

Homework Problems
Physics 417/517
Due November 17, 2009

1. What is the phase advance per pass in a $\nu = 1$ microtron with (stable) synchronous phase $\phi_s = +10^\circ$ after the crest phase?
2. Show explicitly that the matrix representation

$$M_{s',s} = \begin{pmatrix} \sqrt{\frac{\beta(s')}{\beta(s)}} (\cos \Delta\mu_{s',s} + \alpha(s) \sin \Delta\mu_{s',s}) & \sqrt{\beta(s')\beta(s)} \sin \Delta\mu_{s',s} \\ -\frac{1}{\sqrt{\beta(s')\beta(s)}} \left[(1 + \alpha(s')\alpha(s)) \sin \Delta\mu_{s',s} \right] & \sqrt{\frac{\beta(s)}{\beta(s')}} (\cos \Delta\mu_{s',s} - \alpha(s') \sin \Delta\mu_{s',s}) \end{pmatrix}$$

satisfies the composition formula $M_{s'',s} = M_{s'',s'} M_{s',s}$. Also, show from this representation that

$$\tan \Delta\mu_{s',s} = \frac{(M_{s',s})_{12}}{\beta(s)(M_{s',s})_{11} - \alpha(s)(M_{s',s})_{12}}$$

3. As was done in class for the focusing direction, derive the fundamental solution for the dispersion for a region of length L with constant defocusing ($k < 0$)

$$D_{p,0}(s) = \frac{1}{(-k)\rho} (\cosh \sqrt{-k}s - 1)$$

$$D'_{p,0}(s) = \frac{1}{\sqrt{-k}\rho} \sinh \sqrt{-k}s$$

Show the 3×3 dispersion tracking matrix is

$$\begin{pmatrix} \cosh \sqrt{-k}L & \sinh \sqrt{-k}L / \sqrt{-k} & \frac{1}{(-k)\rho} (\cosh \sqrt{-k}L - 1) \\ \sqrt{-k} \sinh \sqrt{-k}L & \cosh \sqrt{-k}L & \frac{1}{\sqrt{-k}\rho} \sinh \sqrt{-k}L \\ 0 & 0 & 1 \end{pmatrix}$$

4. Up to now, we have neglected the phenomenon of edge focusing, which arises any time the design orbit goes into or emerges from a uniform magnet in non-normal incidence. Starting from the 3×3 transfer matrix for the sector dipole bend magnet in the bending plane,

$$\begin{pmatrix} \cos \Theta & \rho \sin \Theta & \rho(1 - \cos \Theta) \\ -\sin \Theta / \rho & \cos \Theta & \sin \Theta \\ 0 & 0 & 1 \end{pmatrix}$$

derive the 3×3 dispersion transfer matrix for a normal configuration rectangular bend in the bend plane, by assuming that the edge focusing on each side of the rectangular bend may be treated as a thin lens. The total bending angle is denoted by Θ , and you may assume that $\Theta \ll 1$. You may find Wille's discussion helpful.