Physics 417/517 Introduction to Particle Accelerator Physics

G. A. Krafft
Jefferson Lab
Jefferson Lab Professor of Physics
Old Dominion University

Energy Units

- When a particle is accelerated, i.e., its energy is changed by an electromagnetic field, it must have fallen through an Electric Field (we show later by very general arguments that Magnetic Fields cannot change particle energy). For electrostatic accelerating fields the energy change is

$$\Delta E = q\Delta\Phi = q\left(\Phi_a - \Phi_b\right)$$

q charge, Φ, the electrostatic potentials before and after the motion through the electric field. Therefore, particle energy can be conveniently expressed in units of the "equivalent" electrostatic potential change needed to accelerate the particle to the given energy. Definition: 1 eV, or 1 electron volt, is the energy acquired by 1 electron falling through a one volt potential difference.

Energy Units

$$1\ \text{eV} = 1.6\times10^{-19}\ \text{C} \times 1\ \text{V} = 1.6\times10^{-19}\ \text{J}$$

$$1\ \text{MeV} = 10^{6}\ \text{eV} = 1.6\times10^{-13}\ \text{J}$$

To convert rest mass to eV use Einstein relation

$$E_0 = mc^2$$

where m is the rest mass. For electrons

$$E_{electron,0} = 9.1\times10^{-31}\ \text{kg}\left(3\times10^{8}\ \text{m/sec}\right)^2 = 81.9\times10^{-15}\ \text{J}$$

$$= 0.512\ \text{MeV}$$

Recent "best fit" value 0.51099906 MeV

Some Needed Relativity

Following Maxwell Equations, which exhibit this symmetry, assume all Laws of Physics must be of form to guarantee the invariance of the space-time interval

$$(ct')^2 - x'^2 - y'^2 - z'^2 = (ct)^2 - x^2 - y^2 - z^2$$

Coordinate transformations that leave interval unchanged are the usual rotations and Lorentz Transformations, e.g. the *z* boost

$$ct' = \gamma(ct - \beta z)$$

$$x' = x$$

$$y' = y$$

$$z' = \gamma(z - \beta ct)$$

Relativistic Factors

where, following Einstein define the relativistic factors

$$\vec{\beta} = \frac{\vec{v}}{c} \qquad \beta = \frac{|v|}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Easy way to accomplish task of defining a *Relativistic Mechanics*: write all laws of physics in terms of 4-vectors and 4-tensors, i.e., quantities that transform under Lorentz transformations in the same way as the coordinate differentials.

Four-vectors

Four-vector transformation under z boost Lorentz Transformation

$$v^{0\,\prime} = \gamma\left(v^0 - \beta v^3\right)$$

$$v^{1\,\prime} = v^1$$

$$v^{2\,\prime} = v^2$$

$$v^{3\,\prime} = \gamma\left(v^3 - \beta v^0\right)$$

Important example: Four-velocity. Note that interval

$$d\tau \equiv \sqrt{1-\beta^2}\,dt$$

Lorentz invariant. So the following is a 4-vector

$$cu^\alpha \equiv \left(\frac{dct}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right) = c\gamma\left(1, \beta_x, \beta_y, \beta_z\right)$$

4-Momentum

Single particle mechanics must be defined in terms of Four-momentum

$$p^{\alpha} \equiv mcu^{\alpha} = mc\gamma\left(1, \beta_x, \beta_y, \beta_z\right)$$

Norms, which must be Lorentz invariant, are

$$\sqrt{u_{\alpha}u^{\alpha}} \equiv 1, \sqrt{p_{\alpha}p^{\alpha}} \equiv mc$$

What happens to Newton's Law $\vec{F} = m\vec{a} = d\vec{p}/dt$?

$$\frac{dp^{\alpha}}{d\tau} \equiv F^{\alpha}$$

But need a Four-force on the RHS!!!

Electromagnetic (Lorentz Force)

Non-relativistic

$$\vec{F} = q\left(\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right)$$

Relativistic Generalization (ν summation implied)

$$F^{\alpha} = qF^{\alpha}{}_{\nu}u^{\nu}$$

Electromagnetic Field

$$F^{\alpha}{}_{\nu} \equiv \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & cB_z & -cB_y \\ E_y & -cB_z & 0 & cB_x \\ E_z & cB_y & -cB_x & 0 \end{pmatrix}$$

Relativistic Mechanics in E-M Field

Energy Exchange Equation (Note: no magnetic field!)

$$\frac{d\gamma}{dt} = \frac{q\vec{E}\cdot\vec{\mathrm{v}}}{mc^2}$$

Relativistic Lorentz Force Equation (you verify in HW!)

$$\frac{d\left(\gamma m\vec{\mathrm{v}}\right)}{dt} = q\left(\vec{E} + \vec{\mathrm{v}}\times\vec{B}\right)$$

Methods of Acceleration

- Acceleration by Static Electric Fields (DC) Acceleration
  - Cockcroft-Walton
  - van de Graaf Accelerators
  - Limited by voltage breakdowns to potentials of under a million volts in 1930, and presently to potentials of tens of millions of volts (in modern van de Graaf accelerators). Not enough to do nuclear physics at the time.
- Radio Frequency (RF) Acceleration
  - Main means to accelerate in most present day accelerators because one can get to 10-100 MV in a meter these days. Reason: alternating fields don't cause breakdown (if you are careful!) until much higher field levels than DC.
  - Ideas started with Ising and Wideröe

Cockcroft-Walton

Proton Source at Fermilab, Beam Energy 750 keV

van de Graaf Accelerator

Generator

Brookhaven
Tandem
van de Graaf
~ 15 MV

Tandem trick multiplies
the output energy