Physics 704/804 Electromagnetic Theory II

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Dispersion

$$u(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}A(k)e^{ikx-\omega(k)t}dk$$

Have exact calculation for modulated Gaussian function

$$u(x,0)=\exp\left(-x^2/2L^2\right)e^{ik_0x}$$

$$A(k)=\int_{-\infty}^{\infty}u(x,0)e^{-ikx}dx$$

$$=\sqrt{2\pi}L\exp\left(-L^2/2\right)\left(k-k_0\right)^2$$

$$\omega(k)=\nu\left(1+\frac{a^2k^2}{2}\right)$$

Pulse Spreading, or Dispersion

$$\mathrm{v}_g = \frac{d\omega}{dk} = \nu a^2 k_0$$

$$u(x,t) = \frac{L}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\left(-L^2/2\right)\left(k-k_0\right)^2} e^{ikx - i\nu t\left[1+\left(a^2 k^2/2\right)\right]} dk$$

$$= \frac{\exp\left[-\dfrac{\left(x-\nu a^2 k_0 t\right)^2}{2L^2\left(1+\dfrac{ia^2\nu t}{L^2}\right)}\right]}{\left(1+\dfrac{ia^2\nu t}{L^2}\right)^{1/2}} \exp\left[ik_0 x - i\nu\left(1+a^2k^2/2\right)t\right]$$

$$L(t) = \frac{d\omega}{dk} = \sqrt{L^2 + \left(\nu a^2 t / L\right)^2}$$

$$\Delta \mathrm{v}_g = \frac{d^2\omega}{dk^2}\Delta k = \frac{\nu a^2}{L}$$

$$\Delta x(t) = \sqrt{\left(\Delta x\right)^2 + \left(\Delta \mathrm{v}_g t\right)^2}$$

Kramers-Kronig Relations

$$\varepsilon(\omega)/\varepsilon_0 - 1 = \int_0^\infty G(\tau)e^{i\omega\tau}\,d\tau$$

Is automatically causal for a wide variety of choices for *G*. Analyticity in UH-ωP implies a relationship between real and imaginary part of the permitivity. Cauchy's theorem for *z* inside a closed curve *C*

$$\varepsilon(z)/\varepsilon_0 = 1 + \frac{1}{2\pi i}\oint_C \frac{\varepsilon(\omega')/\varepsilon_0 - 1}{\omega' - z}\,d\omega'$$

$$= 1 + \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\varepsilon(\omega')/\varepsilon_0 - 1}{\omega' - z}\,d\omega'$$

where the integral is now along the real axis

$$\frac{1}{\omega-\omega'-i\delta} = P\left(\frac{1}{\omega-\omega'}\right) + \pi i \delta\left(\omega'-\omega\right)$$

$$\operatorname{Re}\varepsilon\left(\omega\right)/\varepsilon_0 = 1 + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\operatorname{Im}\varepsilon\left(\omega'\right)/\varepsilon_0}{\omega'-z} d\omega'$$

$$\operatorname{Im}\varepsilon\left(\omega\right)/\varepsilon_0 = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\operatorname{Re}\varepsilon\left(\omega'\right)/\varepsilon_0 - 1}{\omega'-z} d\omega'$$

$$\operatorname{Re}\varepsilon\left(\omega\right)/\varepsilon_0 = 1 + \frac{2}{\pi} P \int_{0}^{\infty} \frac{\omega' \operatorname{Im}\varepsilon\left(\omega'\right)/\varepsilon_0}{\omega'^2-\omega^2} d\omega'$$

$$\operatorname{Im}\varepsilon\left(\omega\right)/\varepsilon_0 = -\frac{2\omega}{\pi} P \int_{0}^{\infty} \frac{\operatorname{Re}\varepsilon\left(\omega'\right)/\varepsilon_0 - 1}{\omega'^2-\omega^2} d\omega'$$

$$\varepsilon\left(-\omega\right) = \varepsilon^*\left(\omega^*\right)$$

Sum Rules

Sum Rules for oscillator strengths

$$\omega_p^2 = 1 + \frac{2}{\pi} P \int_0^\infty \omega \,\mathrm{Im}\,\varepsilon(\omega)/\varepsilon_0 \, d\omega$$

Second Sum Rule

$$\frac{1}{N}\int_0^N \mathrm{Re}\,\varepsilon(\omega)/\varepsilon_0 \, d\omega = 1 + \frac{\omega_p^2}{N^2}$$

Wave Guides: Surface Absorbtion

$$\vec{n}\cdot\left(\vec{B}-\vec{B}_c\right)=0$$

$$\vec{n}\times\left(\vec{E}-\vec{E}_c\right)=0$$

$$\vec{n}\times\left(\vec{H}-\vec{H}_c\right)=0$$

$$\vec{E}_c=\frac{1}{\sigma}\vec{\nabla}\times\vec{H}_c$$

$$\vec{H}_c=-\frac{i}{\mu_c\omega}\vec{\nabla}\times\vec{E}_c$$

$$\left[\frac{\partial^2}{\partial\xi^2}+\frac{2i}{\delta^2}\right]\left(\vec{n}\times\vec{H}_c\right)=0$$

$$\vec{n}\cdot\vec{H}_c=0$$

$$\delta=\left(\frac{2}{\mu_c\omega\sigma}\right)^{1/2}$$

$$\vec{H}_c=\vec{H}_{par}e^{-\xi/\delta}e^{i\xi/\delta}$$

$$\vec{E}_c=\sqrt{\frac{\omega\mu_c}{2\sigma}}(1-i)\left(\vec{n}\times\vec{H}_{par}\right)$$

$$\frac{dP}{da}=\frac{\mu_c\omega\delta}{2}\left|\vec{H}_{par}\right|^2$$