Final Examination
Physics 417/517
Due Friday, December 7, 2007

1. We have used the Advanced Photon Source (APS) at Argonne as an example in many of the homework problems. As was done in the midterm for LHC, let's investigate the parameters of the machine. The APS stores a nominal 100 mA beam current at the design energy of 7 GeV, or 7000 MeV.
   a. What is the relativistic γ of the electrons? What is their relativistic β?
   b. What is their magnetic rigidity?
   c. There are 81 dipole magnets of length 3.06 m, bending in normal configuration. What is the required dipole magnetic field?
   d. What is the bend radius when the particles are in the dipoles?
   e. If the machine circumference is 1104.00 m, what is the revolution frequency?
   f. The RF frequency is reported to be 351.9 MHz. What is the harmonic number to four significant digits?
   g. What is the energy loss per electron per turn from radiation in the bend dipoles?
   h. The total energy gain per pass available from the RF system is 9.5 MV. What must the synchronous phase be in order to have energy equilibrium in the storage ring?
   i. The momentum compaction factor is 2.82×10^{-4}. What is the transition energy?
   j. What is the frequency slip factor η_c at 7 GeV?
   k. What is the small amplitude synchrotron oscillation frequency at the synchronous phase calculated in h?
   l. What is the total power radiated by all the electrons in the bends?

   Note: on part i, use the definition of momentum compaction in the 10/23 lecture. This slide was updated after some of you saw it the first time, to be consistent with Wille.

2. Summarize the properties of bend radiation. In this problem you may assume that the power frequency distribution of bend radiation

$$\frac{dP}{d\omega} = \frac{\sqrt{3}}{8\pi^2\varepsilon_0}\frac{e^2}{\rho}\gamma\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)\,dx$$

   is known.
   a. Derive the total power formula

$$P(t) = \frac{e^2 c}{6\pi\varepsilon_0\rho^2}\beta^4\gamma^4$$

   from the Lienard formula

$$P(t) = \frac{e^2}{6\pi\varepsilon_0 c}\gamma^6\left(\dot{\vec{\beta}}^2 - \left[\vec{\beta}\times\dot{\vec{\beta}}\right]^2\right).$$

b. Show, by integrating the power frequency distribution over all frequencies, that the total power is consistent with the result from part a when $\beta \approx 1$.

c. Calculate the average energy of emitted photons photons in terms of the critical energy $\hbar\omega_c$.

d. Calculate the *rms* spread of the emitted photons in terms of the critical energy $\hbar\omega_c$, where the spread is defined to be $\sqrt{\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2}$ and where E_γ is the energy of the emitted photons.

You need the Bessel function integrals from Homeworks 4 and 5 to complete this problem.

3. The following matrix is unimodular

$$M = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}.$$

a. Find the two eigenvalues of the matrix. Call them λ_+ and λ_-.

b. Are the two eigenvalues real, or do they have some imaginary components?

c. Show $\lambda_+\lambda_- = 1$.

d. What is the phase advance of this matrix?

e. What are the $\alpha, \beta,$ and γ (Twiss parameters) for this matrix?

f. What is M^{100}, i. e., what are the individual matrix elements of the one hundredth power of *M*?

g. What is the one hundredth power of ξM where ξ is any real number, expressed in terms of the matrix found in f and powers of ξ?

h. What condition on $|\xi|$ guarantees that ξM, which is not necessarily unimodular, is stable, i.e., its matrix power remains bounded as the power increases?

4. Recall our expressions from the homework for the transfer matrix of a FODO system. The one-period transfer matrix starting with the middle of the focusing magnet was

$$M_f = \begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - L^2/(2f^2) & 2L + L^2/f \\ -L/(2f^2) + L^2/(4f^3) & 1 - L^2/(2f^2) \end{pmatrix}.$$

and M_d is obtained from M_f by replacing f with $-f$

a. How should one choose *f*, in terms of *L*, so that the phase advance through one period of the FODO system is 45 degrees (corresponding to 1/8 of a transverse oscillation per period)?

b. For the matched phase space (x, x') ellipse in the 45 degree phase advance system, what is the beta-function in the middle of the focusing lens?
c. For the matched phase space ellipse in the 45 degree phase advance system, what is the beta-function in the middle of the defocusing lens?
d. What are the alpha-functions for the matched ellipses at these same two locations?
e. Suppose L is 3 m, and the area of a matched phase space ellipse is $\pi\varepsilon = 2\pi \times 10^{-6}$ m radian, what are the maximum extents ($x_{\max}$) of the matched ellipses in the focusing and defocusing lenses of the 45 degrees phase advance system?

5. The Parabolic-elliptical phase space distribution is

$$\rho(x,x') = A\left(1-\left(\gamma x^2 + 2\alpha x x' + \beta x'^2\right)/\varepsilon\right)\Theta\left(1-\left(\gamma x^2 + 2\alpha x x' + \beta x'^2\right)/\varepsilon\right)$$

assuming $\beta\gamma - \alpha^2 = 1$.

a. Normalize the distribution.
b. Calculate ε_{rms}.
c. Note the statistical average definitions of $\alpha, \beta,$ and γ evaluate to exactly the correct values for this distribution.

Hint: make a variable change to $y = \beta x' + \alpha x$. The distribution function is now proportional to

$$\rho(x,y) \propto \left(1-\frac{x^2}{\varepsilon\beta}-\frac{y^2}{\varepsilon\beta}\right)\Theta\left(1-\frac{x^2}{\varepsilon\beta}-\frac{y^2}{\varepsilon\beta}\right).$$

At least for the distribution normalization, the required two dimensional integrals are one's we've already evaluated in the discussion of uniform elliptical distributions!