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# Introduction to Particle Accelerator Physics

G. A. Krafft
Jefferson Lab
Jefferson Lab Professor of Physics
Old Dominion University

# Synchrotron Radiation

Accelerated particles emit electromagnetic radiation. Emission from very high energy particles has unique properties for a radiation source. As such radiation was first observed at one of the earliest electron synchrotrons, radiation from high energy particles (mainly electrons) is known generically as synchrotron radiation by the accelerator and HENP communities.

The radiation is highly collimated in the beam direction

From relativity

$$ct' = \gamma ct - \gamma\beta z$$
$$x' = x$$
$$y' = y$$
$$z' = -\gamma\beta ct + \gamma z$$

Lorentz invariance of wave phase implies $k^\mu = (\omega/c, k_x, k_y, k_z)$ is a Lorentz 4-vector

$$\omega' = \gamma\omega - \gamma\beta k_z c$$
$$k'_x = k_x$$
$$k'_y = k_y$$
$$k'_z = -\gamma\beta c\omega + \gamma k_z$$

$$\sin\theta = \frac{\sqrt{k_x^2 + k_y^2}}{\omega / c} \qquad \sin\theta' = \frac{\sqrt{k_x'^2 + k_y'^2}}{\omega' / c} \qquad \cos\theta' = \frac{k'_z}{\omega' / c}$$

$$\omega / c = \gamma\omega' / c + \gamma\beta k'_z = \gamma\left(1 + \beta\cos\theta'\right)\left(\omega' / c\right)$$

$$\theta \approx \sin\theta = \frac{\sin\theta'}{\gamma\left(1+\beta\cos\theta'\right)}$$

Therefore all radiation with $\theta' < \pi / 2$, which is roughly ½ of the emission for dipole emission from a transverse acceleration in the beam frame, is Lorentz transformed into an angle less than $1/\gamma$. Because of the strong Doppler shift of the photon energy, higher for $\theta \to 0$, most of the energy in the photons is within a cone of angular extent $1/\gamma$ around the beam direction.

# Larmor's Formula

For a particle executing non-relativistic motion, the total power emitted in electromagnetic radiation is (Larmor)

$$P(t) = \frac{1}{6\pi\varepsilon_0}\frac{q^2}{c^3}\left|\vec{a}\right|^2 = \frac{1}{6\pi\varepsilon_0}\frac{e^2}{m^2c^3}\left|\dot{\vec{p}}\right|^2$$

Lienard's relativistic generalization: Note both *dE* and *dt* are the fourth component of relativistic 4-vectors when one is dealing with photon emission. Therefore, their ratio must be an Lorentz invariant. The invariant that reduces to Larmor's formula in the non-relativistic limit is

$$P = -\frac{e^2}{6\pi\varepsilon_0 c}\frac{du^\mu}{d\tau}\frac{du_\mu}{d\tau}$$

$$P(t)=\frac{e^2}{6\pi\varepsilon_0 c}\gamma^6\left(\dot{\vec{\beta}}^2-\left[\vec{\beta}\times\dot{\vec{\beta}}\right]^2\right)$$

For acceleration along a line, second term is zero and first term for the radiation reaction is small compared to the acceleration as long as gradient less than $10^{14}$ MV/m. Technically impossible.

For transverse bend acceleration $\dot{\vec{\beta}}=-\frac{\beta^2 c}{\rho}\hat{r}$

$$P(t)=\frac{e^2 c}{6\pi\varepsilon_0\rho^2}\beta^4\gamma^4$$

# Fractional Energy Loss

$$\delta E = \frac{e^2}{6\pi\varepsilon_0\rho}\Theta\beta^3\gamma^4$$

For one turn with isomagnetic bending fields

$$\frac{\delta E}{E_{beam}} = \frac{4\pi r_e}{3\rho}\beta^3\gamma^3$$

$r_e$ is the classical electron radius: $2.82\times10^{-13}$ cm

# Radiation Power Distribution

Consulting your favorite Classical E&M text (Jackson, Schwinger, Landau and Lifshitz Classical Theory of Fields)

$$\frac{dP}{d\omega} = \frac{\sqrt{3}}{8\pi^2\varepsilon_0}\frac{e^2}{\rho}\gamma\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)dx$$

# Critical Frequency

Critical (angular) frequency is

$$\omega_c = \frac{3}{2}\gamma^3 \frac{c}{\rho}$$

Energy scaling of critical frequency is understood from $1/\gamma$ emission cone and fact that $1 - \beta \sim 1/(2\gamma^2)$

$$\Delta t = t_B - \left( \frac{\rho}{\gamma c} - \frac{\rho}{2\gamma^3 c} \right) \approx \frac{\rho}{\gamma^3 c}$$

$$t_A = -\frac{\rho}{\gamma\beta c}$$

$$t_B = \frac{\rho}{\gamma\beta c} \approx \frac{\rho}{\gamma c} + \frac{\rho}{2\gamma^3 c}$$