Physics 604

Final Exam

Dec. 16, 2010

1) (30 pts) Four point charges with charge $\pm q$ are arranged as in Figure 1.

a) (5 pts.) What is the charge density function $\rho(r,\theta,\phi)$?

b) (5 pts.) What are the multipoles q_{lm} (do as many as you can; there is useful information in the HW solutions)? In particular, what is the first non-vanishing multipole?

c) (4 pts.) Do multipoles for odd m always vanish? Can you explain why or why not using the rotational symmetry in the source?

d) (5 pts.) What is the potential at locations with $r > a$ and $r < a$?

e) (5 pts.) Suppose the charges are enclosed in a grounded conducting sphere of radius b centered on the origin. What are the potentials for $r > a$ and $r < a$?

f) (4 pts.) For the solution in e), what is the charge density on the inside surface of the sphere?

g) (2 pts.) What is the field outside the sphere?

2) (25 pts.) Solve the square 2-D potential problem given by Figure 2. The following steps may be helpful.

a) (5 pts.) What are the fundamental solutions in Cartesian coordinates?

b) (5 pts) Solve the boundary value problem with the potential Φ at $x = 0$, $0 < y < a$ and $x = a$, $0 < y < a$ equal to V and with the other sides having $\Phi = 0$.

c) (5 pts.) Solve an appropriate boundary value problem to include the faces at $\Phi = -V$.

d) (5 pts.) What is the total potential?

e) (5 pts.) Put a primed coordinate system with its origin at $\vec{x} = (a/2, a/2)$. Near $r' = 0$ what is the θ' dependence of the potential? Near $r' = 0$ what is the θ' dependence of the electric field?

3) (25 pts.) A uniformly charged infinitely thin spherical shell of charge q and radius a is spun around the z - axis with constant angular frequency ω .

a) (5 pts.) Show that the current density function $\vec{J}(r,\theta,\phi)$ is

$$\vec{J}(r,\theta,\phi) = \frac{q\omega}{4\pi a}\delta(r-a)\sin\theta\hat{\phi}.$$

b) (5 pts.) What is the vector potential in all space for this current source assuming the vector potential vanishes as $r \to \infty$?

c) (5 pts.) What is the magnetic induction inside the shell?

d) (5 pts.) What is the magnetic induction outside the shell and the magnetic moment?

e) (5 pts.) What is the magnetic energy inside and outside the shell?

4) (20 pts.) In our final homework set we dealt with accelerator dipole magnets. In this problem, solve a similar problem for 2-D accelerator quadrupole magnets with a model current density

$$J_z(r,\theta) = \frac{NI}{a}\cos 2\theta\,\delta(r-a).$$

a) (4 pts.) What is the vector potential generated by this source assuming the vector potential vanishes as $r \to \infty$?

b) (4 pts.) What is the magnetic induction inside and outside $r = a$?

c) (4 pts.) Write the induction for $r < a$ in terms of the Cartesian coordinates x and y and the Cartesian unit vectors $\hat{x}$ and $\hat{y}$. What are the powers of x and/or y that appear?

d) (4 pts.) What is the magnetic energy per unit length inside and outside $r = a$?

e) (4 pts.) What is the self inductance per unit length of the magnet assuming I is the total current entering (and leaving!) the magnet?

Extra credit (5 pts.): Given what you already know, what is a functional form of the potential as a function of r and θ, and also of x and y for a sextupole magnet? (you've done di(two)poles and quad(four)rupoles already now!)

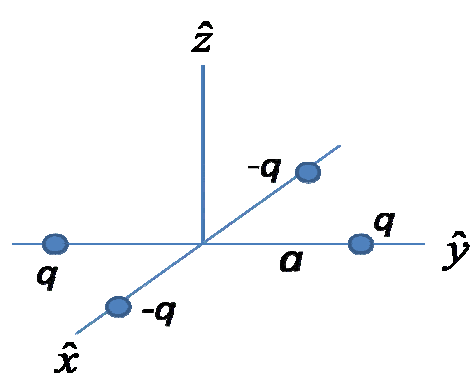

Figure 1

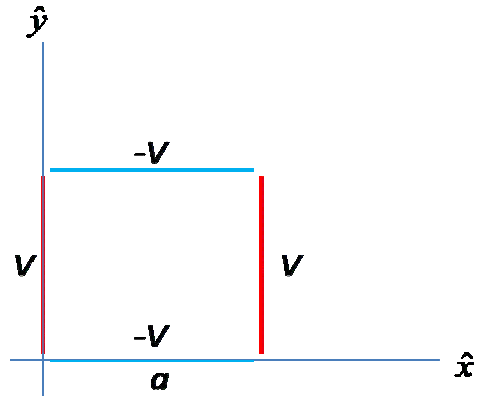

Figure 2