Touschek Scattering

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Beam Lifetime

The beam lifetime in electron ring is limited by a number of factors

- The quantum lifetime τ_q
- Scattering (elastic or inelastic) of electrons by the residual gas atoms τ_g
- Touschek scattering τ_{Touschek}
- Trapped ions in electron beam potential τ_{ion}

The total lifetime is given by

$$\frac{1}{\tau_t} = \frac{1}{\tau_q} + \frac{1}{\tau_g} + \frac{1}{\tau_{\text{Touschek}}} + \frac{1}{\tau_{\text{ion}}}. \tag{1}$$

Touschek Scattering

Touschek Effect

- Particle loss within a bunch due to a single particle-particle collision
- Momentum transfer from transverse into longitudinal phase space causes the scattered particle to cross the momentum aperture of the lattice

Touschek effect is of fundamental importance for operation of collider rings accelerators
$-- >$ Beam lifetime
$-- >$ Beam stability (as the scattered electron will be coasting, *i.e*, cross the stable boundary of the RF-bucket)

Computing Touschek Lifetime

Touschek lifetime: the time needed for beam intensity to decay to half its initial value.
For initial number of particles N_0, the reduced number after time t is given by

$$N(t) = \frac{N_0}{1 + t/\tau} \tag{2}$$

Touschek Lifetime

Flat beam case

$$\frac{1}{\tau} = \frac{N r_0^2 c}{8\pi\gamma^3\sigma_s L}\int_L \frac{D(\xi)ds}{\sigma_x(s)\sigma_y(s)\sigma_{x'}(s)\delta_{acc}^2(s)} \tag{3}$$

r_0 is the classical electron radius
c the speed of light
N the number of particles per bunch
L machine path length
σ_s rms bunch length
$\sigma_{x,y}(s)$ beam size

$$D(\xi) = \int_0^1 \left(\frac{1}{u} - \frac{1}{2}\ln\frac{1}{u} - 1\right) e^{-\xi/u} du \tag{4}$$

$$\xi(s) = \left(\frac{\delta_{acc}(s)}{\gamma\sigma_{x'}(s)}\right)^2 \tag{5}$$

Lattice Functions

Horizontal beam sizes for horizontal bending lattice

$$\sigma_x(s) = \sqrt{\varepsilon_x \beta_x(s) + (\sigma_{\delta_p} \eta_x(s))^2} \quad (6)$$

$$\sigma_y(s) = \sqrt{\varepsilon_y \beta_y(s)} \quad (7)$$

$\varepsilon_{x,y}$ Horizontal (vertical) emittance
η_x, η'_x horizontal dispersion and its derivative
σ_{δ_p} rms relative momentum spread
$\sigma_{x'}(s)$ rms horizontal beam divergence for $x \approx 0$

$$\sigma_{x'}(s) = \frac{\varepsilon_x}{\sigma_x(s)} \sqrt{1 + \frac{H(s)\sigma_{\delta_\mathrm{p}}^2}{\varepsilon_x}} \quad (8)$$

Lattice functions

The chromatic invariant

$$H(s) = \gamma_x(s)\eta_x^2(s) + 2\alpha_x(s)\eta_x(s)\eta_x'(s) + \beta_x(s)\left[\eta_x'(s)\right]^2 \quad (9)$$

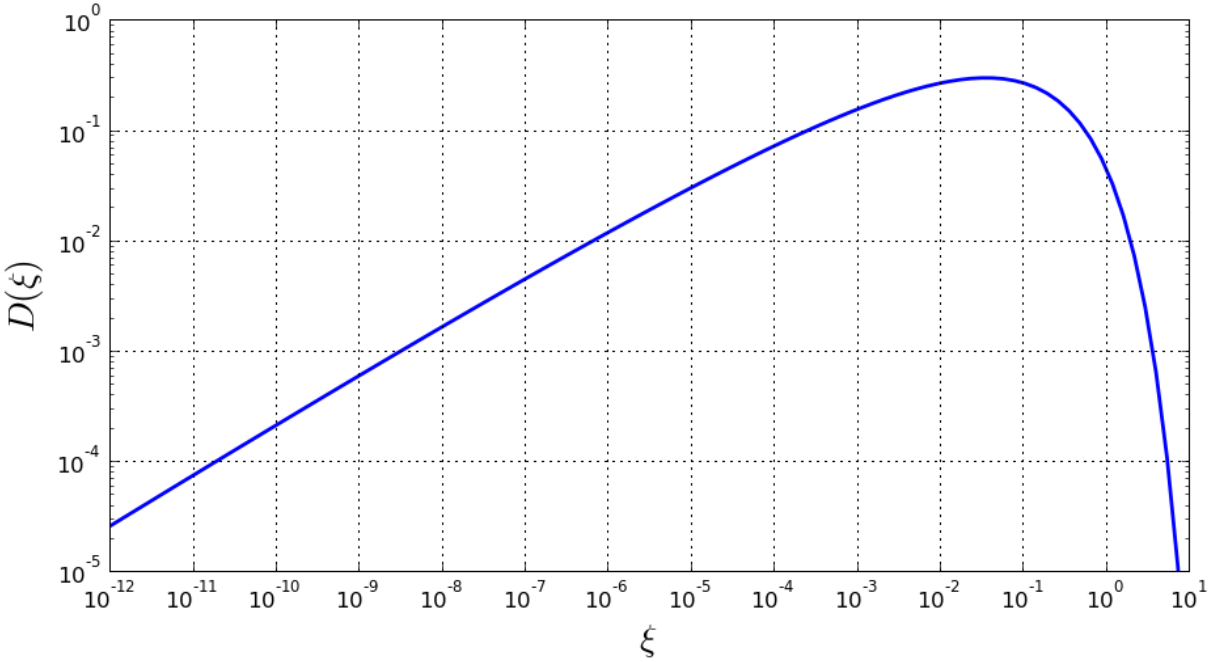
where (10)

$$\gamma_x(s) = \frac{1+\alpha_x^2(s)}{\beta_x(s)}$$

$D(\xi)$ Function

$$\xi(s) = \left(\frac{\delta_{acc}(s)}{\gamma\sigma_{x'}(s)}\right)^2 \tag{11}$$

$$D(\xi) = \int_0^1 \left(\frac{1}{u} - \frac{1}{2}\ln\frac{1}{u} - 1\right) e^{-\xi/u} du \tag{12}$$



Coasting particles

As the particles scattered by Touschek scattering leave the stable RF bucket, they start coasting outside the stable bucket.

N_{coast}

The percentage of particles outside the RF bucket

$$N_{\text{coast}} = \frac{t/\tau}{1 + t/\tau} N_0 \tag{13}$$

Estimating Touschek Lifetime

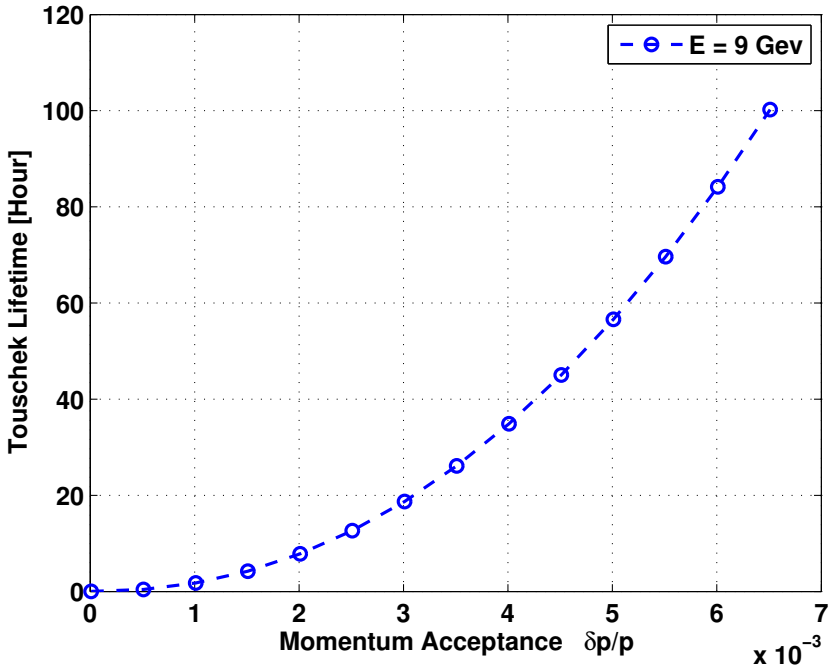
$$\frac{1}{\tau_{\text{Touschek}}} \approx \frac{N r_0^2 c}{8\pi\gamma^3\sigma_s L} \sum_{i=1}^{N} \frac{D(\xi)\Delta s_i}{\sigma_x(s_i)\sigma_y(s_i)\sigma_{x'}(s_i)\delta_{acc}^2(s_i)} \tag{14}$$

$(\beta_{\text{eff}}) = 1/\langle\beta\rangle)$ effective estimate of the β function
$\sigma_{x'} = \sqrt{\varepsilon_x/\beta_{\text{eff}}}$
α, β and η are all assumed constant (therefore yielding constant beam sizes σ_x, σ_y, and divergence $\sigma_{x'}$) in each of the N lattice elements.

Computing Touschek Lifetime

Accurate estimate of the Touschek effect is obtained by estimating the integral in equation (3) as a sum over all N elements in the lattice

We first carry out calculations for the Touschek lifetime with the constant momentum acceptance δ_{acc} in the equation (14)



Touschek lifetime for the 9 GeV MEIC electron ring

Momentum Acceptance

The momentum acceptance of an electron ring is limited by two factors

- The height of the RF bucket
  RF momentum acceptance

$$\delta_{acc}^{RF} = \sqrt{\frac{2U_0\lambda}{\pi E_0 \alpha C}(\cot\phi_s + \phi_s - \frac{\pi}{2})} \tag{15}$$

α Momentum compaction factor
E_o Beam energy
λ RF wavelength

Momentum Acceptance

- In a linear lattice only geometric aperture limits particles excursion
  $-- >$ Dynamic acceptance is infinitely large
- Touschek scattered particle starts its betatron oscillation at the beam core, i.e. approximately at the origin of phase space, with the dispersive orbit as reference, its betatron amplitude after scattering at lattice location $s = 0$ is given by

$$A_x = \gamma_{x0}(\eta_0\delta)^2 + 2\alpha_{x0}(\eta_0\delta)(\eta_0'\delta) + \beta_{x0}(\eta_0'\delta)^2 \tag{16}$$
$$= H_0\delta^2 \tag{17}$$

At another location "s" the particles maximum coordinate

$$x = \sqrt{A_x\beta_{xs}} + \eta_s\delta_s = (\sqrt{H_0\beta_{xs}} + \eta_s)\delta \tag{18}$$

The particle will be lost if $x > a_{xs}$, with a_{xs} the physical aperture (beam pipe half width) at location "s". Thus we find for the lattice MA at location $s = 0$

$$\delta_{acc}(s_0) = \pm \min\left(\frac{a_x(s)}{\sqrt{H(s_0)\beta_x(s)} + |\eta(s)|}\right) \tag{19}$$

$a_x(s)$ is the physical aperture (beam pipe half width) at location s.
"A. Streun - SLS Note 18/97"

Momentum Acceptance

In the case of lattices with high nonlinearities, tracking is needed to determine the transverse acceptance of the lattice.

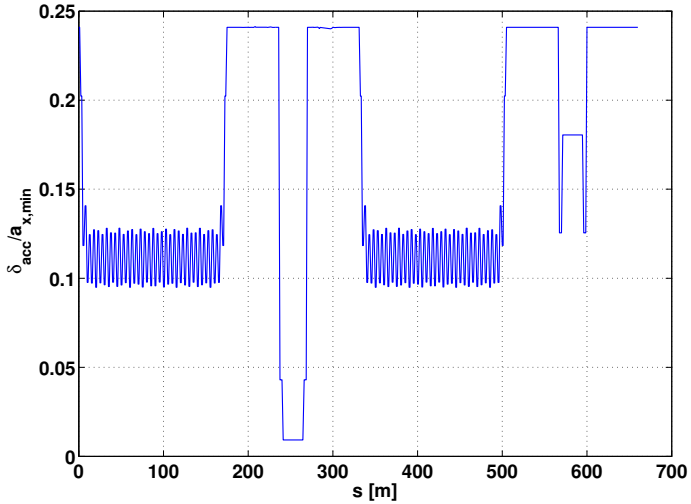


Figure: Normalized (to the minimum beam pipe half width $a_{x,\min}$) transverse dynamic momentum acceptance along MEIC beamline.

It is evident that the momentum acceptance has a constant mean value in the arcs and the extreme values are located in the straight sections: minimum around the interaction region, and maximum in the other parts of the straight.