Physics 704/804 Electromagnetic Theory II

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Wave Guides: Surface Absorbtion

Homogeneous Maxwell Equations imply

$$\vec{n} \cdot \left(\vec{B} - \vec{B}_c \right) = 0$$

$$\vec{n} \times \left(\vec{E} - \vec{E}_c \right) = 0$$

Curl *H* Maxwell Equation and large but finite conductivity implies

$$\vec{n} \times \left(\vec{H} - \vec{H}_c \right) = 0$$

$$\vec{E}_c = \frac{1}{\sigma} \vec{\nabla} \times \vec{H}_c$$

$$\vec{H}_c = -\frac{i}{\mu_c \omega} \vec{\nabla} \times \vec{E}_c$$

$$\left[\frac{\partial^2}{\partial\xi^2}+\frac{2i}{\delta^2}\right]\left(\vec{n}\times\vec{H}_c\right)=0$$

$$\vec{n}\cdot\vec{H}_c=0$$

$$\delta=\left(\frac{2}{\mu_c\omega\sigma}\right)^{1/2}$$

$$\vec{H}_c=\vec{H}_{par}e^{-\xi/\delta}e^{i\xi/\delta}$$

$\vec{H}_{par}$ is the field at the surface

$$\vec{E}_c\left(\xi=0\right)=\sqrt{\frac{\omega\mu_c}{2\sigma}}\left(1-i\right)\left(\vec{n}\times\vec{H}_{par}\right)$$

$$\frac{dP}{da}=\frac{1}{2}\mathrm{Re}\left[\vec{n}\cdot\vec{E}\times\vec{H}^*\right]=\frac{\mu_c\omega\delta}{4}\left|\vec{H}_{par}\right|^2$$

Alternative Calculation

$$\vec{J} = \sigma \vec{E}_c = \frac{1}{\delta}(1-i)\left(\hat{n}\times\vec{H}_{par}\right)e^{-\xi(1-i)/\delta}$$

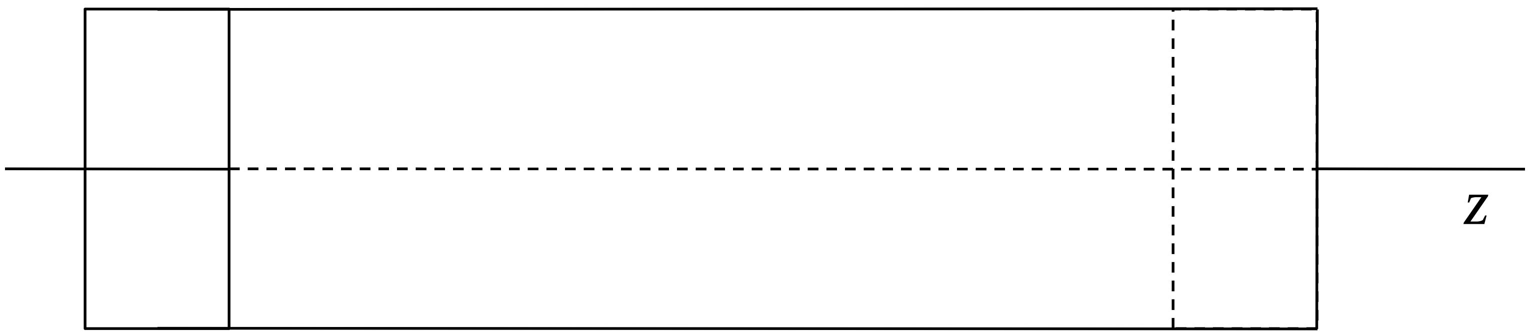
$$\text{power/vol} = \frac{1}{2}\vec{J}\cdot\vec{E}^* = \frac{1}{2\sigma}\left|\vec{J}\right|^2$$

$$K_{eff} = \int_0^\infty J d\xi = \hat{n}\times\vec{H}_{par}$$

surface current for perfect conductor

$$\frac{dP}{da} = \frac{1}{2\sigma\delta}\left|K_{eff}\right|^2 = \frac{\mu_c\omega\delta}{4}\left|K_{eff}\right|^2$$

Cylindrical Systems



z-axis along the cylinder direction

$$\vec{\nabla} \times \vec{E} = i\omega \vec{B} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = -i\mu\varepsilon \vec{E} \qquad \vec{\nabla} \cdot \vec{E} = 0$$

$$\left(\nabla^2 + \mu\varepsilon\omega^2\right)\begin{cases} \vec{E} \\ \vec{B} \end{cases} = 0$$

$$\vec{B} = \vec{B}\left(x, y\right) e^{\pm ikz - \omega t} \qquad \vec{E} = \vec{E}\left(x, y\right) e^{\pm ikz - \omega t}$$

"Transverse" Separation

$$\vec{E} = E_z \hat{z} + \vec{E}_t \qquad \vec{E}_t = \vec{E} - \left(\vec{E}\cdot\hat{z}\right)\hat{z}$$

$$\vec{B} = B_z \hat{z} + \vec{B}_t \qquad \vec{B}_t = \vec{B} - \left(\vec{B}\cdot\hat{z}\right)\hat{z}$$

$$\left[\nabla_t^2 + \left(\mu\varepsilon\omega^2\right) - k^2\right]\begin{cases}\vec{E}\\ \vec{B}\end{cases} = 0$$

$$\frac{\partial \vec{E}_t}{\partial z} + i\omega\hat{z}\times\vec{B}_t = \vec{\nabla}_t E_z \qquad \hat{z}\cdot\left(\vec{\nabla}_t\times\vec{E}_t\right) = i\omega B_z$$

$$\frac{\partial \vec{B}_t}{\partial z} - i\mu\varepsilon\omega\hat{z}\times\vec{E}_t = \vec{\nabla}_t B_z \qquad \hat{z}\cdot\left(\vec{\nabla}_t\times\vec{B}_t\right) = -i\mu\varepsilon\omega E_z$$

$$\vec{\nabla}_t\cdot\vec{E}_t = -\frac{\partial E_z}{\partial z} \qquad \vec{\nabla}_t\cdot\vec{B}_t = -\frac{\partial B_z}{\partial z}$$

TEM Modes

Solutions with transverse field only

$$\vec{E}_{tem} = \vec{E}_{t,tem} \qquad \vec{B}_{tem} = \vec{B}_{t,tem}$$

$$\vec{\nabla}_t \times \vec{E}_{t,tem} = 0 \qquad \vec{\nabla}_t \cdot \vec{E}_{t,tem} = 0$$

$\vec{E}_{tem}$ must solve 2-D *electrostatic* problem

$$k = k_0 = \sqrt{\mu\varepsilon}\,\omega$$

$$\vec{B}_{tem} = \pm\sqrt{\mu\varepsilon}\,\hat{z} \times \vec{E}_{tem}$$

Can there be a solution inside a single closed waveguide?
No, need at least two conductors. No cutoff frequency

More General Case

Transverse field expressible in terms of *z*-field only!

$$\vec{E}_t = \frac{i}{\mu\varepsilon\omega^2 - k^2}\left[k\vec{\nabla}_t E_z - \omega\hat{z}\times\vec{\nabla}_t B_z\right]$$

$$\vec{B}_t = \frac{i}{\mu\varepsilon\omega^2 - k^2}\left[k\vec{\nabla}_t B_z + \mu\varepsilon\omega\hat{z}\times\vec{\nabla}_t E_z\right]$$

Transverse Magnetic (TM)

$$B_z = 0; \qquad \text{Boundary Condition } \left.E_z\right|_S = 0$$

Transverse Electric (TE)

$$E_z = 0; \qquad \text{Boundary Condition } \left.\frac{\partial B_z}{\partial n}\right|_S = 0$$