Homework Problems
Physics 417/517
Due September 15, 2009

1. Evaluate the rest energy (in MeV) for the proton ($m_p = 1.673 \times 10^{-27}$ kg) and the deuteron ($m_d = 1.673 \times 10^{-27}$ kg). Next, assume $\gamma = 2$ for either particle. What is the particle velocity (m/sec)? What are the "kinetic energies" (the total energy minus the rest energy) for both particles in MeV. Repeat for $\gamma = 10$.

2. Suppose a particle orbit as a function of time $\vec{x}(t)$ is given in an inertial frame *K* and the integral
$$\tau = \int_{t_1}^{t_2} \frac{dt}{\gamma(t)} = \int_{t_1}^{t_2} \sqrt{1 - \beta_x^2(t) - \beta_y^2(t) - \beta_z^2(t)}\, dt$$
is evaluated. Using the invariance of the space-time interval, show that if the same calculation is done in a frame *K'* moving uniformly with respect to *K*, then $\tau' = \tau$. In other words, the proper time τ along a particle orbit is a Lorentz invariant quantity.

3. Verify that the relativistic force law $\frac{dp^\alpha}{d\tau} = qF^\alpha{}_\nu u^\nu$ (ν summation implied), where
$$F^\alpha{}_\nu = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix},$$
yields the relativistic Lorentz force equation
$$\frac{d(\gamma m \vec{\mathrm{v}})}{dt} = q\left(\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right)$$
when evaluated on the space components $\alpha = 1, 2, 3$.

4. Calculate the non-relativistic cyclotron angular frequency and cyclotron frequency of the proton and the deutreron in a 0.3 T magnetic field.

5. Repeat, using the relativistic equations of motion, the deriviation in class of the cyclotron frequency. Show the relativistic cyclotron angular frequency is
$$\Omega_c = \frac{qB}{\gamma m}.$$
Show the radius of the cyclotron motion
$$r = \frac{\beta c}{qB/\gamma m}.$$