Accelerator Physics

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Lecture 15

Space Charge and Collective Effects

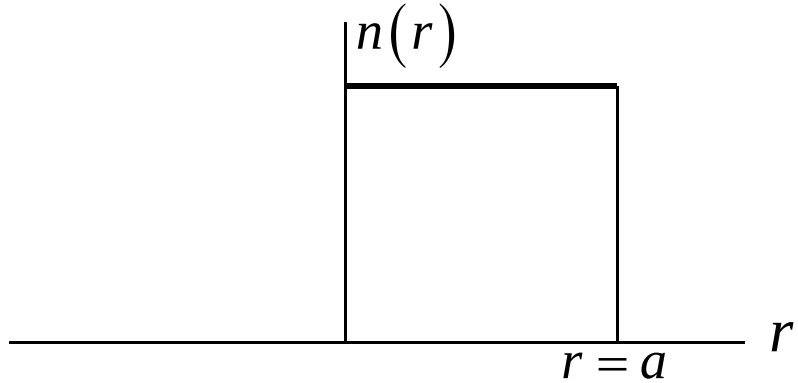
- Collective Effects
  - Brillouin Flow
  - Self-consistent Field
  - KV Equation
  - Bennet Pinch
  - Landau Damping

Simple Problem

- How to account for interactions between particles
- Approach 1: Coulomb sums
  - Use Coulomb's Law to calculate the interaction between each particle in beam
  - Unfavorable N^2 in calculation but perhaps most realistic
  - more and more realistic as computers get better
- Approach 2: Calculate EM field using ME
  - Need procedure to define charge and current densities
  - Track particles in resulting field

Uniform Beam Example

- Assume beam density is uniform and axi-symmetric going into magnetic field



Electric Field

$$\frac{1}{r}\frac{\partial}{\partial r} r E_r = \frac{qn}{\varepsilon_0} \rightarrow E_r = \frac{qn}{2\varepsilon_0} r$$

Self-Magnetic Field by Ampere's Law

$$2\pi r B_\theta = \mu_0 q n \beta c \pi r^2 \rightarrow B_\theta = \mu_0 \frac{qn\beta c}{2} r$$

Brillouin Flow

Total Collective Force on beam particle

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \square \left(\frac{q^2 n}{2\varepsilon_0}\left(1-\beta^2\right)\right) r$$

effective (de)focussing strength

$$k = \frac{\omega_p^2}{2\beta^2 c^2 \gamma^3}$$ where the non-relativistic "plasma frequency" is $\omega_p^2 = \frac{q^2 n}{\varepsilon_0 m}$

By previous work with solenoids in the rotating frame, can have equilibrium (force balance) when

$$\frac{\omega_p^2}{2\gamma} = \Omega_c^2$$ non-relativistic plasma and cyclotron frequencies $\omega_L = \frac{\Omega_c}{2\gamma}$

This state is known as Brillouin Flow and neglects beam temperature (fluid flow)

Comments

- Some authors, Reiser in particular, define a relativistic plasma frequency

$$\omega_p^2 = \frac{q^2 n}{\varepsilon_0 \gamma^3 m}$$

- Lawson's book has a nice discussion about why it is impossible to establish a relativistic Brillouin flow in a device where beam is extracted from a single cathode at an equipotential surface. In this case one needs to have either sheering of the rotation or non-uniform density in the self-consistent solution.

K-V Distribution

- Single value for the transverse Hamiltonian

$$\frac{1}{\varepsilon_x}\left(\frac{x^2+\left(\alpha_x x+\beta_x x'\right)^2}{\beta_x}\right)+\frac{1}{\varepsilon_y}\left(\frac{y^2+\left(\alpha_y y+\beta_y y'\right)^2}{\beta_y}\right)=C$$

$$\psi\left(x,x',y,y'\right)\propto\delta\left(C-1\right)$$

$$\rho\left(z\right)=\frac{I}{\pi\beta cXY}$$

$$K=\frac{I}{I_0}\frac{2}{\beta^3\gamma^3}\qquad I_0=\frac{4\pi\varepsilon_0 mc^3}{q}$$

$$E_x=\frac{I}{\pi\varepsilon_0\beta c}\frac{x}{Y\left(X+Y\right)}$$

K-V Envelope Equation

$$E_x = \frac{I}{\pi\varepsilon_0\beta c}\frac{x}{X\left(X+Y\right)}$$

$$E_y = \frac{I}{\pi\varepsilon_0\beta c}\frac{y}{Y\left(X+Y\right)}$$

$$x'' + k_y x - \frac{2K}{X\left(X+Y\right)}x = 0$$

$$y'' + k_y x - \frac{2K}{Y\left(X+Y\right)}x = 0$$

Envelope Equation

$$X'' + k_x X - \frac{2K}{\left(X+Y\right)} - \frac{\varepsilon_x^2}{X^3} = 0$$

$$Y'' + k_y Y - \frac{2K}{\left(X+Y\right)} - \frac{\varepsilon_y^2}{Y^3} = 0$$