Nonlinear Thomson Scattering

- Many of the the newer Thomson Sources are based on a PULSED Laser (e.g. all of the high-energy lasers are pulsed by their very nature)
- Have developed a general theory to cover radiation calculations in the general case of a pulsed, high field strength laser interacting with electrons in a Thomson scattering arrangement.
- The new theory shows that in many situations the estimates people do to calculate flux and brilliance, based on a constant amplitude models, are just plain wrong.
- The new theory is general enough to cover all “1-D” undulater calculations and all pulsed laser Thomson scattering calculations.
- The main “new physics” that the new calculations include properly is the fact that the electron motion changes based on the local value of the field strength squared. Such ponderomotive forces (i.e., forces proportional to the field strength squared), lead to a detuning of the emission, angle dependent Doppler shifts of the emitted scattered radiation, and additional transverse dipole emission that this theory can calculate.

Ancient History

- Early 1960s: Laser Invented
- Brown and Kibble (1964): Earliest definition of the field strength parameters *K* and/or *a* in the literature that I'm aware of

$$a = \frac{eE_0\lambda_0}{2\pi mc^2} \text{ Thomson Sources} \qquad K = \frac{eB_0\lambda_0}{2\pi mc^2} \text{ Undulators}$$

  Interpreted frequency shifts that occur at high fields as a "relativistic mass shift".
- Sarachik and Schappert (1970): Power into harmonics at high *K* and/or *a* . Full calculation for CW (monochromatic) laser. Later referenced, corrected, and extended by workers in fusion plasma diagnostics.
- Alferov, Bashmakov, and Bessonov (1974): Undulater/Insertion Device theories developed under the assumption of constant field strength. Numerical codes developed to calculate "real" fields in undulaters.
- Coisson (1979): Simplified undulater theory, which works at low *K* and/or *a,* developed to understand the frequency distribution of "edge" emission, or emission from "short" magnets, i.e., including pulse effects

Coisson's Spectrum from a Short Magnet

Coisson low-field strength undulater spectrum*

$$\frac{dE}{d\nu d\Omega} = \frac{r_e^2 c}{\pi}\gamma^2\left(1+\gamma^2\theta^2\right)^2 f^2\left|\tilde{B}\left(\nu\left(1+\gamma^2\theta^2\right)/2\gamma^2\right)\right|^2$$

$$f^2 = f_\sigma^2 + f_\pi^2$$

$$f_\sigma = \frac{1}{\left(1+\gamma^2\theta^2\right)^2}\sin\phi$$

$$f_\pi = \frac{1}{\left(1+\gamma^2\theta^2\right)^2}\left(\frac{1-\gamma^2\theta^2}{1+\gamma^2\theta^2}\right)\cos\phi$$

*R. Coisson, Phys. Rev. A **20**, 524 (1979)

Dipole Radiation

Assume a single charge moves in the x direction

$$\rho(x,y,z,t) = e\delta(x-d(t))\delta(y)\delta(z)$$

$$\vec{J}(x,y,z,t) = e\dot{d}(t)\hat{x}\delta(x-d(t))\delta(y)\delta(z)$$

Introduce scalar and vector potential for fields.
Retarded solution to wave equation (Lorenz gauge), $R = \left|\vec{r} - \vec{r}\,'(t')\right|$

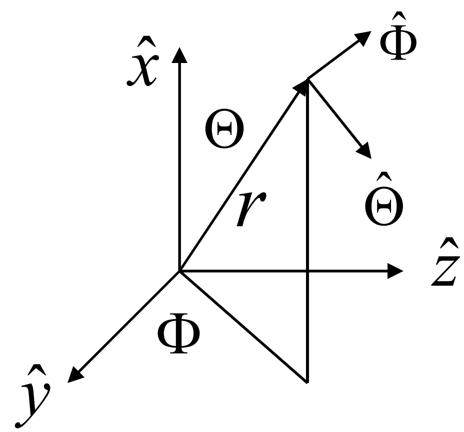
$$\phi(\vec{r},t) = \int \frac{1}{R}\rho\left(\vec{r}\,',t-\frac{R}{c}\right)dx'dy'dz' = e\int \frac{\delta(t'-t+R/c)}{R}dt'$$

$$A_x(\vec{r},t) = \int \frac{1}{Rc}J_x\left(\vec{r}\,',t-\frac{R}{c}\right)dx'dy'dz' = e\int \frac{\dot{d}(t')\delta(t'-t+R/c)}{Rc}dt'$$

Dipole Radiation

$$\vec{B} = \frac{e\ddot{d}\left(t - r/c\right)}{c^2 r}\sin\Theta\hat{\Phi}$$

$$\vec{E} = \frac{e\ddot{d}\left(t - r/c\right)}{c^2 r}\sin\Theta\hat{\Theta}$$



$$I = \frac{c\vec{E}\times\vec{B}}{4\pi} = \frac{1}{4\pi}\frac{e^2\ddot{d}^2\left(t - r/c\right)}{c^3 r^2}\sin^2\Theta\hat{r}$$

$$\frac{dI}{d\Omega} = \frac{1}{4\pi}\frac{e^2\ddot{d}^2\left(t - r/c\right)}{c^3}\sin^2\Theta$$

Polarized in the plane containing $\hat{r} = \vec{n}$ and $\hat{x}$

Dipole Radiation

Define the Fourier Transform

$$\tilde{d}(\omega) = \int d(t) e^{-i\omega t} dt \qquad d(t) = \frac{1}{2\pi} \int \tilde{d}(\omega) e^{i\omega t} d\omega$$

With these conventions Parseval's Theorem is

$$\int d^2(t)\, dt = \frac{1}{2\pi} \int \left|\tilde{d}\right|^2 (\omega)\, d\omega$$

$$\frac{dE}{d\Omega} = \frac{e^2}{4\pi c^3} \int \ddot{d}^2 (t - r/c)\, dt = \frac{e^2}{8\pi^2 c^3} \int \omega^4 \left|\tilde{d}\right|^2 (\omega)\, d\omega$$

$$\frac{dE}{d\omega d\Omega} = \frac{1}{8\pi^2} \frac{e^2 \omega^4 \left|\tilde{d}(\omega)\right|^2}{c^3} \sin^2 \Theta$$

Blue Sky!

This equation does not follow the typical (see Jackson) convention that combines both positive and negative frequencies together in a single positive frequency integral. The reason is that we would like to apply Parseval's Theorem easily. By symmetry, the difference is a factor of two.

Dipole Radiation

For a motion in three dimensions

$$\frac{dE}{d\omega d\Omega} = \frac{1}{8\pi^2} \frac{e^2\omega^4 \left| \vec{n} \times \tilde{\vec{d}}(\omega) \right|^2}{c^3}$$

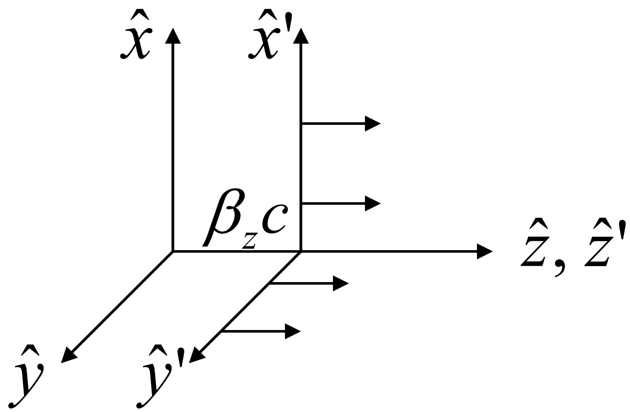
Vector inside absolute value along the magnetic field

$$\frac{dE}{d\omega d\Omega} = \frac{1}{8\pi^2} \frac{e^2\omega^4 \left| \left(\vec{n} \times \tilde{\vec{d}}(\omega) \right) \times \vec{n} \right|^2}{c^3} = \frac{1}{8\pi^2} \frac{e^2\omega^4 \left| \tilde{\vec{d}}(\omega) - \left(\vec{n} \cdot \tilde{\vec{d}}(\omega) \right) \vec{n} \right|^2}{c^3}$$

Vector inside absolute value along the electric field. To get energy into specific polarization, take scaler product with the polarization vector

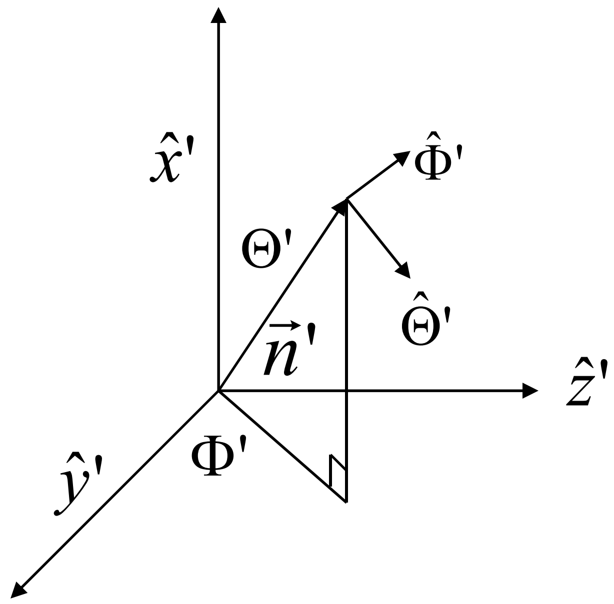
Co-moving Coordinates

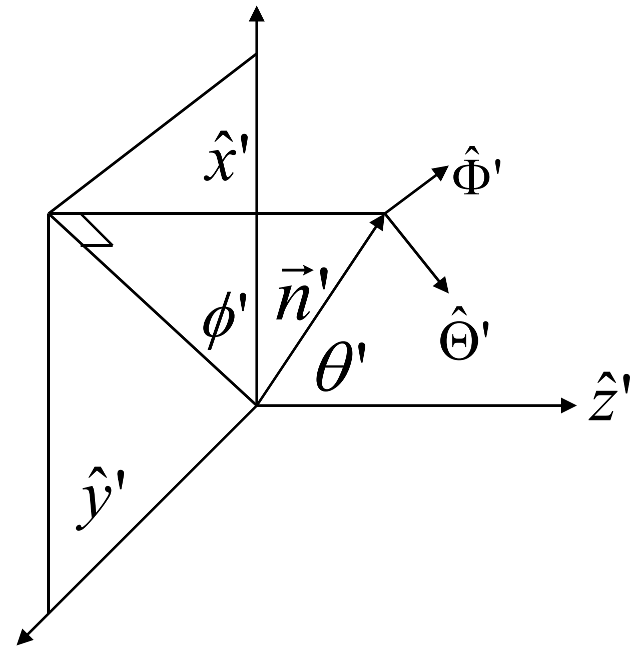
- Assume radiating charge is moving with a velocity close to light in a direction taken to be the *z* axis, and the charge is on average at rest in this coordinate system
- For the remainder of the presentation, quantities referred to the moving coordinates will have primes; unprimed quantities refer to the lab system



- In the co-moving system the dipole radiation pattern applies

New Coordinates





Resolve the polarization of scattered energy into that perpendicular (σ) and that parallel (π) to the scattering plane

$$\vec{n}' = \sin\theta'\cos\phi'\,\hat{x}' + \sin\theta'\sin\phi'\,\hat{y}' + \cos\theta'\,\hat{z}'$$

$$\hat{e}'_\sigma = \vec{n}'\times\hat{z}' / \left|\vec{n}'\times\hat{z}'\right| = \sin\phi'\,\hat{x}' - \cos\phi'\,\hat{y}' = -\hat{\phi}'$$

$$\hat{e}'_\pi = \vec{n}'\times\hat{e}'_\sigma = \cos\theta'\cos\phi'\,\hat{x}' + \cos\theta'\sin\phi'\,\hat{y}' - \sin\theta'\,\hat{z}' = \hat{\theta}'$$

Polarization

It follows that

$$\vec{\tilde{d}}'(\omega')\cdot\hat{e}'_\sigma = \tilde{d}'_x(\omega')\sin\phi' - \tilde{d}'_y(\omega')\cos\phi'$$

$$\vec{\tilde{d}}'(\omega')\cdot\hat{e}'_\pi = \tilde{d}'_x(\omega')\cos\theta'\cos\phi' + \tilde{d}'_y(\omega')\cos\theta'\sin\phi' - \sin\theta'\tilde{d}'_z(\omega')$$

So the energy into the two polarizations in the beam frame is

$$\frac{dE'_\sigma}{d\omega' d\Omega'} = \frac{1}{8\pi^2}\frac{e^2\omega'^4}{c^3}\left|\tilde{d}'_x(\omega')\sin\phi' - \tilde{d}'_y(\omega')\cos\phi'\right|^2$$

$$\frac{dE'_\pi}{d\omega' d\Omega'} = \frac{1}{8\pi^2}\frac{e^2\omega'^4}{c^3}\left|\begin{matrix}\tilde{d}'_x(\omega')\cos\theta'\cos\phi' + \tilde{d}'_y(\omega')\cos\theta'\sin\phi' \\ -\sin\theta'\tilde{d}'_z(\omega')\end{matrix}\right|^2$$

Comments/Sum Rule

- There is no radiation parallel or anti-parallel to the x-axis for x-dipole motion
- In the forward direction $\theta' \to 0$, the radiation polarization is parallel to the x-axis for an x-dipole motion
- One may integrate over all angles to obtain a result for the total energy radiated

$$\frac{dE'_\sigma}{d\omega'} = \frac{1}{8\pi^2}\frac{e^2\omega'^4}{c^3}\left(\left|\tilde{d}'_x(\omega')\right|^2 + \left|\tilde{d}'_y(\omega')\right|^2\right)2\pi$$

$$\frac{dE'_\pi}{d\omega'} = \frac{1}{8\pi^2}\frac{e^2\omega'^4}{c^3}\left[\left(\left|\tilde{d}'_x(\omega')\right|^2 + \left|\tilde{d}'_y(\omega')\right|^2\right)\frac{2\pi}{3} + \left|\tilde{d}'_z(\omega')\right|^2\frac{8\pi}{3}\right]$$

$$\frac{dE'_{tot}}{d\omega'} = \frac{1}{8\pi^2}\frac{e^2\omega'^4\left|\tilde{\vec{d}}'(\omega')\right|^2}{c^3}\frac{8\pi}{3}$$

Generalized Larmor

Sum Rule

Total energy sum rule

$$E'_{tot} = \int_{-\infty}^{\infty} \frac{1}{3\pi} \frac{e^2 \omega'^4 \left| \tilde{\vec{d}}'(\omega') \right|^2}{c^3} d\omega'$$

Parseval's Theorem again gives "standard" Larmor formula

$$P' = \frac{dE'_{tot}}{dt'} = \frac{2}{3} \frac{e^2 \ddot{\vec{d}}'^2 (t')}{c^3} = \frac{2}{3} \frac{e^2 \vec{a}'^2 (t')}{c^3}$$

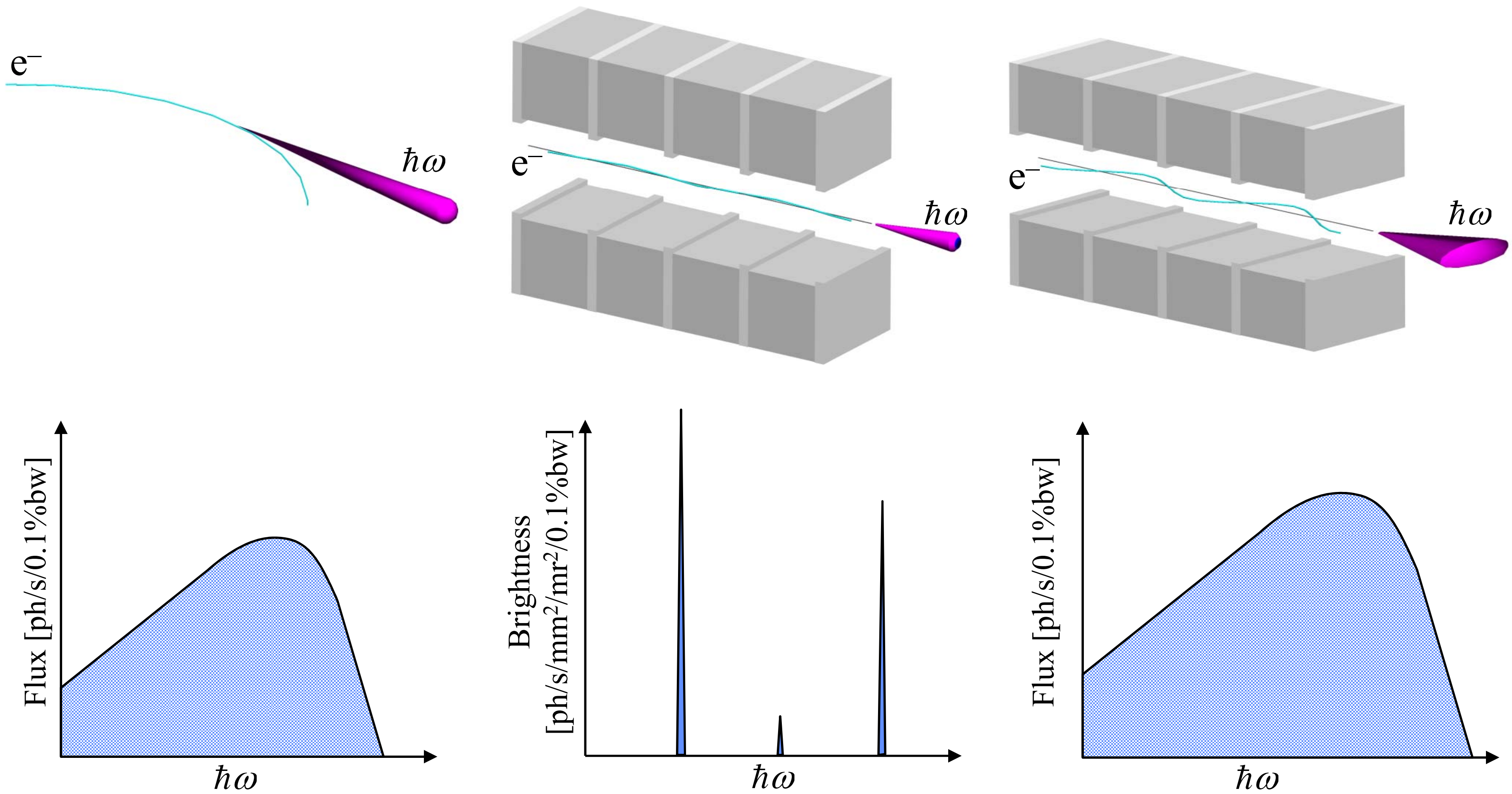
Energy Distribution in Lab Frame

$$\frac{dE_\sigma}{d\omega d\Omega} = \frac{e^2\omega^4\gamma^2(1-\beta\cos\theta)^2}{8\pi^2c^3}\left|\begin{array}{l}\tilde{d}'_x(\omega\gamma(1-\beta\cos\theta))\sin\phi \\ -\tilde{d}'_y(\omega\gamma(1-\beta\cos\theta))\cos\phi\end{array}\right|^2$$

$$\frac{dE_\pi}{d\omega d\Omega} = \frac{e^2\omega^4\gamma^2(1-\beta\cos\theta)^2}{8\pi^2c^3}\left|\begin{array}{l}\tilde{d}'_x(\omega\gamma(1-\beta\cos\theta))\dfrac{\cos\theta-\beta}{1-\beta\cos\theta}\cos\phi \\ +\tilde{d}'_y(\omega\gamma(1-\beta\cos\theta))\dfrac{\cos\theta-\beta}{1-\beta\cos\theta}\sin\phi \\ -\dfrac{\sin\theta}{\gamma(1-\beta\cos\theta)}\tilde{d}'_z(\omega\gamma(1-\beta\cos\theta))\end{array}\right|^2$$

By placing the expression for the Doppler shifted frequency and angles inside the transformed beam frame distribution. Total energy radiated from d'_z is the same as d'_x and d'_y for same dipole strength.

Bend — Undulater — Wiggler



white source

partially coherent source

powerful white source

Weak Field Undulater Spectrum

$$\tilde{\vec{d}}\,'(\omega') = \tilde{d}\,'(\omega')\hat{x} = -\frac{ec}{mc^2}\frac{\tilde{B}(\omega'/c\beta_z\gamma)}{\omega'^2}\hat{x} \qquad \tilde{B}(k) = \int B(z)e^{-ikz}dz$$

$$\frac{dE_\sigma}{d\omega d\Omega} = \frac{1}{8\pi^2}\frac{e^4}{m^2c^5}\frac{\left|\tilde{B}\left(\omega(1-\beta_z\cos\theta)/c\beta_z\right)\right|^2}{\gamma^2(1-\beta_z\cos\theta)^2}\sin^2\phi$$

$$\frac{dE_\pi}{d\omega d\Omega} = \frac{1}{8\pi^2}\frac{e^4}{m^2c^5}\frac{\left|\tilde{B}\left(\omega(1-\beta_z\cos\theta)/c\beta_z\right)\right|^2}{\gamma^2(1-\beta_z\cos\theta)^2}\left(\frac{\cos\theta-\beta_z}{1-\beta_z\cos\theta}\right)^2\cos^2\phi$$

$$r_e^2 \equiv \frac{e^4}{m^2c^4} \qquad \lambda = \frac{\lambda_0}{2\gamma^2} \qquad (1-\beta_z\cos\theta)(1+\beta_z) \approx \frac{1}{\gamma^2}+\theta^2+\ldots \approx \frac{1+\gamma^2\theta^2}{\gamma^2}$$

Generalizes Coisson to arbitrary observation angles

Strong Field Case

$$\frac{d}{dt}\gamma = 0$$

$$\frac{d}{dt}\gamma m\vec{\beta}c = -e\vec{\beta}\times\vec{B}$$

$$\beta_x(z) = \frac{e}{\gamma mc^2}\int_{-\infty}^{z} B(z')dz'$$

High K

$$\beta_z(z) = \sqrt{1 - \frac{1}{\gamma^2} - \beta_x^2(z)}$$

$$\beta_z(z) = \sqrt{1 - \frac{1}{\gamma^2} - \left(\frac{e}{\gamma mc^2} \int_{-\infty}^{z} B(z')dz' \right)^2}$$

$$\beta_z(z) \approx 1 - \frac{1}{2\gamma^2} - \frac{1}{2}\left(\frac{e}{\gamma mc^2} \int_{-\infty}^{z} B(z')dz' \right)^2 = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right) - \frac{K^2}{4\gamma^2}\cos(2k_0 z)$$

High K

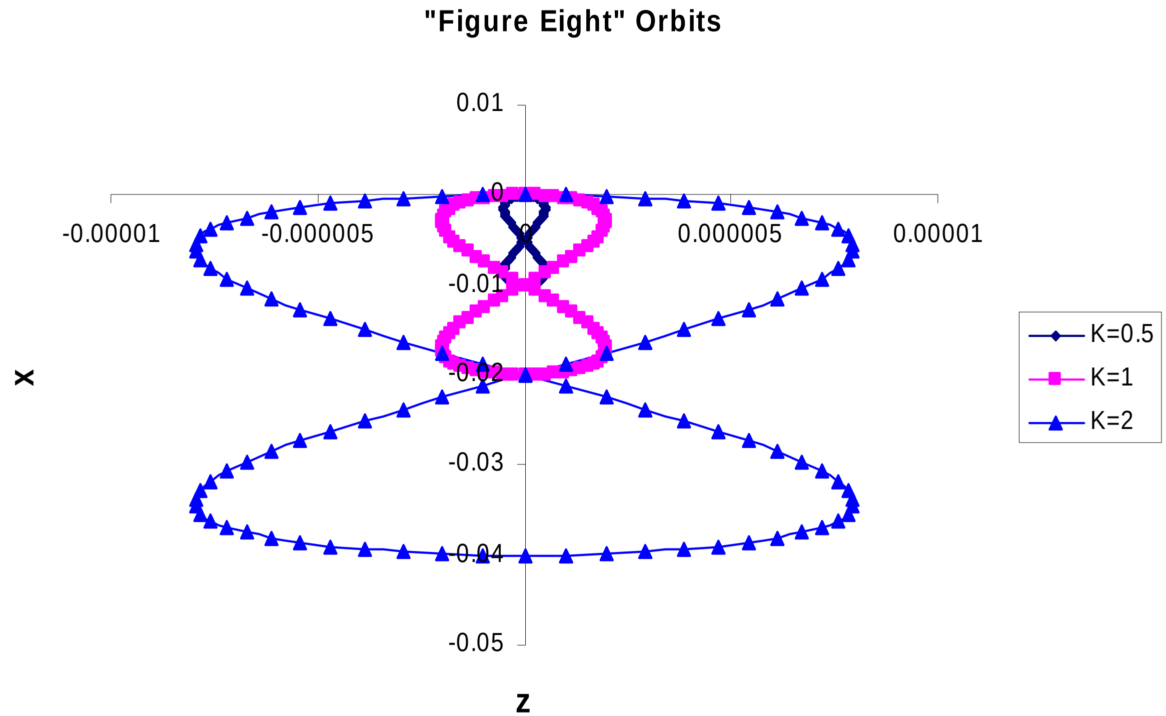
Inside the insertion device the average (z) velocity is

$$\beta^*_z = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

with corresponding

$$\gamma^* = \frac{1}{\sqrt{1-\beta^{*2}_z}} = \frac{\gamma}{\sqrt{1+K^2/2}}$$

To apply dipole distributions, must be in this frame to begin with



$\gamma = 100$, distances are normalized by $\lambda_0 / 2\pi$

Therefore

- Coisson's Theory may be generalized to arbitrary observation angles by using the proper polarization decomposition
- All kinematic parameters, including the angular distribution functions and frequency distributions, are just the same as in the weak field case, except unstarred quantities should be replaced by starred quantities
- In particular, the (FEL) resonance condition becomes

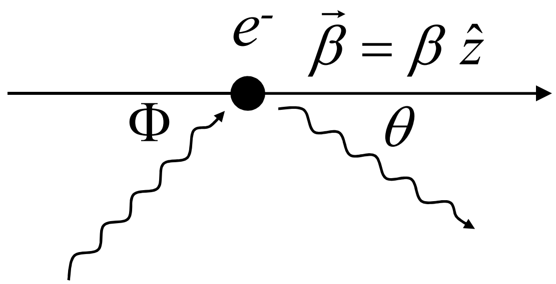
$$\lambda_n = \frac{n\lambda_0}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Thomson Scattering

- Purely "classical" scattering of photons by electrons
- Thomson regime defined by the photon energy in the electron rest frame being small compared to the rest energy of the electron, allowing one to neglect the quantum mechanical "Dirac" recoil on the electron
- In this case electron radiates at the same frequency as incident photon for low enough field strengths
- Classical dipole radiation pattern is generated in beam frame
- Therefore radiation patterns, at low field strength, can be largely copied from textbooks

- Note on terminology: Some authors call any scattering of photons by free electrons Compton Scattering. Compton observed (the so-called Compton effect) frequency shifts in X-ray scattering off (resting!) electrons that depended on scattering angle. Such frequency shifts arise only when the energy of the photon in the rest frame becomes comparable with 0.511 MeV.

Simple Kinematics



Beam Frame

$$p'_{e\mu} = \left(mc^2, 0\right)$$

$$p'_{p\mu} = \left(E'_L, \vec{E}'_L\right)$$

Lab Frame

$$p_{e\mu} = mc^2\left(\gamma, \gamma\beta\,\hat{z}\right)$$

$$p_{p\mu} = E_L\left(1, \sin\Phi\hat{y} + \cos\Phi\hat{z}\right)$$

$$p_e \cdot p_p = mc^2 E'_L = mc^2 E_L \gamma\left(1 - \beta\cos\Phi\right)$$

$$E'_L = E_L \gamma \left(1 - \beta \cos \Phi\right)$$

In beam frame scattered photon radiated with wave vector

$$k'_\mu = \frac{E'_L}{c}\left(1, \sin\theta' \cos\phi', \sin\theta' \sin\phi', \cos\theta'\right)$$

Back in the lab frame, the scattered photon energy E_s is

$$E_s = E'_L\, \gamma\left(1 + \beta \cos\theta'\right) = \frac{E'_L}{\gamma\left(1 - \beta\cos\theta\right)}$$

$$E_s = E_L \frac{\left(1 - \beta \cos \Phi\right)}{\left(1 - \beta \cos\theta\right)}$$

Electron in a Plane Wave

Assume linearly-polarized pulsed laser beam moving in the direction (electron charge is –*e*)

$$\vec{n}_{inc} = \sin\Phi\hat{y} + \cos\Phi\hat{z}$$

$$\vec{A}_{inc}(\vec{x},t) = A_x(ct - \sin\Phi y - \cos\Phi z)\hat{x} \equiv A(\xi)\hat{x}$$

Polarization 4-vector

$$\varepsilon^\mu = (0,1,0,0)$$

Light-like incident propagation 4-vector

$$n^\mu_{inc} = (1,0,\sin\Phi,\cos\Phi)$$

$$\varepsilon \cdot n_{inc} = \varepsilon_\mu n^\mu_{inc} = \vec{\varepsilon} \cdot \vec{n}_{inc} = 0$$

Electromagnetic Field

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \varepsilon^\nu \frac{\partial A}{\partial x_\mu} - \varepsilon^\mu \frac{\partial A}{\partial x_\nu}$$

$$= \left(\varepsilon^\nu n^\mu_{inc} - \varepsilon^\mu n^\nu_{inc}\right)\frac{dA}{d\xi}(\xi)$$

Our goal is to find $x^\mu(\tau)=(ct(\tau),x(\tau),y(\tau),z(\tau))$ when the 4-velocity $u^\mu(\tau)=(cdt/d\tau,dx/d\tau,dy/d\tau,dz/d\tau)(\tau)$ satisfies $du^\mu/d\tau= -eF^{\mu\nu}u_\nu/mc$ where τ is proper time. For any solution to the equations of motion.

$$\frac{d\left(n_{inc\mu}u^\mu\right)}{d\tau} = n_{inc\mu}F^{\mu\nu}u_\nu = 0 \qquad \therefore n_{inc\mu}u^\mu = n_{inc\mu}u^\mu(-\infty)$$

Proportional to amount frequencies up-shifted going to beam frame

ξ is exactly proportional to the proper time!

On the orbit

$$\xi(\tau) = ct(\tau) - \vec{n}_{inc} \cdot \vec{x}(\tau) \qquad d\xi / d\tau = n_{inc\mu} u^{\mu}$$

Integrate with respect to ξ instead of τ. Now

$$\frac{d\left(\varepsilon_{\mu} u^{\mu}\right)}{d\tau} = c\frac{df}{d\xi} n_{inc\mu} u^{\mu} = c\frac{d}{d\tau} f\left(\xi(\tau)\right)$$

where the unitless vector potential is $f(\xi)=-eA(\xi)/mc^2$.

$$\therefore \varepsilon_{\mu} u^{\mu} - cf = \varepsilon_{\mu} u^{\mu}(-\infty)$$

Electron Orbit

$$u^\mu(\xi)=u^\mu(-\infty)+cf(\xi)\left\{\frac{\varepsilon_\nu u^\nu(-\infty)}{n_{inc\nu}u^\nu(-\infty)}n^\mu_{inc}-\varepsilon^\mu\right\}+\frac{c^2 f^2(\xi)}{2\left(n_{inc\nu}u^\nu(-\infty)\right)}n^\mu_{inc}$$

Direct Force from Electric Field

Ponderomotive Force

$$x^\mu(\xi)=\frac{u^\mu(-\infty)\xi}{n_{inc\nu}u^\nu(-\infty)}+\left\{\frac{c\varepsilon_\nu u^\nu(-\infty)}{\left(n_{inc\nu}u^\nu(-\infty)\right)^2}n^\mu_{inc}-\frac{c\varepsilon^\mu}{n_{inc\nu}u^\nu(-\infty)}\right\}\int_{-\infty}^{\xi} f(\xi')d\xi'$$

$$+\frac{c^2 n^\mu_{inc}}{\left(n_{inc\nu}u^\nu(-\infty)\right)^2}\int_{-\infty}^{\xi}\frac{f^2(\xi')}{2}d\xi'$$

Energy Distribution

$$\frac{dE_\sigma}{d\omega d\Omega} = \frac{e^2\omega^2}{8\pi^2 c^3}\left| \begin{array}{c} D_t(\omega;\theta,\phi)\sin\phi \\ -\dfrac{\sin\Phi}{\gamma(1-\beta\cos\Phi)} D_p(\omega;\theta,\phi)\cos\phi \end{array} \right|^2$$

$$\frac{dE_\pi}{d\omega d\Omega} = \frac{e^2\omega^2}{8\pi^2 c^3}\left| \begin{array}{c} D_t(\omega;\theta,\phi)\dfrac{\cos\theta-\beta}{1-\beta\cos\theta}\cos\phi \\ +\dfrac{\sin\Phi}{\gamma(1-\beta\cos\Phi)} D_p(\omega;\theta,\phi)\dfrac{\cos\theta-\beta}{1-\beta\cos\theta}\sin\phi \\ +\dfrac{\beta-\cos\Phi}{1-\beta\cos\Phi} D_p(\omega;\theta,\phi)\dfrac{\sin\theta}{\gamma(1-\beta\cos\theta)} \end{array} \right|^2$$

Effective Dipole Motions: Lab Frame

$$D_t(\omega;\theta,\phi)=\frac{1}{\gamma(1-\beta\cos\Phi)}\int\frac{eA(\xi)}{mc^2}e^{i\varphi(\omega,\xi;\theta,\phi)}d\xi$$

$$D_p(\omega;\theta,\phi)=\frac{1}{\gamma(1-\beta\cos\Phi)}\int\frac{e^2A^2(\xi)}{2m^2c^4}e^{i\varphi(\omega,\xi;\theta,\phi)}d\xi$$

And the (Lorentz invariant!) phase is

$$\varphi(\omega,\xi;\theta,\phi)=\frac{\omega}{c}\left(\begin{array}{l}\xi\dfrac{(1-\beta\cos\theta)}{(1-\beta\cos\Phi)}-\dfrac{\sin\theta\cos\phi}{\gamma(1-\beta\cos\Phi)}\displaystyle\int_{-\infty}^{\xi}\frac{eA(\xi')}{mc^2}d\xi' \\ +\dfrac{1-\sin\theta\sin\phi\sin\Phi-\cos\theta\cos\Phi}{\gamma^2(1-\beta\cos\Phi)^2}\displaystyle\int_{-\infty}^{\xi}\frac{e^2A^2(\xi')}{2m^2c^4}d\xi'\end{array}\right)$$

Summary

- Overall structure of the distributions is very like that from the general dipole motion, only the effective dipole motion, incuding physical effects such as the relativistic motion of the electrons and retardation, must be generalized beyond the straight Fourier transform of the field
- At low field strengths (f <<1), the distributions reduce directly to the classical Fourier transform dipole distributions
- The effective dipole motion from the ponderomotive force involves a simple projection of the incident wave vector in the beam frame onto the axis of interest, multiplied by the general ponderomotive dipole motion integral
- The radiation from the two transverse dipole motions are compressed by the same angular factors going from beam to lab frame as appears in the simple dipole case. The longitudinal dipole radiation is also transformed between beam and lab frame by the same fraction as in the simple longitudinal dipole motion. Thus the usual compression into a $1/\gamma$ cone applies

Weak Field Thomson Backscatter

With $\Phi = \pi$ and f <<1 the result is identical to the weak field undulater result with the replacement of the magnetic field Fourier transform by the electric field Fourier transform

	Undulator	Thomson Backscatter
Driving Field	$\tilde{B}_y\left(\omega\left(1-\beta_z\cos\theta\right)/c\beta_z\right)$	$\tilde{E}_x\left(\omega\left(1-\beta_z\cos\theta\right)/\left(c\left(1+\beta_z\right)\right)\right)$
Forward Frequency	$\lambda \approx \frac{\lambda_0}{2\gamma^2}$	$\lambda \approx \frac{\lambda_0}{4\gamma^2}$
	Lorentz contract + Doppler	Double Doppler

High Field Strength Thomson Backscatter

For a flat incident laser pulse the main results are very similar to those from undulaters with the following correspondences

	Undulater	Thomson Backscatter
Field Strength	K	a
Forward Frequency	$\lambda \approx \frac{\lambda_0}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$	$\lambda \approx \frac{\lambda_0}{4\gamma^2}\left(1+\frac{a^2}{2}\right)$
Transverse Pattern	$\beta^*_z + \cos\theta'$	$1+\cos\theta'$

NB, be careful with the radiation pattern, it is the same at small angles, but quite a bit different at large angles

Realistic Pulse Distribution at High a

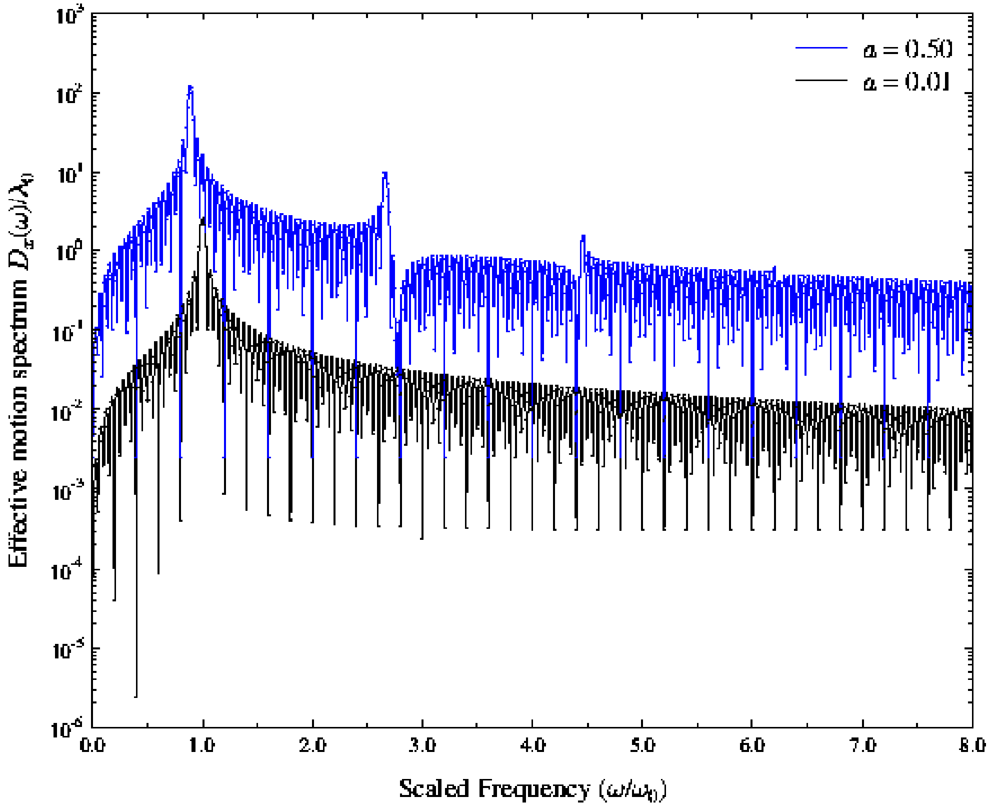
In general, it's easiest to just numerically integrate the lab-frame expression for the spectrum in terms of D_x , D_y, and D_z. A 10^5 to 10^6 point Simpson integration is adequate for most purposes. We've done two types of pulses, flat pulses to reproduce the previous results and to evaluate numerical error, and Gaussian Laser pulses.

One may utilize a two-timing approximation (i.e., the laser pulse is a slowly varying sinusoid with amplitude $a(\xi)$), and the fundamental expressions, to write the energy distribution at any angle in terms of Bessel function expansions and a ξ integral over the modulation amplitude. This approach actually has a limited domain of applicability (K,a<0.1)

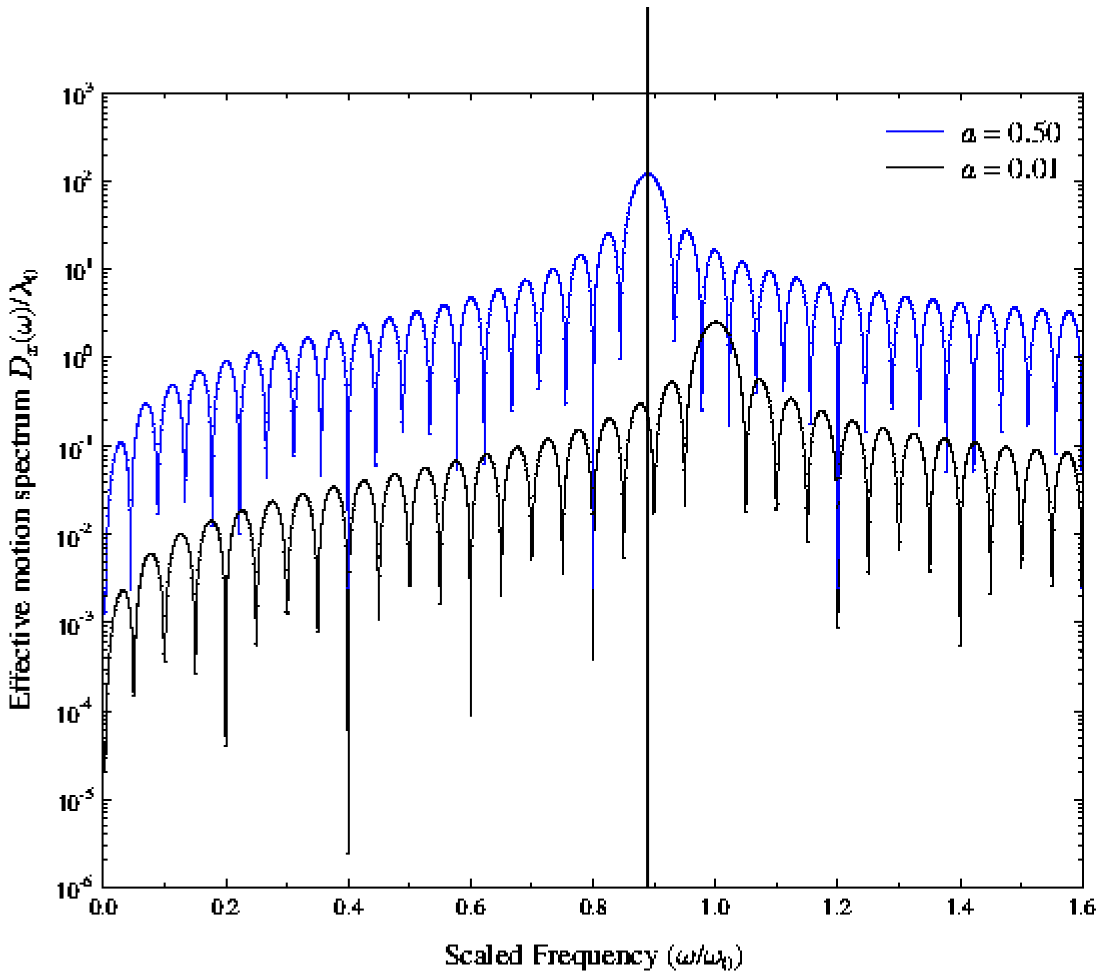
Forward Direction: Flat Pulse

20-period
equivalent undulater: $A_x(\xi) = A_0 \cos(2\pi\xi/\lambda_0)[\Theta(\xi) - \Theta(\xi - 20\lambda_0)]$

$$\omega_0 \equiv (1+\beta_z)^2 \gamma^2 2\pi c/\lambda_0 \approx 4\gamma^2 2\pi c/\lambda_0, \; a = eA_0/mc^2$$



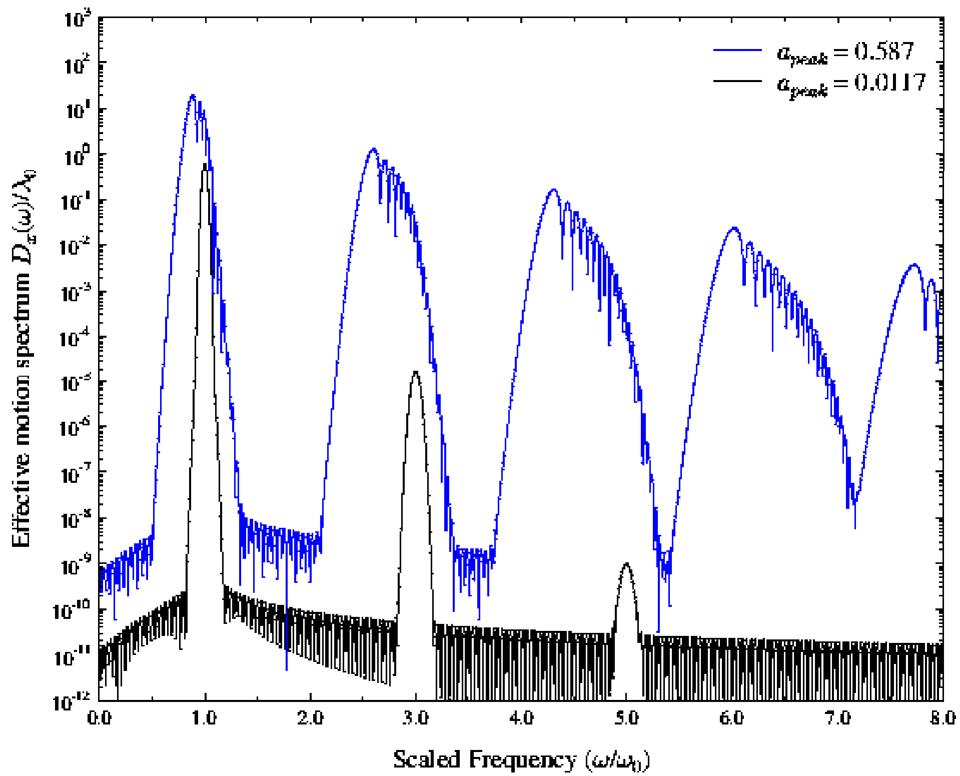
$$1/\left(1+a^2/2\right)$$



Forward Direction: Gaussian Pulse

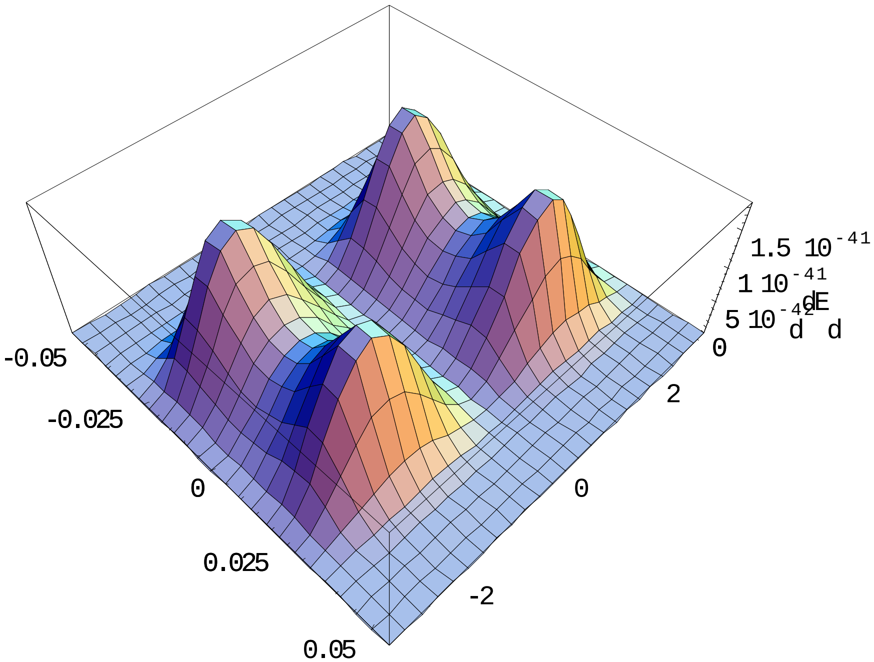
$$A_x(\xi) = A_{peak} \exp\left(-z^2 / 2(8.156\lambda_0)^2\right)\cos(2\pi\xi / \lambda_0) \qquad a_{peak} = eA_{peak} / mc^2$$

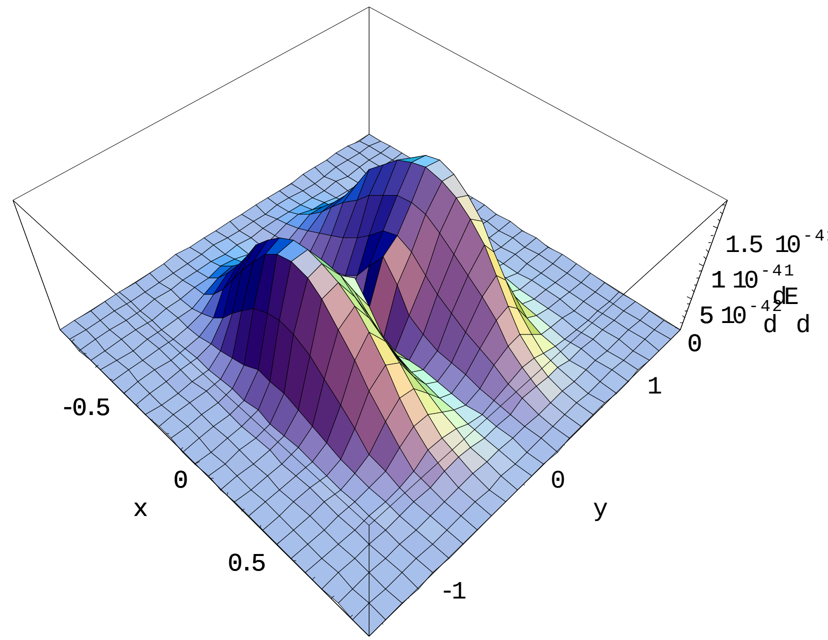
A_{peak} and λ_0 chosen for same intensity and same *rms* pulse length as previous slide



Radiation Distributions: Backscatter

Gaussian Pulse σ at first harmonic peak

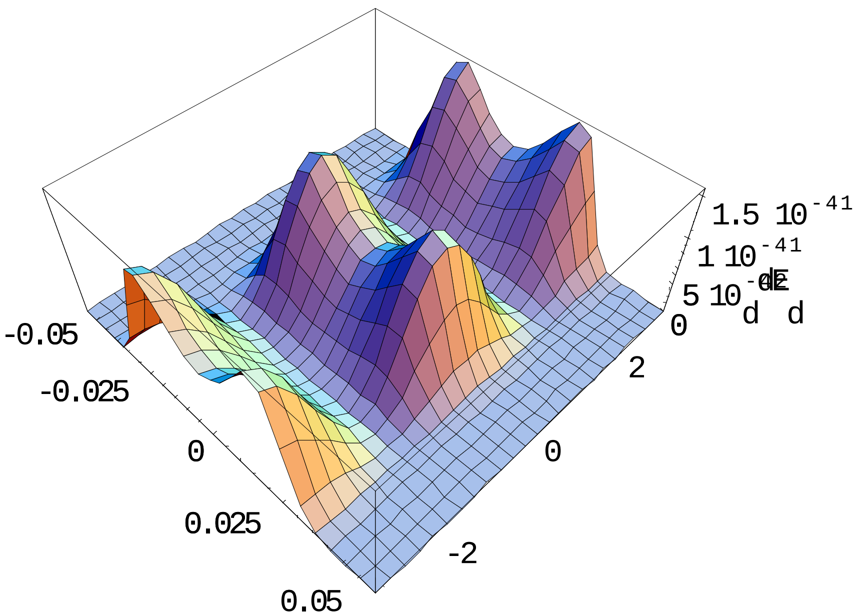


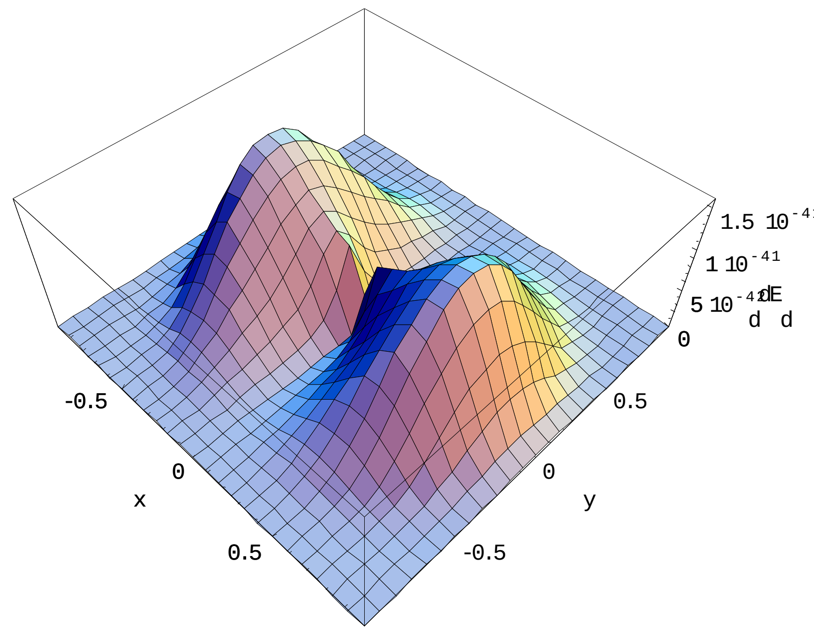


Courtesy: Adnan Doyuran (UCLA)

Radiation Distributions: Backscatter

Gaussian π at first harmonic peak

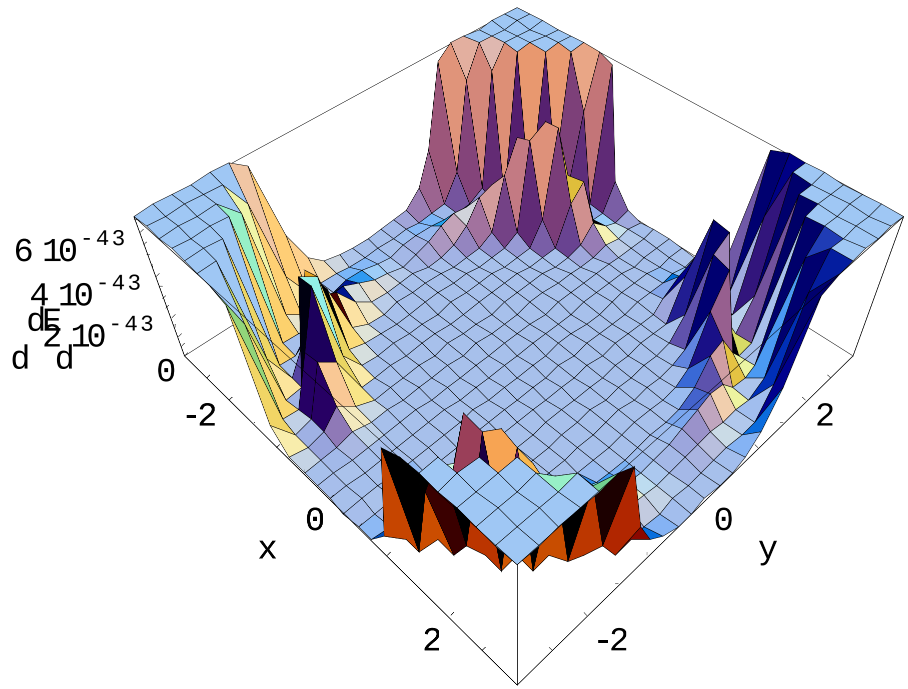




Courtesy: Adnan Doyuran (UCLA)

Radiation Distributions: Backscatter

Gaussian σ at second harmonic peak



Courtesy: Adnan Doyuran (UCLA)

90 Degree Scattering

$$\frac{dE_\sigma}{d\omega d\Omega} = \frac{e^2\omega^2}{8\pi^2 c^3}\left| D_t(\omega;\theta,\phi)\sin\phi - \frac{1}{\gamma} D_p(\omega;\theta,\phi)\cos\phi \right|^2$$

$$\frac{dE_\pi}{d\omega d\Omega} = \frac{e^2\omega^2}{8\pi^2 c^3}\left| \begin{array}{c} D_t(\omega;\theta,\phi)\dfrac{\cos\theta - \beta}{1-\beta\cos\theta}\cos\phi \\ +\dfrac{1}{\gamma} D_p(\omega;\theta,\phi)\dfrac{\cos\theta-\beta}{1-\beta\cos\theta}\sin\phi \\ + D_p(\omega;\theta,\phi)\dfrac{\beta\sin\theta}{\gamma(1-\beta\cos\theta)} \end{array} \right|^2$$

90 Degree Scattering

$$D_t(\omega;\theta,\phi)=\frac{1}{\gamma}\int\frac{eA(\xi)}{mc^2}e^{i\varphi(\omega,\xi;\theta,\phi)}d\xi$$

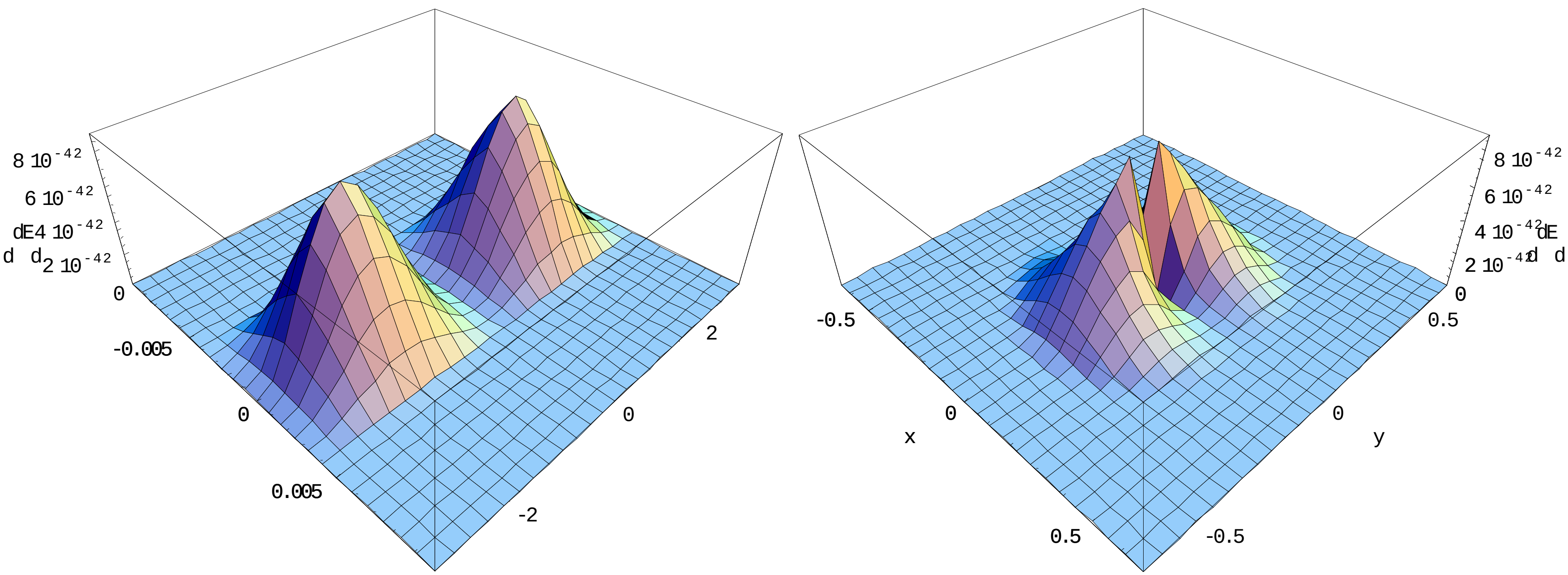
$$D_p(\omega;\theta,\phi)=\frac{1}{\gamma}\int\frac{e^2A^2(\xi)}{2m^2c^4}e^{i\varphi(\omega,\xi;\theta,\phi)}d\xi$$

And the phase is

$$\varphi(\omega,\xi;\theta,\phi)=\frac{\omega}{c}\left(\begin{array}{c}\xi(1-\beta\cos\theta)-\dfrac{\sin\theta\cos\phi}{\gamma}\displaystyle\int_{-\infty}^{\xi}\frac{eA(\xi')}{mc^2}d\xi' \\ +\dfrac{1-\sin\theta\sin\phi}{\gamma^2}\displaystyle\int_{-\infty}^{\xi}\frac{e^2A^2(\xi')}{2m^2c^4}d\xi'\end{array}\right)$$

Radiation Distribution: 90 Degree

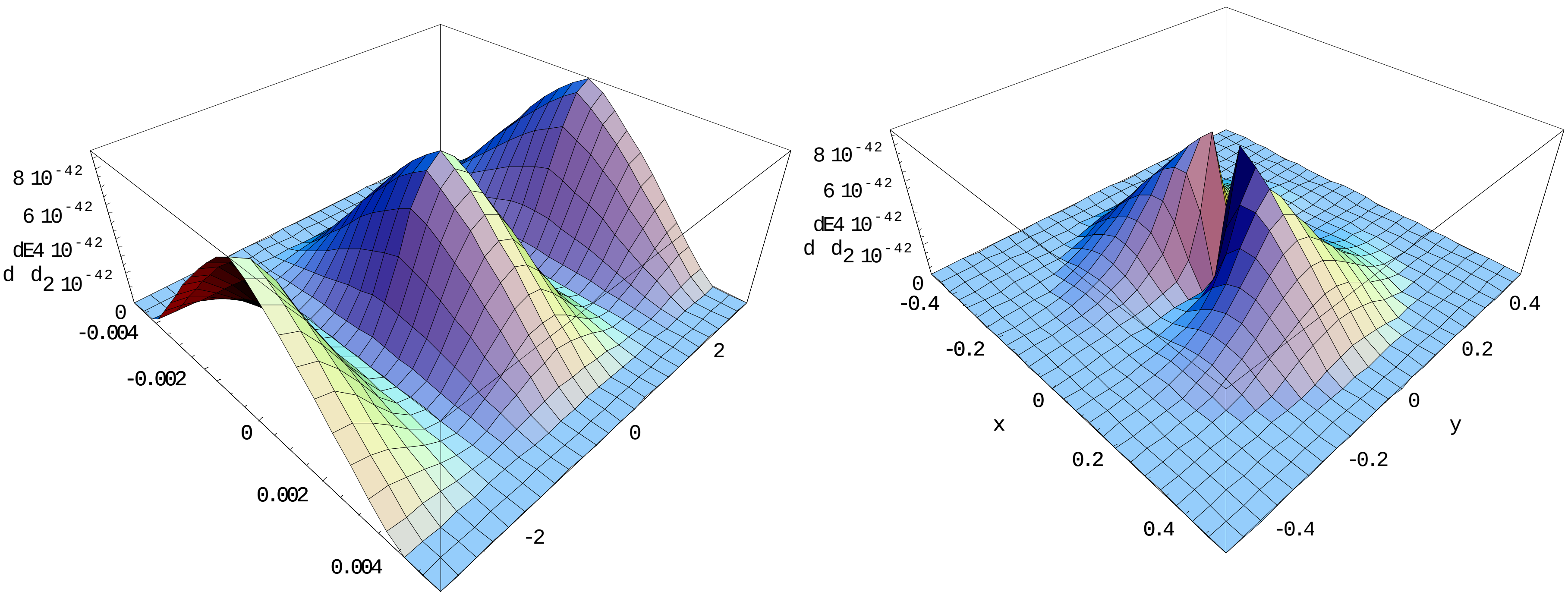
Gaussian Pulse σ at first harmonic peak



Courtesy: Adnan Doyuran (UCLA)

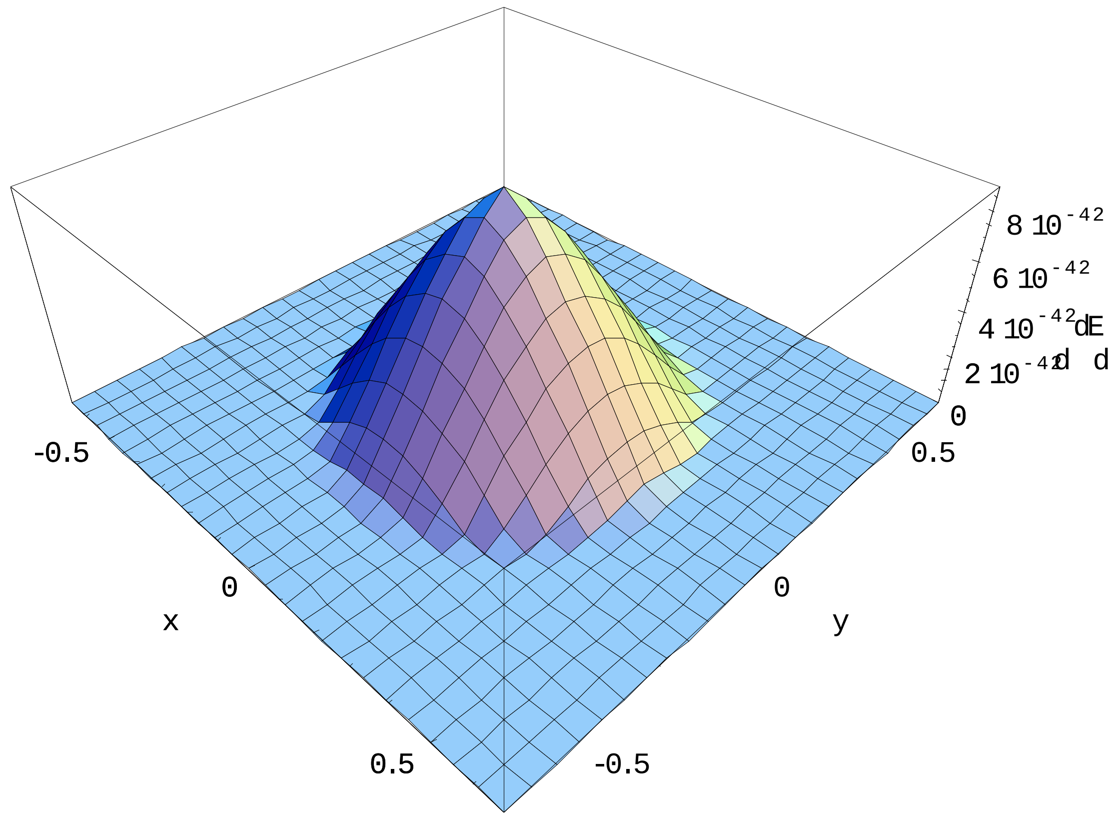
Radiation Distributions: 90 Degree

Gaussian Pulse π at first harmonic peak



Courtesy: Adnan Doyuran (UCLA)

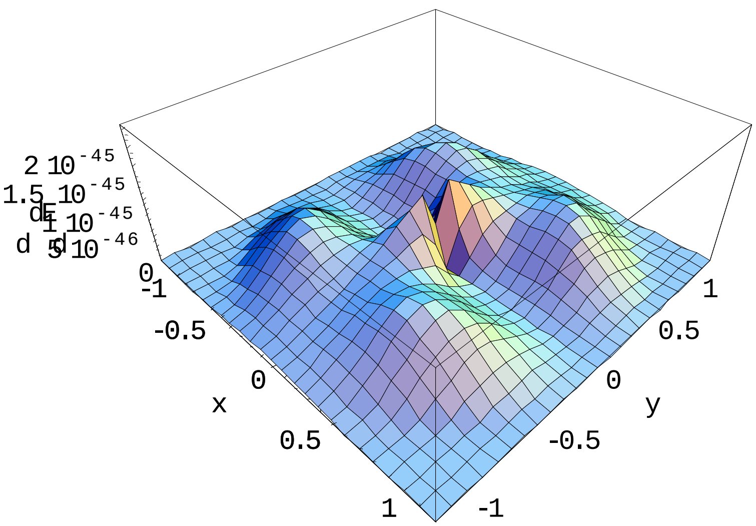
Polarization Sum: Gaussian 90 Degree



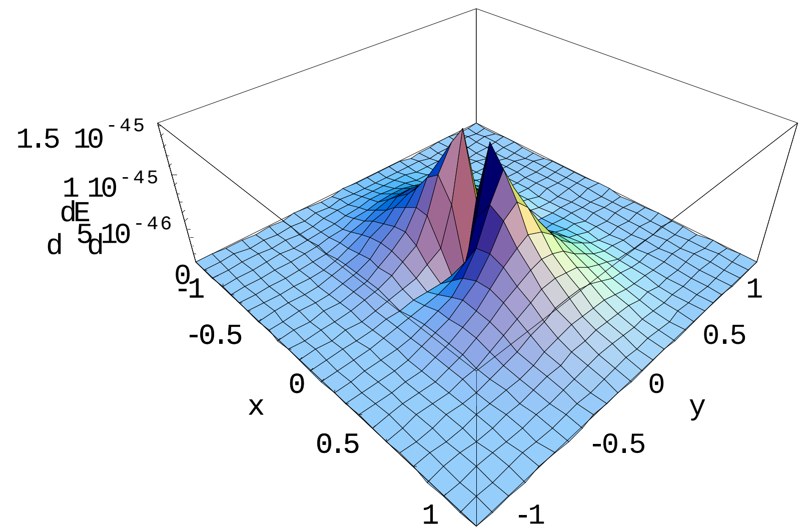
Courtesy: Adnan Doyuran (UCLA)

Radiation Distributions: 90 Degree

Gaussian Pulse second harmonic peak



σ



π

Second harmonic emission on axis from ponderomotive dipole!

Courtesy: Adnan Doyuran (UCLA)

Total Energy Radiated

Lienard's Generalization of Larmor Formula (1898!)

$$\frac{dE}{dt}=\frac{2}{3}\frac{e^2}{c}\gamma^6\left[\left(\dot{\vec{\beta}}\right)^2+\left(\vec{\beta}\times\dot{\vec{\beta}}\right)^2\right]=\frac{2}{3}\frac{e^2}{c}\gamma^4\left[\left(\dot{\vec{\beta}}\right)^2+\gamma^2\left(\vec{\beta}\cdot\dot{\vec{\beta}}\right)^2\right]$$

Barut's Version

$$\frac{dE}{d\tau}=\frac{2}{3}\frac{e^2}{c^3}\frac{dt}{d\tau}\frac{d^2x^\mu}{d\tau^2}\frac{d^2x_\mu}{d\tau^2}$$

$$E=\frac{2e^2}{3}\int_{-\infty}^{\infty}\left[\gamma^2(1-\beta\cos\Phi)\left(\frac{df}{d\xi}\right)^2+\frac{f^2}{2}\left(\frac{df}{d\xi}\right)^2\right]d\xi$$

Usual Larmor term

From ponderomotive dipole

Some Cases

Total radiation from electron initially at rest

$$E = \frac{2e^2}{3} \int_{-\infty}^{\infty} \left[\left(\frac{df}{d\xi} \right)^2 + \frac{f^2}{2} \left(\frac{df}{d\xi} \right)^2 \right] d\xi$$

For a flat pulse exactly (Sarachik and Schappert)

$$\frac{dE}{dt} = \frac{1}{3} \frac{e^2 \omega^2}{c} a^2 \left(1 + a^2 / 8 \right)$$

For Circular Polarization

$$\vec{A}_{inc}(\xi) = A(\xi)\{\cos(2\pi\xi/\lambda)\hat{x} \pm \sin(2\pi\xi/\lambda)[-\cos\Phi\hat{y} + \sin\Phi\hat{z}]\}$$

$$E = \frac{2e^2}{3}\int_{-\infty}^{\infty}\left[\gamma\left(n_{inc\mu}u^{\mu}(-\infty)\right) - f_{\pm}\beta\gamma\sin\Phi + \frac{\hat{A}^2}{2}\right]$$

$$\hat{A} = -eA/mc^2 \qquad \times\left[\left(\frac{d\hat{A}}{d\xi}\right)^2 + \left(\frac{2\pi}{\lambda}\right)^2\hat{A}^2\right]d\xi$$

Only specific case I can find in literature completely calculated has $\sin\Phi = 0$ and flat pulses ($dA/d\xi = 0$). The orbits are then pure circles

For zero average velocity in middle of pulse

$$\gamma\vec{\beta}(-\infty) = -\frac{\vec{n}_{inc}}{n_{inc\,\nu}u^{\nu}(-\infty)}\frac{c\hat{A}^2}{2} \rightarrow \gamma^2 n_{inc\,\nu}u^{\nu}(-\infty)/c = 1 + \frac{\hat{A}^2}{2}$$

Sokolov and Ternov, in *Radiation from Relativistic Electrons*, give

$$\frac{dE'}{dt'} = \frac{2}{3}\frac{e^2\omega'^2}{c}a^2\left(1 + a^2\right)$$

and the general formula (which goes back to Schott and the turn of the 20th century!) checks out

Conclusions

- Recent development of superconducting cavities has enabled CW operation at energy gains in excess of 20 MV/m, and acceleration of average beam currents of 10s of mA.
- The ideas of Beam Recirculation and Energy Recovery have been introduced; how these concepts may be combined to yield a new class of accelerators that can be used in many interesting applications has been discussed. I've given you some indication about the historical development of recirculating SRF linacs.
- The present knowledge on beam recirculation and its limitations in a superconducting environment, leads us to think that recirculating accelerators of several GeV energy, and with beam currents approaching those in storage ring light sources, are possible.

Conclusions

- I've shown how dipole solutions to the Maxwell equations can be used to obtain and understand very general expressions for the spectral angular energy distributions for weak field undulaters and general weak field Thomson Scattering photon sources
- A "new" calculation scheme for high intensity pulsed laser Thomson Scattering has been developed. This same scheme can be applied to calculate spectral properties of "short", high-K wigglers.
- Due to ponderomotive broadening, it is simply wrong to use single-frequency estimates of flux and brilliance in situations where the square of the field strength parameter becomes comparable to or exceeds the ($1/N$) spectral width of the induced electron wiggle
- The new theory is especially useful when considering Thomson scattering of Table Top TeraWatt lasers, which have exceedingly high field and short pulses. Any calculation that does not include ponderomotive broadening is incorrect.

Conclusions

- Because the laser beam in a Thomson scatter source can interact with the electron beam non-colinearly with the beam motion (a piece of physics that cannot happen in an undulater), ponderomotively driven transverse dipole motion is now possible
- This motion can generate radiation at the second harmonic of the up-shifted incident frequency on axis. The dipole direction is in the direction of laser incidence.
- Because of Doppler shifts generated by the ponderomotive displacement velocity induced in the electron by the intense laser, the frequency of the emitted radiation has an angular asymmetry.
- Sum rules for the total energy radiated, which generalize the usual Larmor/Lenard sum rule, have been obtained.