma_x x_\beta^2 + 2\alpha_x x_\beta x'_\beta + \beta_x x'^2_\beta$

$$\boxed{\left\langle \delta a_x^2 \right\rangle = \frac{E_{ph}^2}{E^2} H_x(s)}$$

here $H_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta'^2_x$

Emittance growth in bend

$$\sigma_x^2 = \left\langle \left(\sqrt{\beta_x(s)} e^{i\psi_x(s)} \right)^2 \right\rangle_s = \frac{1}{2} a_x^2 \beta_x$$

$$\boxed{\varepsilon_x = \frac{1}{2} \Delta \left\langle a_x^2 \right\rangle = \frac{1}{2cE^2} \int ds \int E_{ph}^2 \dot{N}_{ph}(E_{ph}) H(s) dE_{ph}}$$

In bends:

$$\varepsilon_x = \frac{55 C_\gamma \hbar c (mc^2)^2}{64 \pi \sqrt{3}} \gamma^5 \int \frac{H ds}{\rho^3}$$

$$\varepsilon_x(\mathrm{m\text{-}rad}) = 1.3 \cdot 10^{-10} \frac{E^5(\mathrm{GeV}^5) \langle H \rangle (\mathrm{m})}{\rho^2 (\mathrm{m}^2)} \frac{\Theta}{2\pi}$$

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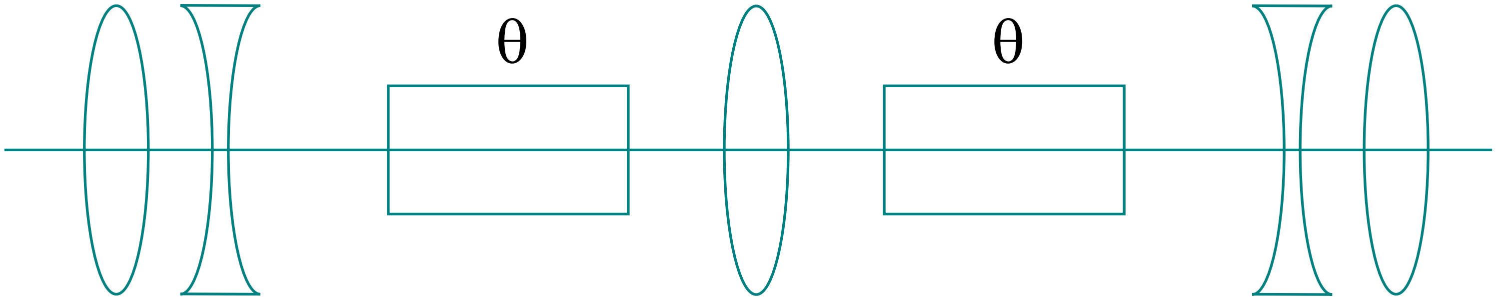
H-function

As we have seen, lattice function H in dipoles ($1/\rho \neq 0$) matters for low emittance.

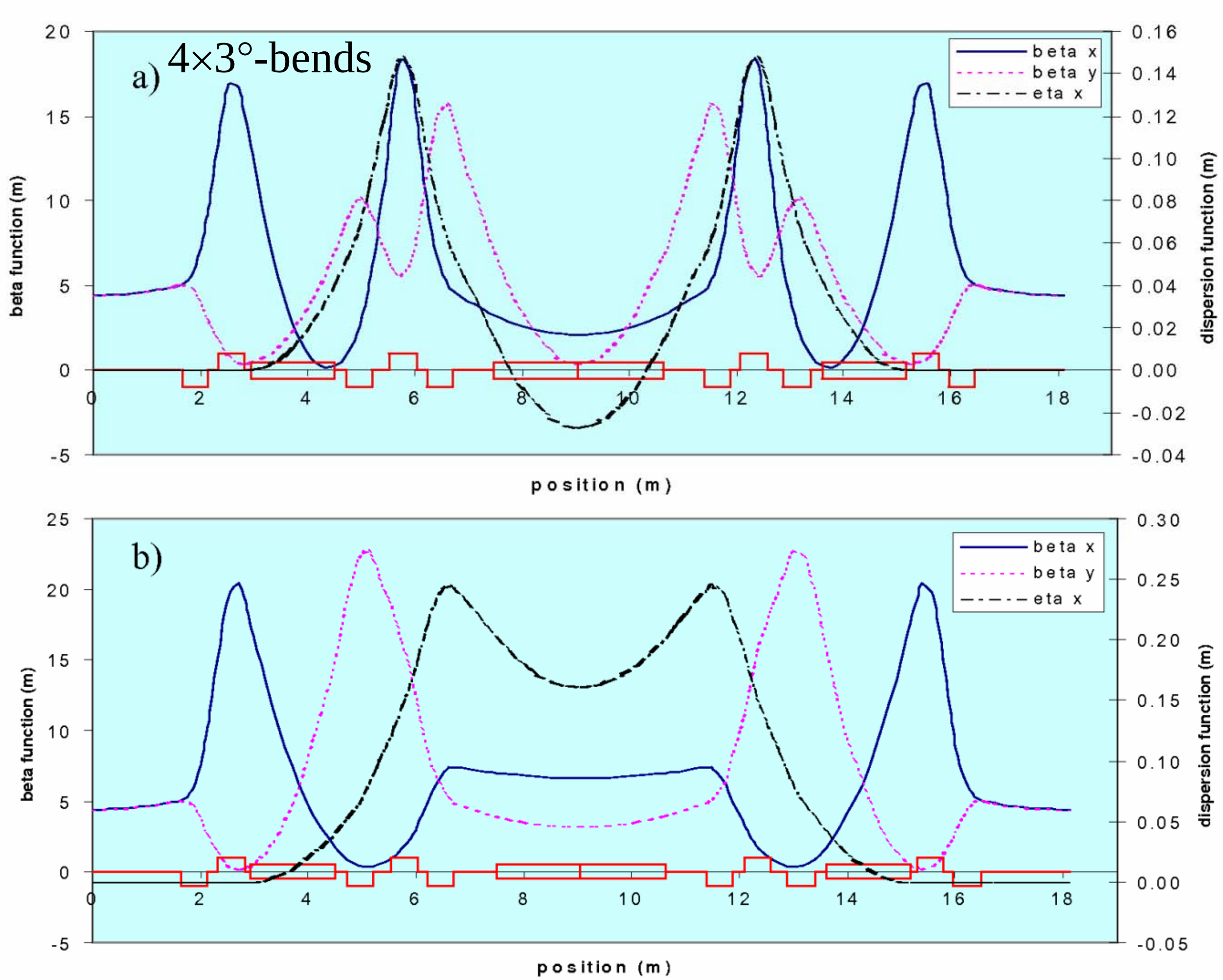
In the simplest achromatic cell (two identical dipole magnets with lens in between), dispersion is defined in the bends. One can show that an optimum Twiss parameters (α, β) exist that minimize $\langle H \rangle$



Such optimized double bend achromat is known as a Chasman Green lattice, and H is given by

$$\langle H \rangle = \frac{1}{4\sqrt{15}} \rho \theta^3$$

Example of triple bend achromat



$$\langle H \rangle = 3.6\,\mathrm{mm}$$

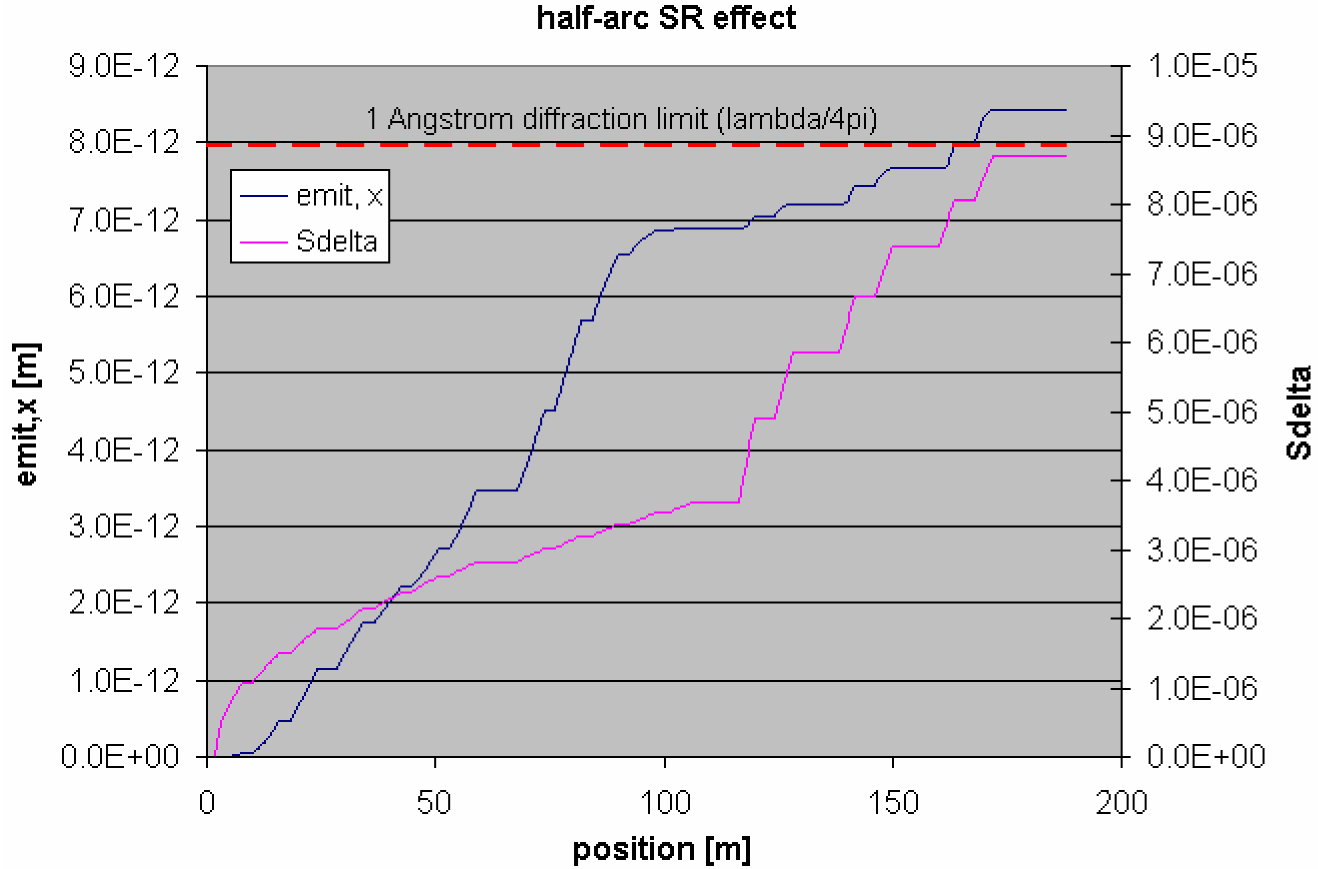
$$\langle H \rangle = 9.1\,\mathrm{mm}$$

A TBA cell for an ERL recirculating arc: a) $R_{56} = 0$; b) $R_{56} = 2\,\mathrm{cm}$

$$\langle H \rangle_{C-G} \approx 0.66\,\mathrm{mm}$$

Emittance and energy spread in ¼ CESR

energy = 5 GeV



large dispersion section for bunch compression

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Emittance growth in undulator

$$\varepsilon_x \approx \frac{1}{2}\frac{\sigma_E^2}{E^2}\langle H \rangle$$

with energy spread σ_E/E calculated earlier.

$$\langle H \rangle = \frac{1}{L_u}\int_0^{L_u} (\beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2)ds$$

For sinusoidal undulator field $B(s) = B_0 \cos k_p s$ with $k_p = 2\pi / \lambda_p$

Differential equation for dispersion $\eta'' = \frac{1}{\rho} = \frac{1}{\rho_0}\cos k_p s$

with $\rho_0 = \frac{\gamma}{k_p K}$

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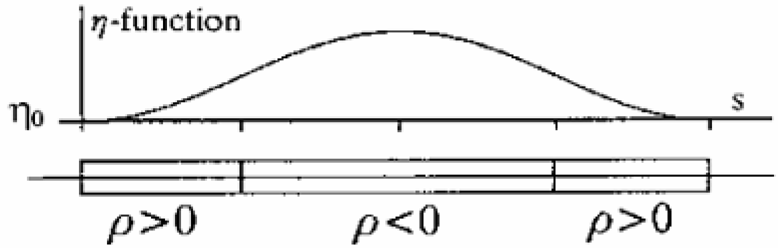
Emittance growth in undulator (contd.)



Dispersion function in one period of a undulator magnet.

$$\eta(s) = \frac{1}{k_p^2 \rho_0}(1-\cos k_p s) + \eta_0 \qquad \eta'(s) = \frac{1}{k_p \rho_0} \sin k_p s$$

For an undulator with beam waist (β^*) located at its center

$$\langle H \rangle \approx \frac{\beta^*}{2k_p^2\rho_0^2}\left(1 + \frac{L_u^2}{12\beta^{*2}} + \frac{2\eta_0^2 k_p^2 \rho_0^2}{\beta^{*2}} + \frac{8\eta_0\rho_0}{\beta^{*2}} + \frac{11}{2\beta^{*2}k_p^2}\right)$$

$$\approx \frac{\beta^* K^2}{2\gamma^2}\left(1 + \frac{L_u^2}{12\beta^{*2}} + \frac{2\eta_0^2\gamma^2}{\beta^{*2}K^2} + \frac{8\gamma\eta_0}{k_p K \beta^{*2}}\right)$$

Unless undulator is placed in high dispersion region, contribution to emittance remains small.