Physics 704/804 Electromagnetic Theory II

G. A. Krafft

Jefferson Lab

Jefferson Lab Professor of Physics

Old Dominion University

"Transverse" Separation

$$\vec{E} = E_z \hat{z} + \vec{E}_t \qquad \vec{E}_t = \vec{E} - \left(\vec{E}\cdot\hat{z}\right)\hat{z}$$

$$\vec{B} = B_z \hat{z} + \vec{B}_t \qquad \vec{B}_t = \vec{B} - \left(\vec{B}\cdot\hat{z}\right)\hat{z}$$

$$\left[\nabla_t^2 + \left(\mu\varepsilon\omega^2\right) - k^2\right]\begin{cases}\vec{E} \\ \vec{B}\end{cases} = 0$$

$$\frac{\partial \vec{E}_t}{\partial z} + i\omega\hat{z}\times\vec{B}_t = \vec{\nabla}_t E_z \qquad \hat{z}\cdot\left(\vec{\nabla}_t\times\vec{E}_t\right) = i\omega B_z$$

$$\frac{\partial \vec{B}_t}{\partial z} - i\mu\varepsilon\omega\hat{z}\times\vec{E}_t = \vec{\nabla}_t B_z \qquad \hat{z}\cdot\left(\vec{\nabla}_t\times\vec{B}_t\right) = -i\mu\varepsilon\omega E_z$$

$$\vec{\nabla}_t\cdot\vec{E}_t = -\frac{\partial E_z}{\partial z} \qquad \vec{\nabla}_t\cdot\vec{B}_t = -\frac{\partial B_z}{\partial z}$$

More General Case

Transverse field expressible in terms of *z*-field only!

$$\vec{E}_t = \frac{i}{\mu\varepsilon\omega^2 - k^2}\left[k\vec{\nabla}_t E_z - \omega\hat{z}\times\vec{\nabla}_t B_z\right]$$

$$\vec{B}_t = \frac{i}{\mu\varepsilon\omega^2 - k^2}\left[k\vec{\nabla}_t B_z + \mu\varepsilon\omega\hat{z}\times\vec{\nabla}_t E_z\right]$$

Transverse Magnetic (TM)

$$B_z = 0; \qquad \text{Boundary Condition } \left.E_z\right|_S = 0$$

Transverse Electric (TE)

$$E_z = 0; \qquad \text{Boundary Condition } \left.\frac{\partial B_z}{\partial n}\right|_S = 0$$

Waveguides

$$\vec{H}_t = \frac{\pm 1}{Z} \hat{z} \times \vec{E}_t$$

Wave Impedance

$$Z = \begin{cases} \dfrac{k}{\varepsilon\omega} = \dfrac{k}{k_0}\sqrt{\dfrac{\mu}{\varepsilon}} & \text{(TM)} \\[2ex] \dfrac{\mu\omega}{k} = \dfrac{k_0}{k}\sqrt{\dfrac{\mu}{\varepsilon}} & \text{(TE)} \end{cases}$$

Eigenvalue Problem

TM Waves

$$\vec{E}_t = \pm \frac{ik}{\gamma^2} \vec{\nabla}_t \psi$$

TE Waves

$$\vec{H}_t = \pm \frac{ik}{\gamma^2} \vec{\nabla}_t \psi$$

Transverse Helmholz Equation

$$\left(\nabla_t^2 + \gamma^2\right)\psi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2\right)\psi = 0$$

Boundary Conditions

$$\psi\Big|_S = 0 \qquad (\mathrm{TM}) \qquad \left.\frac{\partial \psi}{\partial n}\right|_S = 0 \qquad (\mathrm{TE})$$

Spectrum of eigenvalues and eigenfunctions

$$\gamma_\lambda \qquad \psi_\lambda(x, y)$$

Wavelength in mode λ

$$k_\lambda^2 = \mu\varepsilon\omega^2 - \gamma_\lambda^2$$

Cutoff Frequency

$$\omega_\lambda^2 = \frac{\gamma_\lambda^2}{\mu\varepsilon}$$

No propagation at frequencies below cutoff

$$k_\lambda^2 = \mu\varepsilon\left(\omega^2 - \omega_\lambda^2\right)$$

For single-mode propagation choose frequency to be above cutoff for lowest mode and below cutoff for all other modes.
Phase velocity infinite at cutoff!