Physics 704/804 Electromagnetic Theory II

G. A. Krafft

Jefferson Lab

Jefferson Lab Professor of Physics

Old Dominion University

Energy Flow

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*$$

$$\vec{S} = \frac{\omega k}{2\gamma^4}\begin{cases}\varepsilon\left[\hat{z}\left|\vec{\nabla}_t\psi\right|^2 + i\dfrac{\gamma^2}{k}\psi\vec{\nabla}_t\psi^*\right]\\[2ex] \mu\left[\hat{z}\left|\vec{\nabla}_t\psi\right|^2 - i\dfrac{\gamma^2}{k}\psi^*\vec{\nabla}_t\psi\right]\end{cases}$$

$$P = \int_A \vec{S}\cdot\hat{z}\,da = \frac{\omega k}{2\gamma^4}\begin{Bmatrix}\varepsilon\\ \mu\end{Bmatrix}\int_A \vec{\nabla}_t\psi^*\cdot\vec{\nabla}_t\psi\,da$$

$$P = \frac{\omega k}{2\gamma^4}\begin{Bmatrix}\varepsilon\\ \mu\end{Bmatrix}\left[\oint_C \psi^*\frac{\partial\psi}{\partial n}dl + \int_A \psi^*\cdot\nabla_t^2\psi\,da\right]$$

Energy and Group Velocity

$$P = \frac{1}{2\sqrt{\mu\varepsilon}}\left(\frac{\omega}{\omega_\lambda}\right)^2\left(1-\frac{\omega_\lambda^2}{\omega^2}\right)^{1/2}\begin{Bmatrix}\varepsilon\\ \mu\end{Bmatrix}\int_A \psi^*\psi\, da$$

$$U = \frac{1}{2}\left(\frac{\omega}{\omega_\lambda}\right)^2\begin{Bmatrix}\varepsilon\\ \mu\end{Bmatrix}\int_A \psi^*\psi\, da$$

$$\frac{P}{U} = \frac{k}{\omega}\frac{1}{\mu\varepsilon} = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{1-\frac{\omega_\lambda^2}{\omega^2}} = \mathrm{v}_g$$

$$\mathrm{v}_p\mathrm{v}_g = \frac{1}{\mu\varepsilon}$$

Attenuation

Field attenuation constant given by β_λ

$$P(z) = P_0 e^{-2\beta_\lambda z}$$

$$\beta_\lambda = -\frac{1}{2P}\frac{dP}{dz}$$

$$\frac{dP}{dz} = -\frac{1}{2\sigma\delta}\oint_C \left|\hat{n}\times\vec{H}\right|^2 dl$$

$$= -\frac{1}{2\sigma\delta}\left(\frac{\omega}{\omega_\lambda}\right)^2 \oint_C \left\{ \begin{array}{c} \dfrac{1}{\mu^2\omega_\lambda^2}\left|\dfrac{\partial\psi}{\partial n}\right|^2 \\ \dfrac{1}{\mu\varepsilon\omega_\lambda^2}\left(1-\dfrac{\omega_\lambda^2}{\omega^2}\right)\left|\hat{n}\times\vec{\nabla}_t\psi\right|^2 + \dfrac{\omega_\lambda^2}{\omega^2}\left|\psi\right|^2 \end{array} \right\} dl$$

$$\left\langle \left| \frac{\partial \psi}{\partial n} \right|^2 \right\rangle \sim \left\langle \left| \hat{n} \times \vec{\nabla}_t \psi \right|^2 \right\rangle \sim \mu \varepsilon \omega_\lambda^2 \left\langle \left| \psi \right|^2 \right\rangle$$

$$\oint_C \frac{1}{\omega_\lambda^2} \left| \frac{\partial \psi}{\partial n} \right|^2 dl = \varsigma_\lambda \mu \varepsilon \frac{C}{A} \int_A \left| \psi \right|^2 da$$

$$\beta_\lambda = \sqrt{\frac{\varepsilon}{\mu}} \frac{1}{\sigma \delta_\lambda} \left(\frac{C}{2A} \right) \frac{\left(\omega / \omega_\lambda \right)^{1/2}}{\left(1 - \omega^2 / \omega_\lambda^2 \right)^{1/2}} \left[\varsigma_\lambda + \eta_\lambda \left(\frac{\omega_\lambda}{\omega} \right)^2 \right]$$

Cavity Losses

$$Q = \omega_0 \frac{\text{Stored energy}}{\text{Power loss}}$$ both proportional to $|\psi|^2$

$$\frac{dU}{dt} = -\frac{\omega_0}{Q} U$$

$$U(t) = U_0 e^{-\omega_0 t/Q}$$

$$E(t) = E_0 e^{-\omega_0 t/2Q} e^{-i(\omega_0 + \Delta\omega)t}$$