Physics 604 Electromagnetic Theory I

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Ponderable Media

$\vec{\nabla} \times \vec{E}_{micro} = 0 \rightarrow \vec{\nabla} \times \vec{E} = 0$ after averaging

Polarization vector (dipole moment/V)

$\vec{P}(\vec{x}) = \sum_i n_i \langle \vec{p}_i \rangle$ n_i number density of type i

by superposition

$$\Delta\Phi(\vec{x}, \vec{x}') = \frac{1}{4\pi\varepsilon_0}\left[\frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|}\Delta V + \frac{\vec{P}(\vec{x}') \cdot (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}\Delta V\right]$$

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int d^3x'\left[\frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} + \vec{P}(\vec{x}') \cdot \vec{\nabla}'\frac{1}{|\vec{x} - \vec{x}'|}\right]$$

Electric Displacement

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int d^3x' \frac{1}{|\vec{x}-\vec{x}'|}\left[\rho(\vec{x}') + \vec{\nabla}'\cdot\vec{P}(\vec{x}')\right]$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\left[\rho - \vec{\nabla}\cdot\vec{P}\right] \qquad \vec{\nabla}\cdot\vec{D} = \rho$$

Electric Displacement $\vec{D}$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

isotropic media electric susceptibility

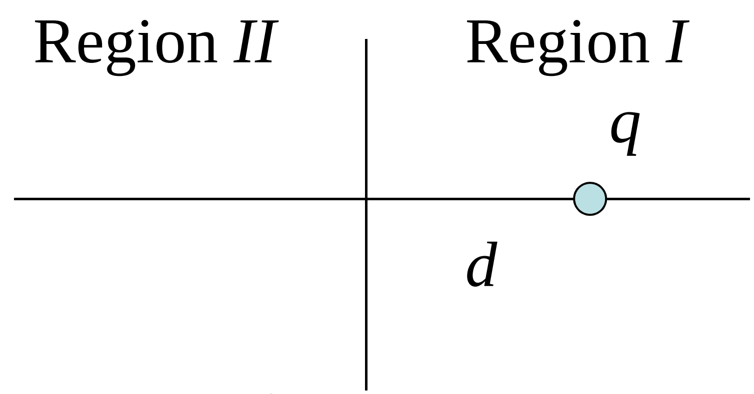
$$\vec{P} = \varepsilon_0\chi_E\vec{E}$$

$$\varepsilon = \varepsilon_0(1+\chi_E) \qquad \vec{D} = \varepsilon\vec{E}$$

boundary conditions

$$\left(\vec{D}_{II} - \vec{D}_I\right)\cdot\vec{n}_{II,I} = \sigma_{free} \qquad \left(\vec{E}_{II} - \vec{E}_I\right)\times\vec{n}_{II,I} = 0$$

Two Regions Problem



$$\varepsilon_I \vec{\nabla} \cdot \vec{E} = \rho \qquad z > 0$$

$$\varepsilon_{II} \vec{\nabla} \cdot \vec{E} = 0 \qquad z < 0$$

$$\vec{\nabla} \times \vec{E} = 0 \qquad \text{everywhere}$$

on boundary

$$\varepsilon_I E_{I,z} = \varepsilon_{II} E_{II,z} \quad E_{I,x} = E_{II,x} \quad E_{I,y} = E_{II,y}$$

$$\Phi = \frac{1}{4\pi\varepsilon_I}\left(\frac{q}{R_1} + \frac{q'}{R_2}\right) \quad z > 0 \quad \Phi = \frac{1}{4\pi\varepsilon_{II}} \frac{q''}{R_1} \quad z < 0$$

BCs imply

$$q - q' = q'' \qquad \frac{1}{\varepsilon_I}\left(q + q'\right) = \frac{q''}{\varepsilon_{II}}$$

$$q' = -\left(\frac{\varepsilon_{II} - \varepsilon_I}{\varepsilon_{II} + \varepsilon_I}\right) q \qquad q'' = \left(\frac{2\varepsilon_{II}}{\varepsilon_{II} + \varepsilon_I}\right) q$$

on boundary

$$\sigma_{pol} = -\left(\vec{P}_{II} - \vec{P}_I\right) \cdot \hat{z}$$

$$\vec{P}_i = \left(\varepsilon_i - \varepsilon_0\right) E_i = -\left(\varepsilon_i - \varepsilon_0\right) \vec{\nabla} \Phi$$

$$\sigma_{pol} = -\frac{q}{2\pi} \frac{\varepsilon_0}{\varepsilon_I} \left(\frac{\varepsilon_{II} - \varepsilon_I}{\varepsilon_{II} + \varepsilon_I}\right) \frac{d}{\left(r^2 + d^2\right)^{3/2}}$$

Dielectric Sphere

$$\Phi_{\text{in}} = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

$$\Phi_{\text{out}} = \sum_{l=0}^{\infty} \left[B_l r^l + C_l r^{-(l+1)} \right] P_l(\cos\theta)$$

tangential E

$$-\frac{1}{a}\frac{\partial \Phi_{\text{in}}}{\partial \theta} = -\frac{1}{a}\frac{\partial \Phi_{\text{out}}}{\partial \theta}$$

Normal D

$$-\varepsilon \frac{\partial \Phi_{\text{in}}}{\partial r} = -\varepsilon_0 \frac{\partial \Phi_{\text{out}}}{\partial r}$$

BCs imply

$$A_1 = -E_0 + \frac{C_1}{a^3} \qquad A_l = \frac{C_l}{a^{2l+1}} \quad l \neq 1$$

$$\frac{\varepsilon}{\varepsilon_0} A_1 = -E_0 - 2\frac{C_1}{a^3} \qquad \frac{\varepsilon}{\varepsilon_0} l A_l = -\left(l+1\right)\frac{C_l}{a^{2l+1}} \quad l \neq 1$$

only non-zero

$$A_1 = -\left(\frac{3}{2+\varepsilon/\varepsilon_0}\right) \qquad C_1 = -\left(\frac{\varepsilon/\varepsilon_0 - 1}{2+\varepsilon/\varepsilon_0}\right) E_0$$

$$\Phi_{\text{in}} = -\left(\frac{3}{2+\varepsilon/\varepsilon_0}\right) E_0 r \cos\theta$$

$$\Phi_{\text{out}} = -E_0 r\cos\theta + \left(\frac{\varepsilon/\varepsilon_0 - 1}{2+\varepsilon/\varepsilon_0}\right) E_0 \frac{a^3}{r^2}\cos\theta$$

$$E_{\text{in}} = \left(\frac{3}{2 + \varepsilon / \varepsilon_0} \right) E_0 < E_0 \quad \text{if } \varepsilon > \varepsilon_0$$

effective dipole moment

$$p = 4\pi\varepsilon_0 \left(\frac{\varepsilon / \varepsilon_0 - 1}{2 + \varepsilon / \varepsilon_0} \right) a^3 E_0$$

$$\vec{P} = \left(\varepsilon - \varepsilon_0\right) \vec{E} = 3\varepsilon_0 \left(\frac{\varepsilon / \varepsilon_0 - 1}{2 + \varepsilon / \varepsilon_0} \right) \vec{E}_0$$

$$\sigma_{\text{pol}} = 3\varepsilon_0 \left(\frac{\varepsilon / \varepsilon_0 - 1}{2 + \varepsilon / \varepsilon_0} \right) \vec{E}_0 \cos\theta$$

for dielectric hole $\varepsilon / \varepsilon_0 \to \varepsilon_0 / \varepsilon$

$$E_{\text{in}} = \frac{2\varepsilon}{2\varepsilon + \varepsilon_0} \qquad p = 4\pi\varepsilon_0 \left(\frac{\varepsilon / \varepsilon_0 - 1}{1 + 2\varepsilon / \varepsilon_0} \right) a^3 E_0$$

Clausius-Mossotti

$$\vec{E}_i = -\vec{E}_P \qquad \vec{p} = \frac{4\pi R^3}{3}\vec{P}$$

$$E_P = \frac{3}{4\pi R^3}\int_{r<R} \vec{E} d^3x = \frac{-\vec{P}}{3\varepsilon_0} = -\vec{E}_i$$

$$\vec{P} = n\left\langle \vec{p}_{mol} \right\rangle \qquad \left\langle \vec{p}_{mol} \right\rangle = \varepsilon_0 \gamma_{mol}\left(\vec{E} + \vec{E}_i\right)$$

$$\vec{P} = n\gamma_{mol}\left(\varepsilon_0 \vec{E} + \frac{\vec{P}}{3}\right) \qquad \chi_e = \frac{n\gamma_{mol}}{1 - \frac{1}{3}n\gamma_{mol}}$$

$$\gamma_{mol} = \frac{3}{n}\left(\frac{\varepsilon / \varepsilon_0 - 1}{\varepsilon / \varepsilon_0 + 2}\right)$$

Electrostatic Energy

$$W = \frac{1}{2}\int \rho(\vec{x})\Phi(\vec{x})\,d^3x$$

$$\delta W = \int \delta\rho(\vec{x})\Phi(\vec{x})\,d^3x$$

$$\delta\rho(\vec{x}) = \vec{\nabla}\cdot\delta\vec{D}$$

$$\delta W = \int \delta\vec{D}\cdot\vec{E}\,d^3x$$

$$W = \int\int_0^D \vec{D}\cdot\vec{E}\,d^3x$$

for linear media

$$\vec{E}\cdot\delta\vec{D} = \frac{1}{2}\delta\left(\vec{E}\cdot\vec{D}\right) \qquad W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\,d^3x$$