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Introduction to Particle Accelerator Physics

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Radiation Power Distribution

Consulting your favorite Classical E&M text (Jackson, Schwinger, Landau and Lifshitz *Classical Theory of Fields*), and quoted in Chapter 2 of Wille

$$\frac{dP}{d\omega} = \frac{\sqrt{3}}{8\pi^2\varepsilon_0}\frac{e^2}{\rho}\gamma\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)dx$$

Critical Frequency

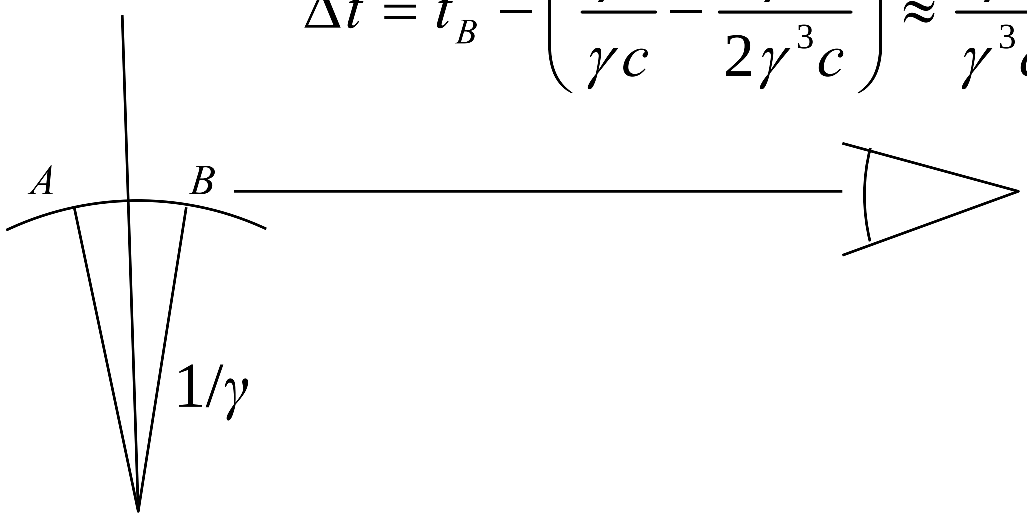
Critical (angular) frequency is

$$\omega_c = \frac{3}{2}\gamma^3 \frac{c}{\rho}$$

Energy scaling of critical frequency is understood from $1/\gamma$ emission cone and fact that $1 - \beta \sim 1/(2\gamma^2)$

$$\Delta t = t_B - \left(\frac{\rho}{\gamma c} - \frac{\rho}{2\gamma^3 c} \right) \approx \frac{\rho}{\gamma^3 c}$$

$$t_A = -\frac{\rho}{\gamma\beta c}$$



$$t_B = \frac{\rho}{\gamma\beta c} \approx \frac{\rho}{\gamma c} + \frac{\rho}{2\gamma^3 c}$$

Photon Number

$$P = \int_0^\infty \frac{dP}{d\omega} d\omega = \frac{\sqrt{3}}{8\pi^2 \varepsilon_0} \frac{e^2}{\rho} \omega_c \gamma \int_0^\infty \xi \int_\xi^\infty K_{5/3}(x)\, dx\, d\xi = \frac{e^2 c}{6\pi\varepsilon_0 \rho^2} \gamma^4$$

$$\frac{d\dot{n}}{d\omega} = \frac{1}{\hbar\omega} \frac{dP}{d\omega}$$

$$\langle \hbar\omega \rangle = \frac{\int_0^\infty \hbar\omega \frac{d\dot{n}}{d\omega} d\omega}{\int_0^\infty \frac{d\dot{n}}{d\omega} d\omega} = \frac{8}{15\sqrt{3}} \hbar\omega_c$$

$$\dot{n} = \frac{5\alpha}{2\sqrt{3}} \frac{c}{\rho} \gamma \qquad \delta n = \frac{5\alpha}{2\sqrt{3}} \Theta \gamma \qquad \alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}$$

Insertion Devices

Often periodic magnetic field magnets are placed in beam path of high energy storage rings. The radiation generated by electrons passing through such insertion devices has unique properties.

Field of the insertion device magnet

$$\vec{B}(x,y,z) = B(z)\hat{y} \qquad B(z) \approx B_0 \cos(2\pi z/\lambda_{ID})$$

Vector potential for magnet (1 dimensional approximation)

$$\vec{A}(x,y,z) = A(z)\hat{x} \qquad A(z) \approx \frac{B_0 \lambda_{ID}}{2\pi} \sin(2\pi z/\lambda_{ID})$$

Electron Orbit

Uniformity in x-direction means that canonical momentum in the x-direction is conserved

$$\mathrm{v}_x\left(z\right)=\frac{eA\left(z\right)}{\gamma m}=\frac{K}{\gamma}c\sin\left(2\pi z/\lambda_{ID}\right)$$

$$x\left(z\right)=\int\frac{\mathrm{v}_x}{\mathrm{v}_z}dz\approx-\frac{1}{\left\langle\beta_z\right\rangle}\frac{K}{\gamma}\frac{\lambda_{ID}}{2\pi}\cos\left(2\pi z/\lambda_{ID}\right)$$

Field Strength Parameter

$$K\equiv\frac{eB_0\lambda_{ID}}{2\pi mc}$$