Physics 704/804 Electromagnetic Theory II

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Cavity Losses

$$Q = \omega_0 \frac{\text{Stored energy}}{\text{Power loss}} \qquad \text{both proportional to } |\psi|^2$$

$$\frac{dU}{dt} = -\frac{\omega_0}{Q} U$$

$$U(t) = U_0 e^{-\omega_0 t/Q}$$

$$E(t) = E_0 e^{-\omega_0 t/2Q} e^{-i(\omega_0 + \Delta\omega)t}$$

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(\omega) e^{-i\omega t} dt$$

$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} \Theta(t) E_0 e^{-\omega_0 t/2Q} e^{-i(\omega_0 + \Delta\omega)t} e^{i\omega t} dt$$

Lorentzian Line Shape

$$= \int_0^\infty E_0 e^{-\omega_0 t/2Q} e^{-i(\omega_0+\Delta\omega)t} e^{i\omega t} dt$$

$$\left|\tilde{E}(\omega)\right|^2 \propto \frac{1}{(\omega-\omega_0-\Delta\omega)^2+(\omega_0/2Q)^2}$$

$$\mathrm{FWHM} = \Gamma = \omega_0 / Q$$

$$U = \frac{d}{4}\begin{Bmatrix}\varepsilon \\ \mu\end{Bmatrix}\left[1+\left(\frac{p\pi}{\gamma_\lambda d}\right)\right]\int_A |\psi|^2 da$$

$$P_{loss} = \frac{1}{2\sigma\delta}\left[\oint_C dl \int_0^d dz \left|\vec{n}\times H\right|^2_{sides} + 2\int_A da \left|\vec{n}\times H\right|^2_{ends}\right]$$

Q Calculation

For TM

$$P_{loss} = \frac{\varepsilon}{\sigma\delta\mu}\left[1+\left(\frac{p\pi}{\gamma_\lambda d}\right)\right]\left(1+\xi_\lambda\frac{Cd}{4A}\right)\int_A |\psi|^2 da$$

$$Q = \frac{\mu}{\mu_c}\frac{d}{\delta}\frac{1}{2\left(1+\xi_\lambda\frac{Cd}{4A}\right)} = \frac{\mu}{\mu_c}\left(\frac{V}{S\delta}\right)\times(\text{Geometrical factor})$$

For right circular $\text{TE}_{1,1,1}$

$$GF = \left(1+\frac{d}{R}\right)\frac{\left(1+0.343\frac{d^2}{R^2}\right)}{\left(1+0.209\frac{d}{R}+0.244\frac{d^3}{R^3}\right)} \qquad \text{Max at } \frac{d}{R}=1.91$$

Schumann Resonances

$$\frac{\omega^2}{c^2}\vec{B} - \vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = 0$$

$$B_\phi\left(r,\theta\right) = \frac{u_l\left(r\right)}{r}P_l^1\left(\cos\theta\right)$$

Spherical Bessel Functions

$$\frac{d^2u_l\left(r\right)}{dr^2} + \left[\frac{\omega^2}{c^2} - \frac{l\left(l+1\right)}{r^2}\right]u_l\left(r\right) = 0$$

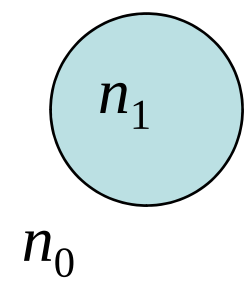
$$\frac{du_l}{dr} = 0 \qquad \text{at boundaries}$$

$$u_l \approx A\cos\left(q\left(r-a\right)\right) \qquad q = n\pi / h$$

$$\omega_l = \sqrt{l\left(l+1\right)}\frac{c}{a} \qquad n = 0 \text{ sequence}$$

Optical Fibers

$$\Delta = \frac{n_1^2 - n_0^2}{2n_1^2} \approx 1 - \frac{n_0}{n_1} \qquad \theta_{\max} = \cos^{-1}\left(n_0 / n_1\right)$$



$$dN = \pi a^2 \frac{d^2 k}{\left(2\pi\right)^2} 2$$

$$N \approx a^2 k^2 \int_0^{\theta_{\max}} \theta \, d\theta = \frac{1}{2}\left(ka\sqrt{2\Delta}\right)^2 = \frac{1}{2} V^2$$

fibre parameter $\quad V = ka\sqrt{2\Delta}$

$\lambda = 0.85\text{mm},\ n_1 = 1.4,\ a = 25$ micron

$N = 335,\quad$ many modes excited