Physics 604 Electromagnetic Theory I

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Classical Electrodynamics

- A main physics discovery of the last half of the 20th century is that electrodynamics is a part of an overarching quantum theory of nature called the *standard model*
- In the standard model, electromagnetism is closely linked with the weak interaction force, through the so-called electroweak unification, in a non-Abelian gauge theory that relies on and utilizes the usual phase invariance of quantum mechanics
- As part of the standard model, electromagnetic forces are distinguished in that they couple to electric charge, and they are long (∞ ?) range because to the best of present knowledge, the photon (EM force carrier) is (nearly?) massless
- This fact "explains" their prevalence in our day-to-day life

Yukawa/Coulomb Potential

- General form for the interaction potential

$$V_{yukawa}(r) \propto \frac{e^{-r/\lambda_c}}{r}$$

$$\lambda_c = \frac{\hbar}{mc}$$

- Usual Coulomb interaction when $m \to 0$

$$V_{coulomb}(r) \propto \frac{1}{r}$$

- Because we can see (optically!) across the universe, photon very close to massless

Range of Validity of Classical Description

- Large numbers of quanta (quantum mechanical fluctuations are relatively small, leading to classical results)

$$N_{photons} >> 1$$

- Low photon energy

$$E_{photons} = \hbar\omega_{photons} << m_{electron}c^2$$

Higher energies can lead to (quantum mechanical) effects like Compton Effect. Not sufficient (photoelectric effect!)

- Macroscopic charge, i. e., large numbers of individual electrons. Electron discreteness not important.
- Does not work for atoms and new nano-scale devices. Need Quantum Mechanics for them!

Line and Surface Integrals

- Line integral

$$\int_L \vec{V} \cdot d\vec{l} \equiv \int_a^b \vec{V}\left(\vec{L}(\tau)\right) \cdot \frac{d\vec{L}(\tau)}{d\tau} d\tau$$

for *any* parameterization of L (invariant to choice!)

- Surface (Flux) integral

$$\int_S \vec{V} \cdot \vec{n} dA \equiv \iint_{S'} \vec{V}\left(\vec{S}(\sigma,\tau)\right) \cdot \left[\frac{\partial \vec{S}}{\partial \sigma} \times \frac{\partial \vec{S}}{\partial \tau}\right]\left(\vec{S}(\sigma,\tau)\right) d\sigma\, d\tau$$

$$\frac{\partial \vec{S}}{\partial \sigma}(\sigma,\tau) \equiv \lim_{\delta\sigma \to 0} \frac{\vec{S}(\sigma+\delta\sigma,\tau) - \vec{S}(\sigma,\tau)}{\delta\sigma}$$

again independent of parameters up to sign (Jacobian!).
Can usually integrate easily in high-symmetry situations.

Vector Calculus Theorems

- Stokes Theorem

$$\int_S \vec{\nabla} \times \vec{A} \cdot \vec{n} dA = \int_{L=\partial S} \vec{A} \cdot d\vec{l}$$

- Divergence Theorem

$$\int_V \vec{\nabla} \cdot \vec{A} dV = \int_{S=\partial V} \vec{A} \cdot \vec{n} dA$$