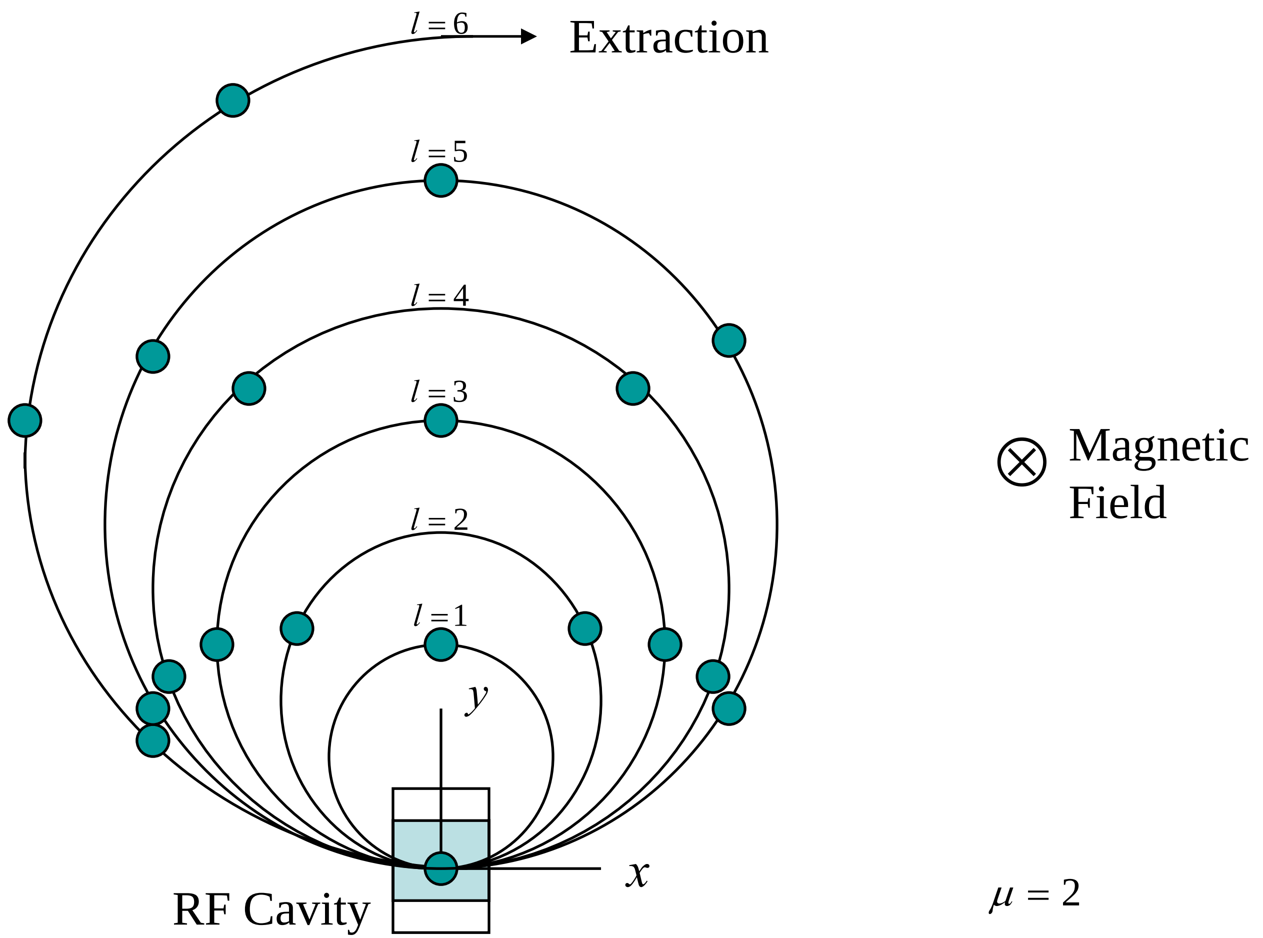
Physics 417/517 Introduction to Particle Accelerator Physics

G. A. Krafft
Jefferson Lab
Jefferson Lab Professor of Physics
Old Dominion University

Classical Microtron: Veksler (1945)



$\mu = 2$

$\nu = 1$

Phase Stability Condition

"Synchronous" electron has

$$\text{Phase} = \phi_s \qquad\qquad E_l = E_o + leV_c \cos\phi_s$$

Difference equation for differences after passing through cavity pass $l + 1$:

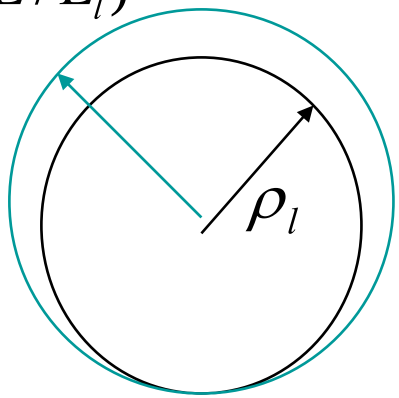
$$\begin{pmatrix} \Delta\phi_{l+1} \\ \Delta E_{l+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -eV_c \sin\phi_s & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{2\pi M_{56}}{\lambda E_l} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

Because for an electron passing the cavity

$$\Delta E_{after} = \Delta E_{before} + eV_c\left(\cos(\phi_s + \Delta\phi) - \cos\phi_s\right)$$

Phase Stability Condition

$\rho_l(1+\Delta E/E_l)$



$$K_i = 1/\rho_i^2$$

$$D_{x,p,0} = \rho_i\left(1-\cos\left(s/\rho_i\right)\right) \qquad 0 \le s \le 2\pi\rho_i$$

$$\therefore\ M_{56} = \int \frac{D}{\rho} ds = \int_0^{2\pi\rho_l} \left(1-\cos s/\rho_l\right) ds$$

$$= 2\pi\rho_l$$

$$\begin{pmatrix} \Delta\phi_{l+1} \\ \Delta E_{l+1} \end{pmatrix} \approx \begin{pmatrix} 1 & \dfrac{4\pi^2\rho_l}{\lambda E_l} \\ -eV_c \sin\phi_s & 1-\dfrac{4\pi^2\rho_l eV_c}{\lambda E_l}\sin\phi_s \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

Phase Stability Condition

Have Phase Stability if

$$-1 < \left(\frac{\mathrm{Tr}\, M}{2} \right) < 1 \rightarrow -1 < 1 - \frac{2\pi^2 \rho_l e V_c}{\lambda E_l} \sin\phi_s < 1$$

$$\frac{2\pi^2 \rho_l e V_c}{\lambda E_l} \sin\phi_s = \frac{\pi f_{RF} e V_c}{f_c m c^2} \cos\phi_s \tan\phi_s = \frac{\pi f_{RF} \Delta\gamma}{f_c} \tan\phi_s$$

i.e.,

$$\boxed{0 < \nu\pi \tan\phi_s < 2}$$

Phase Stability Condition

Have Phase Stability if

$$\left(\frac{\mathrm{Tr}\, M}{2}\right)^2 < 1$$

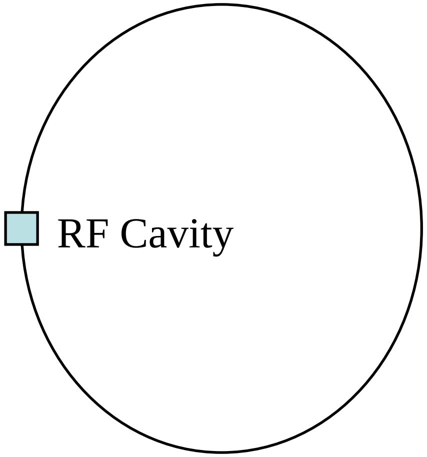
i.e.,

$$\boxed{0 < \nu\pi \tan\phi_s < 2}$$

Synchrotrons

Two basic generalizations needed

- Acceleration of non-relativistic particles
- Difference equation describing per turn dynamics becomes a differential equation with solution involving a new frequency, the synchrotron frequency



Acceleration of non-relativistic particles

For microtron, racetrack microtron and other polytrons, electron speed is at the speed of light. For non-relativistic particles the recirculation time also depends on the longitudinal velocity $v_z = \beta_z c$.

$$t_{recirc} = L / \beta_z c$$

$$\Delta t = \frac{\partial L}{\partial p}\frac{\Delta p}{\beta_z c} + \frac{L}{c}\frac{\partial}{\partial \beta_z}\frac{1}{\beta_z}\Delta\beta_z$$

$$\frac{\Delta t}{t_{recirc}} = \frac{M_{56}}{L}\frac{\Delta p}{p} - \frac{\Delta\beta_z}{\beta_z} = \frac{M_{56}}{L}\frac{\Delta p}{p} - \frac{1}{\gamma^2}\frac{\Delta p}{p}$$

Momentum Compaction $\alpha = (\Delta L / L)/(\Delta p / p) = M_{56} / L$

$$\frac{\Delta t}{t_{recirc}} = -\eta_c \frac{\Delta p}{p} \rightarrow \eta_c = \frac{1}{\gamma^2} - \frac{M_{56}}{L} = \frac{1}{\gamma^2} - \alpha$$

$$2 p \Delta p c^2 = 2 E \Delta E \rightarrow \frac{\Delta p}{p} = \frac{1}{\beta_z^2} \frac{\Delta E}{E} \rightarrow \frac{\Delta t}{t_{recirc}} = -\frac{\eta_c}{\beta_z^2} \frac{\Delta E}{E}$$

Transition Energy: Energy at which the change in the once around time becomes independent of momentum (energy)

$$\eta_c = 0 \rightarrow \frac{1}{\gamma_t^2} = \frac{M_{56}}{L} = \alpha$$

No Phase Focusing at this energy!

Equation for Synchrotron Oscillations

$$\begin{pmatrix} \Delta\phi_{l+1} \\ \Delta E_{l+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -eV_c \sin\phi_s & 1 \end{pmatrix} \begin{pmatrix} 1 & -\dfrac{2\pi L \eta_c}{\lambda \beta_z^2 E_l} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -\dfrac{2\pi L \eta_c}{\lambda \beta_z^2 E_l} \\ -eV_c \sin\phi_s & 1 + \dfrac{2\pi L \eta_c}{\lambda \beta_z^2 E_l} eV_c \sin\phi_s \end{pmatrix} \begin{pmatrix} \Delta\phi_l \\ \Delta E_l \end{pmatrix}$$

Assume momentum slowly changing (adiabatic acceleration)
Phase advance per turn is

$$\cos\Delta\mu = 1 + \frac{\pi L \eta_c}{\lambda \beta_z^2 E_l} eV_c \sin\phi_s \rightarrow \Delta\mu^2 \approx -\frac{2\pi L \eta_c}{\lambda \beta_z^2 E_l} eV_c \sin\phi_s$$

So change in phase per unit time is

$$\frac{\Delta\mu}{T_0} \approx \frac{1}{T_0}\sqrt{-\frac{2\pi L\eta_c}{\lambda\beta_z pc} eV_c \sin\phi_s}$$

yielding synchrotron oscillations with frequency

$$\omega_s = \omega_{rev}\sqrt{-\frac{h\eta_c}{2\pi}\frac{eV_c}{pc}\sin\phi_s}$$

where the *harmonic number* $h = L / \beta_z \lambda$, gives the integer number of *RF* oscillations in one turn

Phase Stable Acceleration

At energies below transition, $\eta_c > 0$. To achieve acceleration with phase stability need $\phi_s < 0$

$$\therefore \omega_s = \omega_{rev} \sqrt{\frac{h\eta_c}{2\pi} \frac{eV_c}{pc} \sin\left(-\phi_s\right)}$$

At energies above transition, $\eta_c < 0$, which corresponds to the case we're used to from electrons. To achieve acceleration with phase stability need $\phi_s > 0$

$$\therefore \omega_s = \omega_{rev} \sqrt{\frac{h\left(-\eta_c\right)}{2\pi} \frac{eV_c}{pc} \sin\phi_s}$$