Physics 417/517 Introduction to Particle Accelerator Physics

G. A. Krafft
Jefferson Lab
Jefferson Lab Professor of Physics
Old Dominion University

Insertion Device (FEL) Resonance Condition

Angle transforms as

$$\cos\theta^* = \frac{k_z^*}{k^*} = \frac{\left(\cos\theta - \beta^*\right)}{\left(1-\beta^*\cos\theta\right)}$$

Wave vector in lab frame has

$$k = \frac{k^*}{\gamma^*\left(1-\beta^*\cos\theta\right)} = \frac{2\pi\beta^* c}{\lambda_{ID}\left(1-\beta^*\cos\theta\right)}$$

In the forward direction $\cos\theta = 1$

$$\lambda_e \approx \frac{\lambda_{ID}}{2\gamma^{*2}} = \frac{\lambda_{ID}}{2\gamma^2}\left(1+K^2/2\right)$$

Power Emitted Lab Frame

Larmor/Lienard calculation in the lab frame yields

$$\langle P \rangle = \frac{e^2}{6\pi\varepsilon_0}\gamma^4\beta^{*2}c\left(\frac{K}{\gamma}\right)^2\left(\frac{2\pi}{\lambda_{ID}}\right)^2\frac{1}{2}$$

Total energy radiated after one passage of the insertion device

$$\delta E = 2\pi^2\frac{e^2}{6\pi\varepsilon_0\lambda_{ID}}\gamma^2\beta^* NK^2$$

Power Emitted Beam Frame

Larmor/Lienard calculation in the beam frame yields

$$\left\langle P^* \right\rangle = \frac{e^2}{6\pi\varepsilon_0} cK^2 \left(\frac{2\pi}{\lambda^*} \right)^2 \frac{1}{2}$$

Total energy of each photon is $\hbar 2\pi c/\lambda^*$, therefore the total number of photons radiated after one passage of the insertion device

$$N_\gamma = \frac{2\pi}{3} \alpha N K^2$$

Spectral Distribution in Beam Frame

Begin with power distribution in beam frame: dipole radiation pattern (single harmonic only when K<<1; replace γ^* by γ, β^* by β)

$$\frac{dP^*}{d\Omega^*} = \frac{e^2 c}{32\pi^2 \varepsilon_0} k^{*4} a^2 \sin^2 \Theta^*$$

Number distribution in terms of wave number

$$\frac{dN_\gamma}{d\Omega^*} = \frac{\alpha}{4} N K^2 \frac{k_y^{*2} + k_z^{*2}}{k^{*2}}$$

Solid angle transformation

$$d\Omega^* = \frac{d\Omega}{\gamma^2 \left(1 - \beta \cos\theta\right)^2}$$