Physics 417/517 Introduction to Particle Accelerator Physics

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Insertion Devices

Often periodic magnetic field magnets are placed in beam path of high energy storage rings. The radiation generated by electrons passing through such insertion devices has unique properties.

Field of the insertion device magnet

$$\vec{B}(x,y,z) = B(z)\hat{y} \qquad B(z) \approx B_0 \cos\left(2\pi z / \lambda_{ID}\right)$$

Vector potential for magnet (1 dimensional approximation)

$$\vec{A}(x,y,z) = A(z)\hat{x} \qquad A(z) \approx \frac{B_0 \lambda_{ID}}{2\pi} \sin\left(2\pi z / \lambda_{ID}\right)$$

Electron Orbit

Uniformity in x-direction means that canonical momentum in the x-direction is conserved

$$\mathrm{v}_x(z) = \frac{eA(z)}{\gamma m} = \frac{K}{\gamma} c \sin\left(2\pi z / \lambda_{ID}\right)$$

$$x(z) = \int \frac{\mathrm{v}_x}{\mathrm{v}_z} dz \approx -\frac{1}{\langle \beta_z \rangle} \frac{K}{\gamma} \frac{\lambda_{ID}}{2\pi} \cos\left(2\pi z / \lambda_{ID}\right)$$

Field Strength Parameter

$$K \equiv \frac{eB_0 \lambda_{ID}}{2\pi mc}$$

Average Velocity

Energy conservation gives that γ is a constant of the motion

$$\beta_z(z) = \sqrt{1 - \frac{1}{\gamma^2} - \beta_x^2(z)}$$

Average longitudinal velocity in the insertion device is

$$\beta^{*2} = \langle \beta_z \rangle^2 = 1 - \frac{1}{\gamma^2} - \frac{K^2}{2\gamma^2}$$

Average rest frame has

$$\gamma^{*2} = \frac{1}{1 - \beta^{*2}} = \frac{\gamma^2}{1 + K^2/2}$$

Relativistic Kinematics

In average rest frame the insertion device is Lorentz contracted, and so its wavelength is

$$\lambda^* = \lambda_{ID} / \beta^* \gamma^*$$

The sinusoidal wiggling motion emits with angular frequency

$$\omega^* = 2\pi c / \lambda^*$$

Lorentz transformation formulas for the wave vector

$$k^* = \gamma^* k\left(1 - \beta^* \cos\theta\right)$$
$$k_x^* = k_x = k \sin\theta \cos\varphi$$
$$k_y^* = k_y = k \sin\theta \sin\varphi$$
$$k_z^* = \gamma^* k\left(\cos\theta - \beta^*\right)$$

Insertion Device (FEL) Resonance Condition

Angle transforms as

$$\cos\theta^* = \frac{k_z^*}{k^*} = \frac{\left(\cos\theta - \beta^*\right)}{\left(1 - \beta^* \cos\theta\right)}$$

Wave vector in lab frame has

$$k = \frac{k^*}{\gamma^*\left(1 - \beta^* \cos\theta\right)} = \frac{2\pi\beta^* c}{\lambda_{ID}\left(1 - \beta^* \cos\theta\right)}$$

In the forward direction cos θ = 1

$$\lambda_e \approx \frac{\lambda_{ID}}{2\gamma^{*2}} = \frac{\lambda_{ID}}{2\gamma^2}\left(1 + K^2/2\right)$$

Power Emitted Lab Frame

Larmor/Lienard calculation in the lab frame yields

$$\langle P \rangle = \frac{e^2}{6\pi\varepsilon_0}\gamma^4\beta^{*2}c\left(\frac{K}{\gamma}\right)^2\left(\frac{2\pi}{\lambda_{ID}}\right)^2\frac{1}{2}$$

Total energy radiated after one passage of the insertion device

$$\delta E = 2\pi^2\frac{e^2}{6\pi\varepsilon_0\lambda_{ID}}\gamma^2\beta^* NK^2$$

Power Emitted Beam Frame

Larmor/Lienard calculation in the beam frame yields

$$\left\langle P^* \right\rangle = \frac{2e^2}{6\pi\varepsilon_0} cK^2 \left(\frac{2\pi}{\lambda^*} \right)^2 \frac{1}{2}$$

Total energy of each photon is $\hbar 2\pi c/\lambda^*$, therefore the total number of photons radiated after one passage of the insertion device

$$N_\gamma = \frac{2\pi}{3} \alpha N K^2$$