Accelerator Physics

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Lecture 12

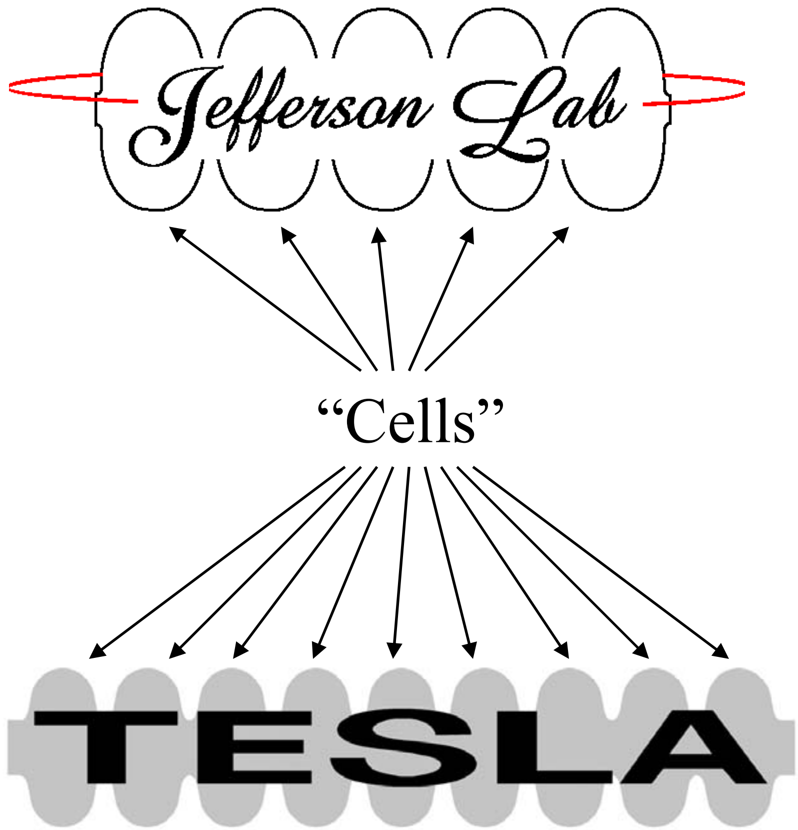
RF Acceleration

- Characterizing Superconducting RF (SRF) Accelerating Structures
  - Terminology
  - Energy Gain, R/Q, Q_0, Q_L and Q_{ext}
- RF Equations and Control
  - Coupling Ports
  - Beam Loading
- RF Focusing
- Betatron Damping and Anti-damping

Terminology

1 CEBAF Cavity

5 Cell Cavity



1 DESY Cavity

9 Cell Cavity

Modern Jefferson Lab Cavities (1.497 GHz) are optimized around a 7 cell design

Typical cell longitudinal dimension: $\lambda_{RF}/2$

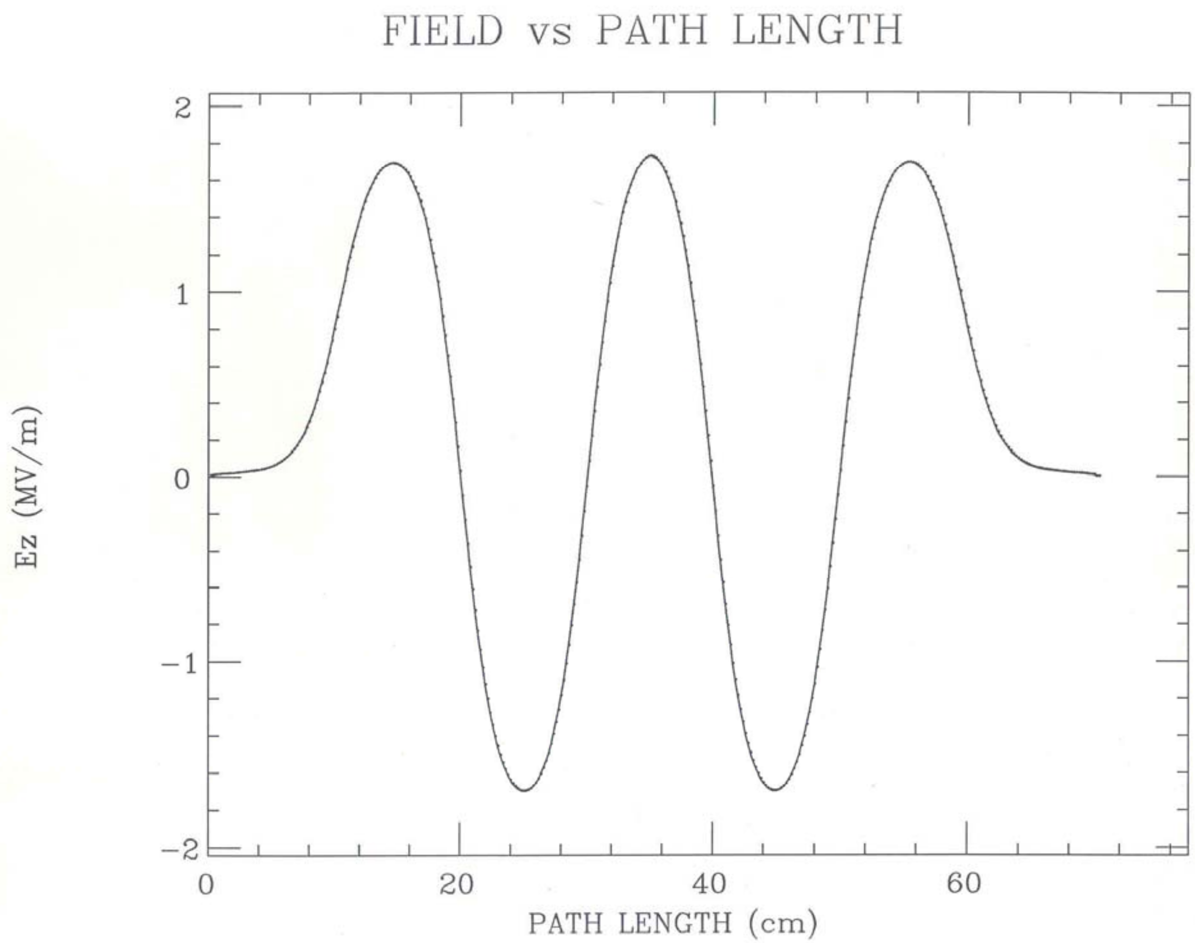
Phase shift between cells: π

Cavities usually have, in addition to the resonant structure in picture:

(1) At least 1 input coupler to feed RF into the structure

(2) Non-fundamental high order mode (HOM) damping

(3) Small output coupler for RF feedback control


FIELD vs PATH LENGTH
Ez (MV/m)
PATH LENGTH (cm)
2
1
0
−1
−2
0
20
40
60

Some Fundamental Cavity Parameters

- Energy Gain

$$\frac{d\left(\gamma mc^2\right)}{dt} = -e\vec{E}\left(\vec{x}\left(t\right),t\right)\cdot\vec{\mathrm{v}}$$

- For standing wave RF fields and velocity of light particles

$$\vec{E}\left(\vec{x},t\right) = \vec{E}\left(\vec{x}\right)\cos\left(\omega_{RF}t+\delta\right) \to \Delta\left(\gamma mc^2\right) \approx -e\int_{-\infty}^{\infty} E_z\left(0,0,z\right)\cos\left(z/\lambda_{RF}+\delta\right)dz$$

$$= \frac{e\tilde{E}_z\left(2\pi/\lambda_{RF}\right)e^{-i\delta}+\mathrm{c.c.}}{2} \qquad V_c \equiv \left|e\tilde{E}_z\left(2\pi/\lambda_{RF}\right)\right|$$

- Normalize by the cavity length L for gradient

$$\mathrm{E}_{\mathrm{acc}}\left(\mathrm{MV/m}\right) = \frac{V_c}{L}$$

Shunt Impedance *R/Q*

- Ratio between the square of the maximum voltage delivered by a cavity and the product of ω_{RF} and the energy stored in a cavity

$$\frac{R}{Q} \equiv \frac{V_c^2}{\omega_{RF}\left(\text{stored energy}\right)}$$

- Depends only on the cavity geometry, independent of frequency when uniformly scale structure in 3D
- Piel's rule: R/Q ~100 Ω/cell

CEBAF 5 Cell	**480 Ω**
CEBAF 7 Cell	**760 Ω**
DESY 9 Cell	**1051 Ω**

Unloaded Quality Factor

- As is usual in damped harmonic motion define a quality factor by

$$Q \equiv \frac{2\pi\left(\text{energy stored in oscillation}\right)}{\text{energy dissipated in 1 cycle}}$$

- Unloaded Quality Factor Q_0 of a cavity

$$Q_0 \equiv \frac{\omega_{RF}\left(\text{stored energy}\right)}{\text{heating power in walls}}$$

- Quantifies heat flow directly into cavity walls from AC resistance of superconductor, and wall heating from other sources.

Loaded Quality Factor

- When add the *input* coupling port, must account for the energy loss through the port on the oscillation

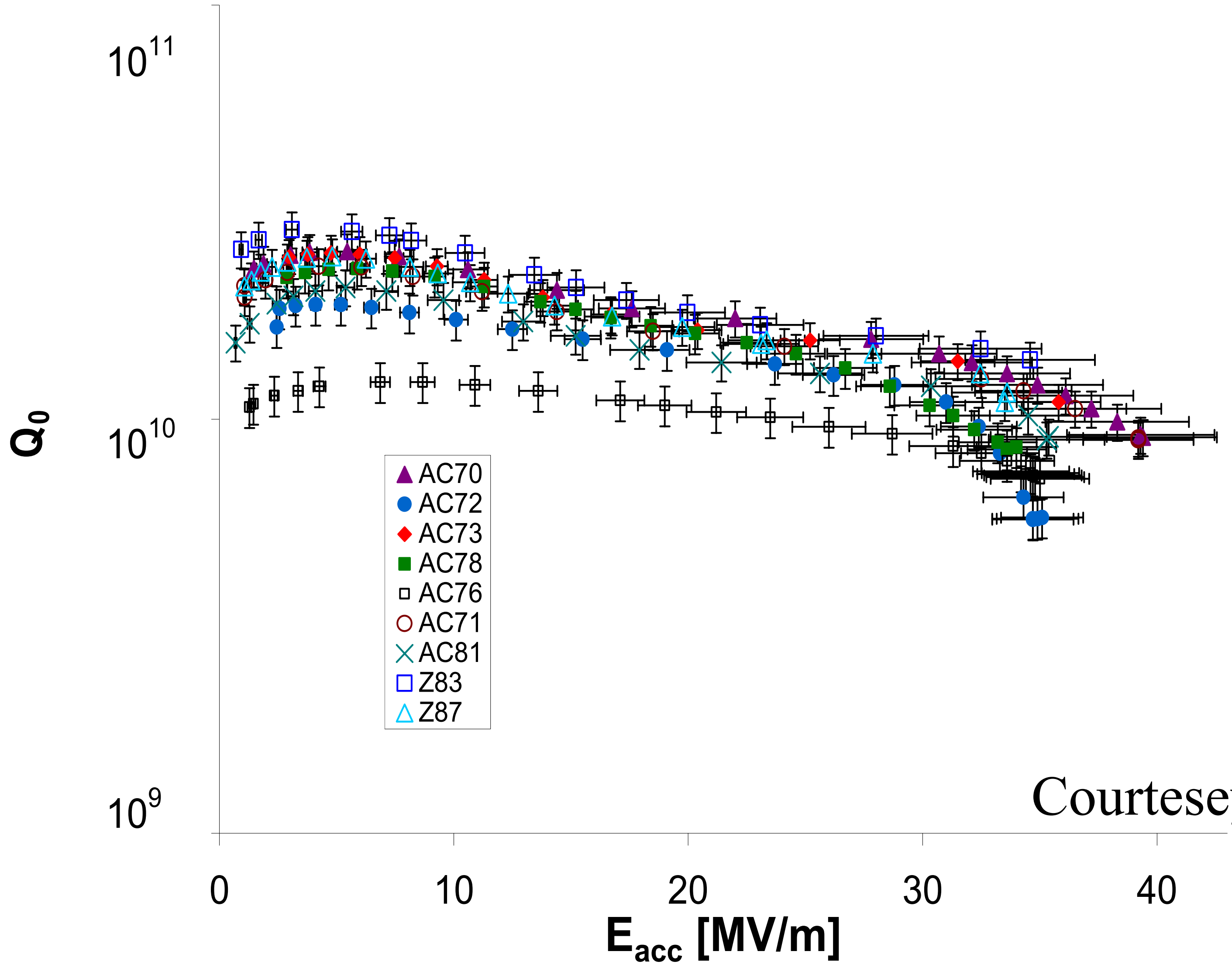
$$\frac{1}{Q_{tot}} \equiv \frac{1}{Q_L} = \frac{\text{total power lost}}{\omega_{RF}\left(\text{stored energy}\right)} = \frac{1}{Q_{ext}} + \frac{1}{Q_0}$$

- Coupling Factor

$$\beta \equiv \frac{Q_0}{Q_{ext}} \gg 1 \quad \text{for present day SRF cavities,} \qquad Q_L = \frac{Q_0}{1+\beta}$$

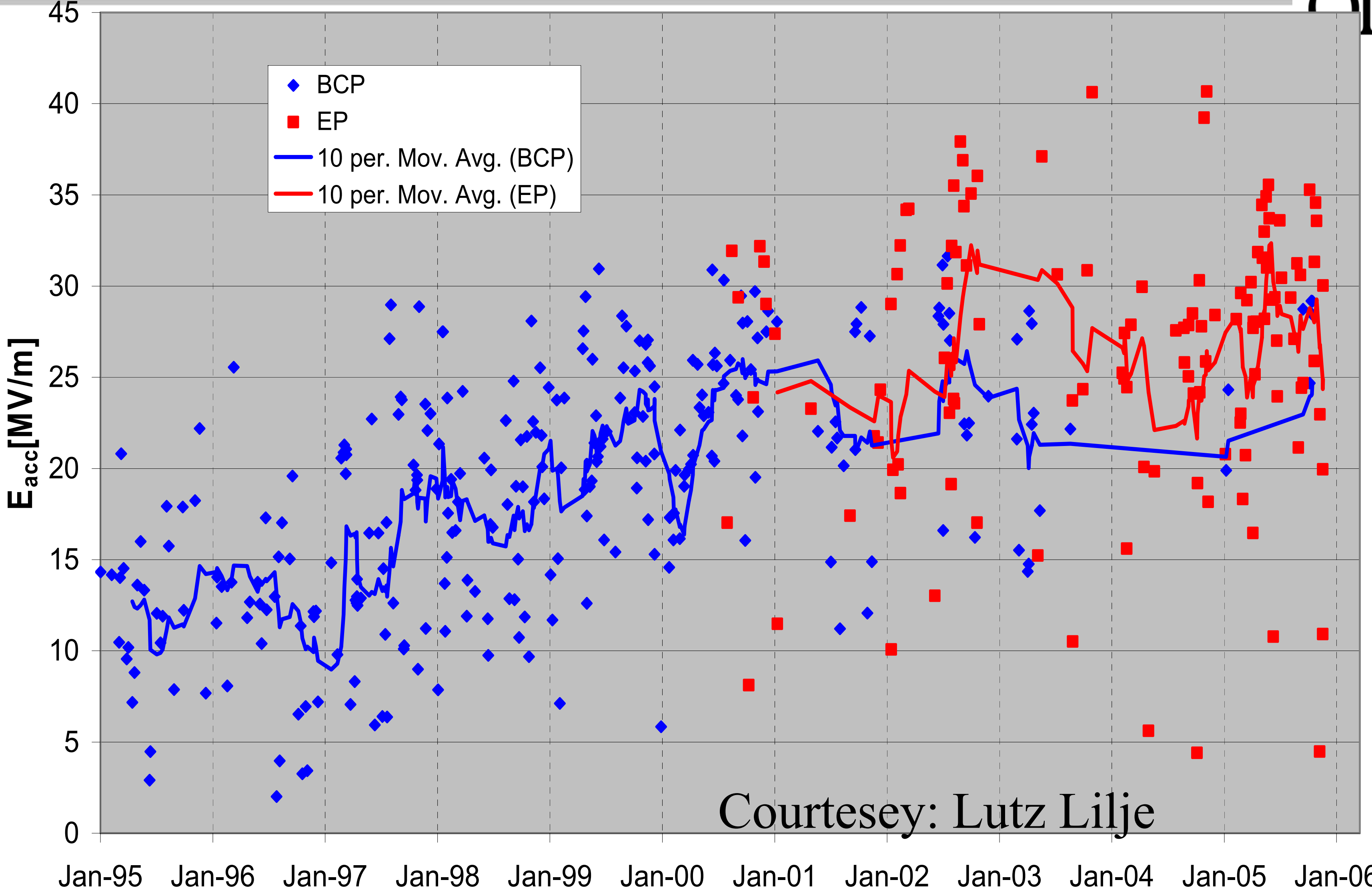
- It's the loaded quality factor that gives the effective resonance width that the RF system, and its controls, see from the superconducting cavity
- Chosen to minimize operating RF power: current matching (CEBAF, FEL), rf control performance and microphonics (SNS, ERLs)

Q_0 vs. Gradient for Several 1300 MHz Cavities



Courtesey: Lutz Lilje

E_{acc} vs. time



BCP
EP
10 per. Mov. Avg. (BCP)
10 per. Mov. Avg. (EP)
Eacc[MV/m]
0
5
10
15
20
25
30
35
40
45
Jan-95
Jan-96
Jan-97
Jan-98
Jan-99
Jan-00
Jan-01
Jan-02
Jan-03
Jan-04
Jan-05
Jan-06
Courtesey: Lutz Lilje

RF Cavity Equations

- Introduction
- Cavity Fundamental Parameters
- RF Cavity as a Parallel LCR Circuit
- Coupling of Cavity to an rf Generator
- Equivalent Circuit for a Cavity with Beam Loading
  - On Crest and on Resonance Operation
  - Off Crest and off Resonance Operation
    - Optimum Tuning
    - Optimum Coupling
- RF cavity with Beam and Microphonics
- Q_{ext} Optimization under Beam Loading and Microphonics
- RF Modeling
- Conclusions

Introduction

- Goal: Ability to predict rf cavity's steady-state response and develop a differential equation for the transient response

- We will construct an equivalent circuit and analyze it

- We will write the quantities that characterize an rf cavity and relate them to the circuit parameters, for

  a) a cavity

  b) a cavity coupled to an rf generator

  c) a cavity with beam

RF Cavity Fundamental Quantities

- Quality Factor *Q*0:

$$Q_0 \equiv \frac{\omega_0 W}{P_{diss}} = \frac{\text{Energy stored in cavity}}{\text{Energy dissipated in cavity walls per radian}}$$

- Shunt impedance *Ra*:

$$R_a \equiv \frac{V_a^2}{P_{diss}} \quad \text{in ohms per cell}$$

(accelerator definition); *Va* = accelerating voltage

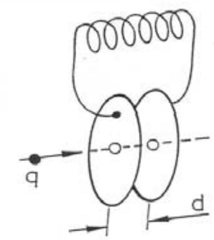
- Note: Voltages and currents will be represented as complex quantities, denoted by a tilde. For example:

$$V_c(t) = \mathrm{Re}\left\{\tilde{V}_c(t)e^{i\omega t}\right\} \qquad \tilde{V}_c(t) = V_c e^{i\phi(t)}$$

where $V_c = \left|\tilde{V}_c\right|$ is the magnitude of $\tilde{V}_c$ and ϕ is a slowly varying phase.

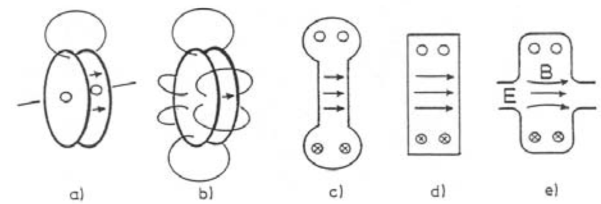
Equivalent Circuit for an rf Cavity

Simple LC circuit representing an accelerating resonator.



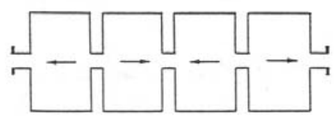
Simple lumped L-C circuit repesenting an accelerating resonator. $\omega_0^2 = 1/LC$

Metamorphosis of the LC circuit into an accelerating cavity.

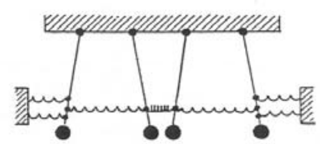


Metamorphosis of the L-C circuit of Fig. 1 into an accelerating cavity (after R.P.Feynman[33]). Fig. 5d shows the cylindrical "pillbox cavity" and Fig. 5e a slightly modified pillbox cavity with beam holes (typical β between 0.5 and 1.0). Fig. 5c resembles a low β version of the pillbox variety ($0.2<\beta<0.5$).

Chain of weakly coupled pillbox cavities representing an accelerating cavity.
Chain of coupled pendula as its mechanical analogue.



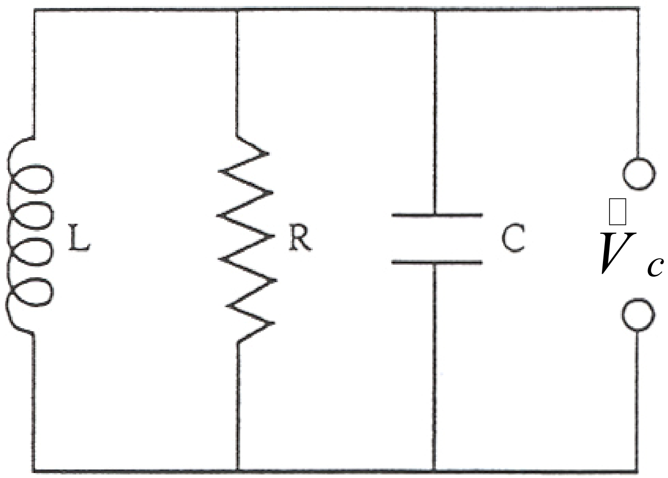
Chain of weakly-coupled pillbox cavities representing an accelerating module



Chain of coupled pendula as a mechanical analogue to Fig. 6a

Equivalent Circuit for an rf Cavity (cont'd)

- An rf cavity can be represented by a parallel LCR circuit:



$$\vec{V}_c(t) = \vec{v}_c \, e^{i\omega t}$$

- Impedance Z of the equivalent circuit: $\tilde{Z} = \left[\frac{1}{R} + \frac{1}{iL\omega} + iC\omega \right]^{-1}$

- Resonant frequency of the circuit: $\omega_0 = 1/\sqrt{LC}$

- Stored energy W: $W = \frac{1}{2} C V_c^2$

Equivalent Circuit for an rf Cavity (cont'd)

- Power dissipated in resistor *R*: $$P_{diss} = \frac{1}{2}\frac{V_c^2}{R}$$

$$R_a \equiv \frac{V_a^2}{P_{diss}} \qquad \therefore\ R_a = 2R$$

- From definition of shunt impedance

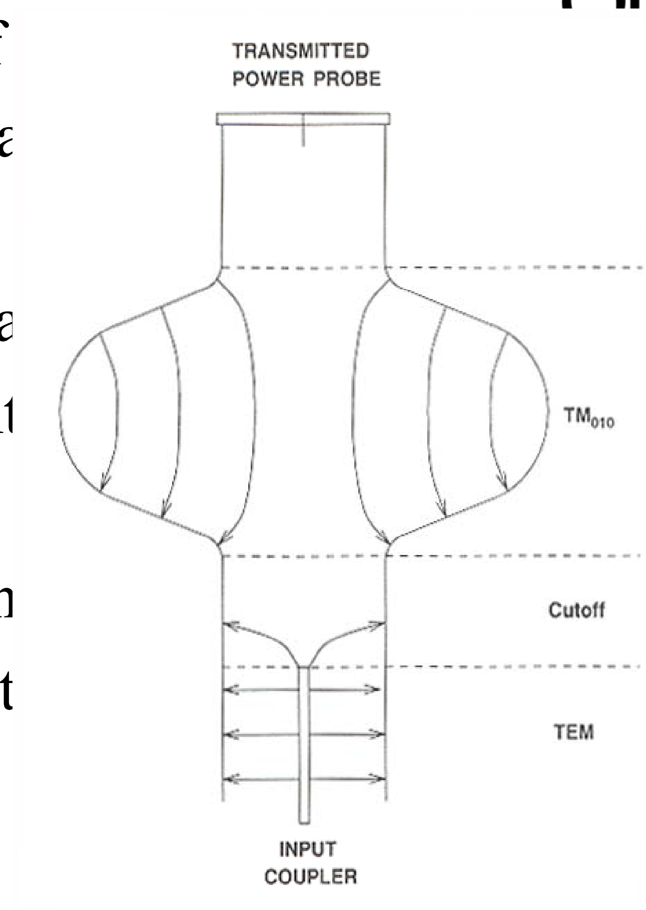
$$Q_0 \equiv \frac{\omega_0 W}{P_{diss}} = \omega_0 CR$$

- Quality factor of resonator:

- Note: $$\tilde{Z} = R\left[1 + iQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]^{-1}$$ For $\omega \approx \omega_0$, $$\tilde{Z} \approx R\left[1 + 2iQ_0\left(\frac{\omega - \omega_0}{\omega_0}\right)\right]^{-1}$$

Cavity with External Coupling

- Consider a cavity connected to an rf
- A coaxial cable carries power from a
  to the cavity
- The strength of the input coupler is a
  changing the penetration of the cent
- There is a fixed output coupler,
  the *transmitted power probe*, which
  power transmitted through the cavit



Cavity with External Coupling (cont'd)

Consider the rf cavity after the rf is turned off.

Stored energy W satisfies the equation: $\frac{dW}{dt} = -P_{tot}$

Total power being lost, $P_{tot,}$ is: $P_{tot} = P_{diss} + P_e + P_t$

P_e is the power leaking back out the input coupler. P_t is the power coming out the transmitted power coupler. Typically P_t is very small $\Rightarrow P_{tot} \approx P_{diss} + P_e$

$$Q_0 \equiv \frac{\omega_0 W}{P_{diss}}$$

Recall $\frac{dW}{dt} = -\frac{\omega_0 W}{Q_L} \Rightarrow W = W_0 e^{-\frac{\omega_0 t}{Q_L}}$

Similarly define a "loaded" quality factor Q_L: $Q_L \equiv \frac{\omega_0 W}{P_{tot}}$

No

$$\tau_L = \frac{Q_L}{\omega_0}$$

∴ energy in the cavity decays exponentially with time constant:

Cavity with External Coupling (cont'd)

Equation

$$\frac{P_{tot}}{\omega_0 W} = \frac{P_{diss} + P_e}{\omega_0 W}$$

suggests that we can assign a quality factor to each loss mechanism, such that

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e}$$

where, by definition,

$$Q_e \equiv \frac{\omega_0 W}{P_e}$$

Typical values for CEBAF 7-cell cavities: $Q0=1\text{x}10^{10}$, $Qe \approx QL=2\text{x}10^{7}$.

Cavity with External Coupling (cont'd)

- Define "coupling parameter":

$$\beta \equiv \frac{Q_0}{Q_e}$$

therefore

$$\frac{1}{Q_L} = \frac{(1+\beta)}{Q_0}$$

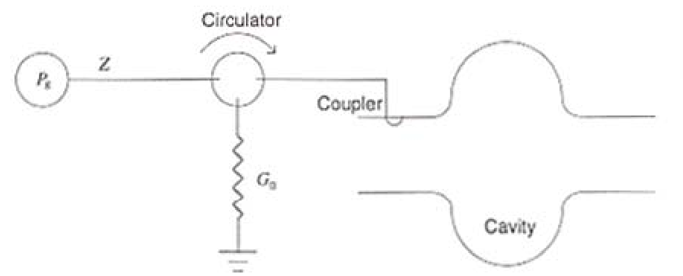
- β is equal to:

$$\beta = \frac{P_e}{P_{diss}}$$

It tells us how strongly the couplers interact with the cavity. Large β implies that the power leaking out of the coupler is large compared to the power dissipated in the cavity walls.

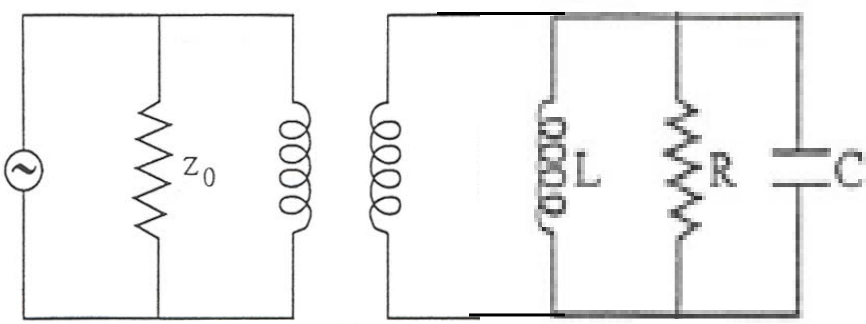
Equivalent Circuit of a Cavity Coupled to an rf Source

- The system we want to mod



- Between the rf generator and the cavity is an isolator – a circulator connected to a load. Circulator ensures that signals coming from the cavity are terminated in a matched load.
- Equivalent circuit:

$$\tilde{I}_k(t) = \tilde{i}_k\, e^{i\omega t}$$



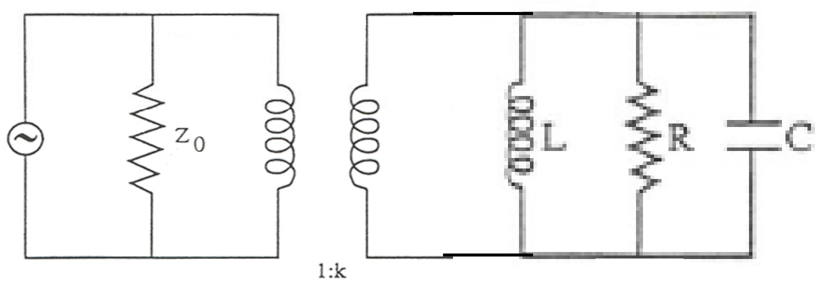
RF Generator + Circulator Coupler

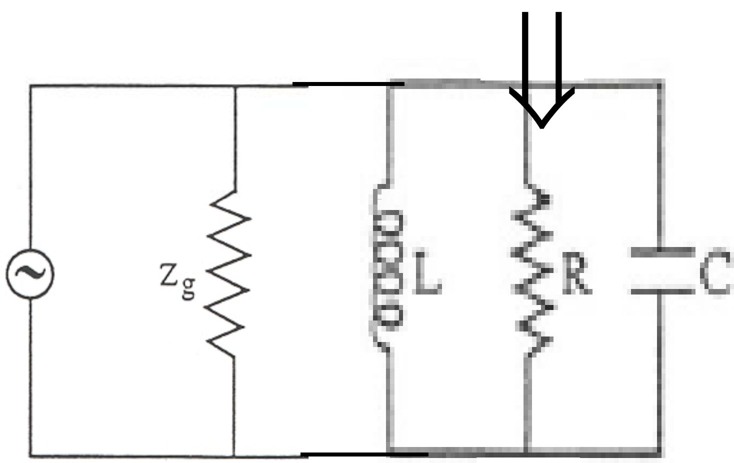
Cavity

- Coupling is represented by an ideal transformer of turn ratio 1:k

Equivalent Circuit of a Cavity Coupled to an rf Source

$$\tilde{I}_k(t) = \tilde{i}_k e^{i\omega t}$$





$$\tilde{I}_g(t) = \tilde{i}_g e^{i\omega t}$$

$$I_g = \frac{I_k}{k}$$

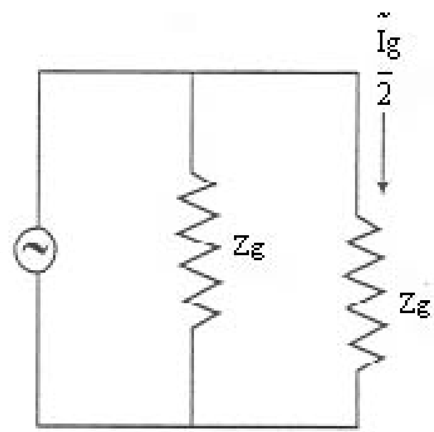
$$Z_g = k^2 Z_0$$

By definition, $\beta \equiv \dfrac{R}{Z_g} = \dfrac{R}{k^2 Z_0} \quad \therefore \quad Z_g = \dfrac{R}{\beta}$

Generator Power

- When the cavity is matched to the input circuit, the power dissipation in the cavity is maximized.

$$\tilde{I}_g(t) = \tilde{i}_g e^{i\omega t}$$



$$P_{diss}^{\max} = \frac{1}{2} Z_g \left(\frac{I_g}{2} \right)^2 \quad \text{or} \quad P_{diss}^{\max} = \frac{1}{16\beta} R_a I_g^2 \equiv P_g$$

- We define the available generator power P_g at a given generator current $\tilde{I}_g$ to be equal to P_{diss}^{max}.

Some Useful Expressions

- We derive expressions for W, P_{diss}, P_{refl}, in terms of cavity parameters

$$\frac{W}{P_g} = \frac{\dfrac{Q_0}{\omega_0} P_{diss}}{\dfrac{1}{16\beta} R_a I_g^2} = \frac{\dfrac{Q_0}{\omega_0} \dfrac{V_c^2}{R_a}}{\dfrac{1}{16\beta} R_a I_g^2} = \frac{16\beta}{R_a^2} \frac{Q_0}{\omega_0} \frac{V_c^2}{I_g^2}$$

$$V_c = I_g Z_{TOT}$$

$$Z_{TOT} = \left[\frac{1}{Z_g} + \frac{1}{Z} \right]^{-1}$$

$$Z_{TOT} = \frac{R_a}{2} \left[(1+\beta) + iQ_0 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right]^{-1}$$

$$\therefore \quad W = 4\beta \frac{Q_0}{\omega_0} \frac{1}{(1+\beta)^2 + Q_0^2 \left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega} \right)^2} P_g$$

$$\text{For} \quad \omega \approx \omega_0 \Rightarrow$$

$$W \approx \frac{4\beta}{(1+\beta)^2} \frac{Q_0}{\omega_0} \frac{1}{1 + \left[2 \dfrac{Q_0}{(1+\beta)} \dfrac{\omega - \omega_0}{\omega_0} \right]^2} P_g$$

Some Useful Expressions (cont’d)

$$W \approx \frac{4\beta}{(1+\beta)^2}\frac{Q_0}{\omega_0}\frac{1}{1+\left[2\frac{Q_0}{(1+\beta)}\frac{\omega-\omega_0}{\omega_0}\right]^2}P_g$$

- Define “Tuning angle” Ψ:

$$\tan\Psi \equiv -Q_L\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) \approx -2Q_L\frac{\omega-\omega_0}{\omega_0} \quad \text{for} \quad \omega \approx \omega_0$$

$$\therefore$$

$$W = \frac{4\beta}{(1+\beta)^2}\frac{\mathrm{Q}_0}{\omega_0}\frac{1}{1+\tan^2\Psi}\mathrm{P}_\mathrm{g}$$

$$P_{diss} = \frac{\omega_0 W}{Q_0}$$

$$\therefore$$

- Recall:

$$P_{diss} = \frac{4\beta}{(1+\beta)^2}\frac{1}{1+\tan^2\Psi}P_g$$

Some Useful Expressions (cont'd)

- Optimal coupling: W/Pg maximum or $Pdiss = Pg$
  which implies $\Delta\omega = 0,\ \beta = 1$
  this is the case of critical coupling
- Reflected power is calculated from energy conservation:

$$P_{refl} = P_g - P_{diss}$$

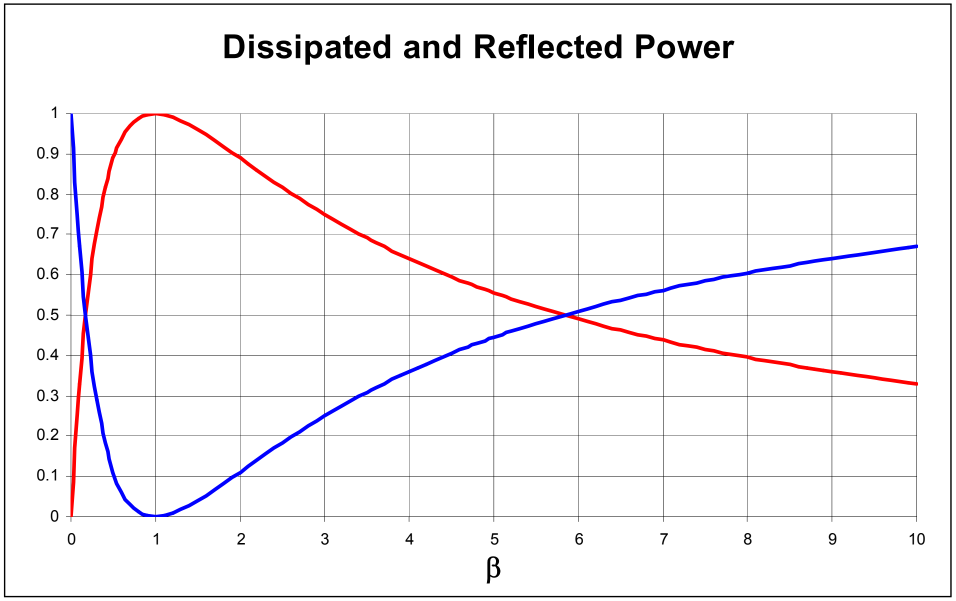
$$P_{refl} = P_g\left[1 - \frac{4\beta}{(1+\beta)^2}\frac{1}{1+\tan^2\Psi}\right]$$

$$W = \frac{4\beta}{(1+\beta)^2}\frac{Q_0}{\omega_0}P_g$$

- On resonance:

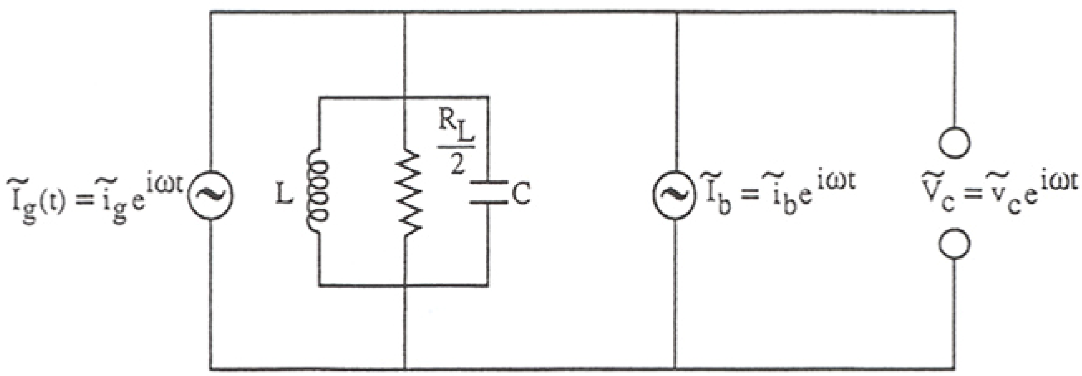
$$P_{diss} = \frac{4\beta}{(1+\beta)^2}P_g$$

$$P_{refl} = \left(\frac{1-\beta}{1+\beta}\right)^2 P_g$$



Equivalent Circuit for a Cavity with Beam

- Beam in the rf cavity is represented by a current generator.
- Equivalent circuit:



$$i_C = C\frac{dv_C}{dt}, \quad i_R = \frac{v_C}{R_L/2}, \quad v_C = L\frac{di_L}{dt}$$

- Differential equation that describes the dynamics of the system:

- R_L is the loaded impedance defined as:

$$R_L = \frac{R_a}{(1+\beta)}$$

Equivalent Circuit for a Cavity with Beam (cont'd)

- Kirchoff's law:

$$\tilde{i}_L + \tilde{i}_R + \tilde{i}_C = \tilde{i}_g - \tilde{i}_b$$

- Total current is a superposition of generator current and beam current and beam current opposes the generator current.

$$\frac{d^2\tilde{v}_c}{dt^2} + \frac{\omega_0}{Q_L}\frac{d\tilde{v}_c}{dt} + \omega_0^2\tilde{v}_c = \frac{\omega_0 R_L}{2Q_L}\frac{d}{dt}\left(\tilde{i}_g - \tilde{i}_b\right)$$

- Assume that $\tilde{v}_c, \tilde{i}_g, \tilde{i}_b$ have a fast (rf) time-varying component and a slow varying component:

$$\tilde{v}_c = \tilde{V}_c e^{i\omega t}$$

$$\tilde{i}_g = \tilde{I}_g e^{i\omega t}$$

$$\tilde{i}_b = \tilde{I}_b e^{i\omega t}$$

where ω is the generator angular frequency and $\tilde{V}_c, \tilde{I}_g, \tilde{I}_b$ are complex quantities.

Equivalent Circuit for a Cavity with Beam (cont'd)

- Neglecting terms of order $\frac{d^2\tilde{V}_c}{dt^2}, \frac{d\tilde{I}}{dt}, \frac{1}{Q_L}\frac{d\tilde{V}_c}{dt}$ we arrive at:

$$\boxed{\frac{d\tilde{V}_c}{dt} + \frac{\omega_0}{2Q_L}(1 - i\tan\Psi)\tilde{V}_c = \frac{\omega_0 R_L}{4Q_L}(\tilde{I}_g - \tilde{I}_b)}$$

where Ψ is the tuning angle.

- For short bunches: $|\tilde{I}_b| \approx 2I_0$ where I_0 is the average beam current.

Equivalent Circuit for a Cavity with Beam (cont'd)

$$\frac{d\tilde{V}_c}{dt} + \frac{\omega_0}{2Q_L}(1 - i\tan\Psi)\tilde{V}_c = \frac{\omega_0 R_L}{4Q_L}(\tilde{I}_g - \tilde{I}_b)$$

- At steady-state: $\tilde{V}_c = \dfrac{R_L/2}{(1 - i\tan\Psi)}\tilde{I}_g - \dfrac{R_L/2}{(1 - i\tan\Psi)}\tilde{I}_b$

or $\tilde{V}_c = \dfrac{R_L}{2}\tilde{I}_g \cos\Psi e^{i\Psi} - \dfrac{R_L}{2}\tilde{I}_b \cos\Psi e^{i\Psi}$

or $\tilde{V}_c = \boxed{\tilde{V}_{gr}\cos\Psi e^{i\Psi}} + \boxed{\tilde{V}_{br}\cos\Psi e^{i\Psi}}$

or $\tilde{V}_c = \tilde{V}_g + \tilde{V}_b$

$$\left\{\begin{aligned} \tilde{V}_{gr} &= \frac{R_L}{2}\tilde{I}_g \\ \tilde{V}_{br} &= -\frac{R_L}{2}\tilde{I}_b \end{aligned}\right\}$$

are the generator and beam-loading voltages on resonance

and $\left\{\begin{matrix} \tilde{V}_g \\ \tilde{V}_b \end{matrix}\right\}$ are the generator and beam-loading voltages.

Equivalent Circuit for a Cavity with Beam (cont'd)

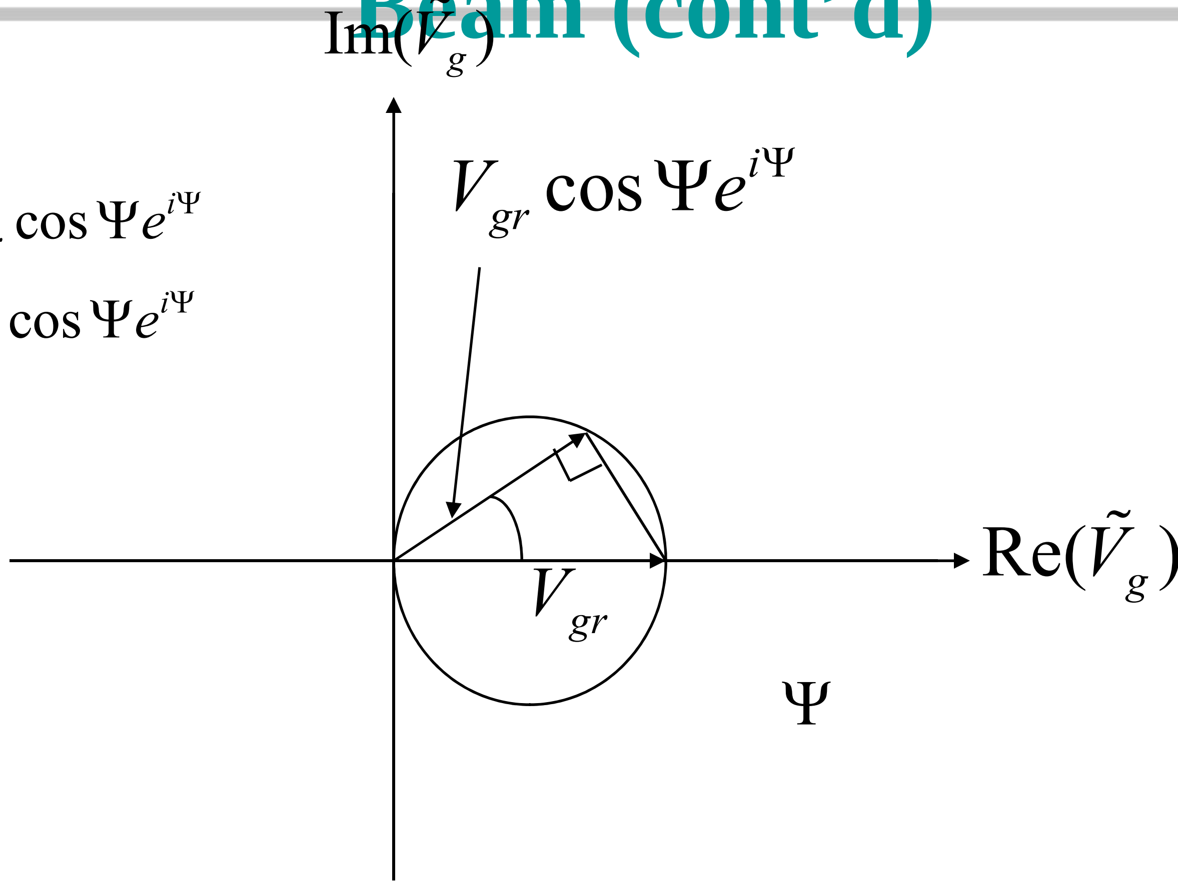
- Note that:

$$|\tilde{V}_{gr}| = \frac{2\sqrt{\beta}}{\sqrt{1+\beta}}\sqrt{P_g R_L} \approx 2\sqrt{P_g R_L} \quad \text{for large } \beta$$

$$|\tilde{V}_{br}| = R_L I_0$$

Equivalent Circuit for a Cavity with Beam (cont'd)

$$\tilde{V}_g = \tilde{V}_{gr} \cos\Psi e^{i\Psi}$$
$$\tilde{V}_b = \tilde{V}_{br} \cos\Psi e^{i\Psi}$$



As Ψ increases the magnitude of both V_g and V_b decreases while their phases rotate by Ψ.

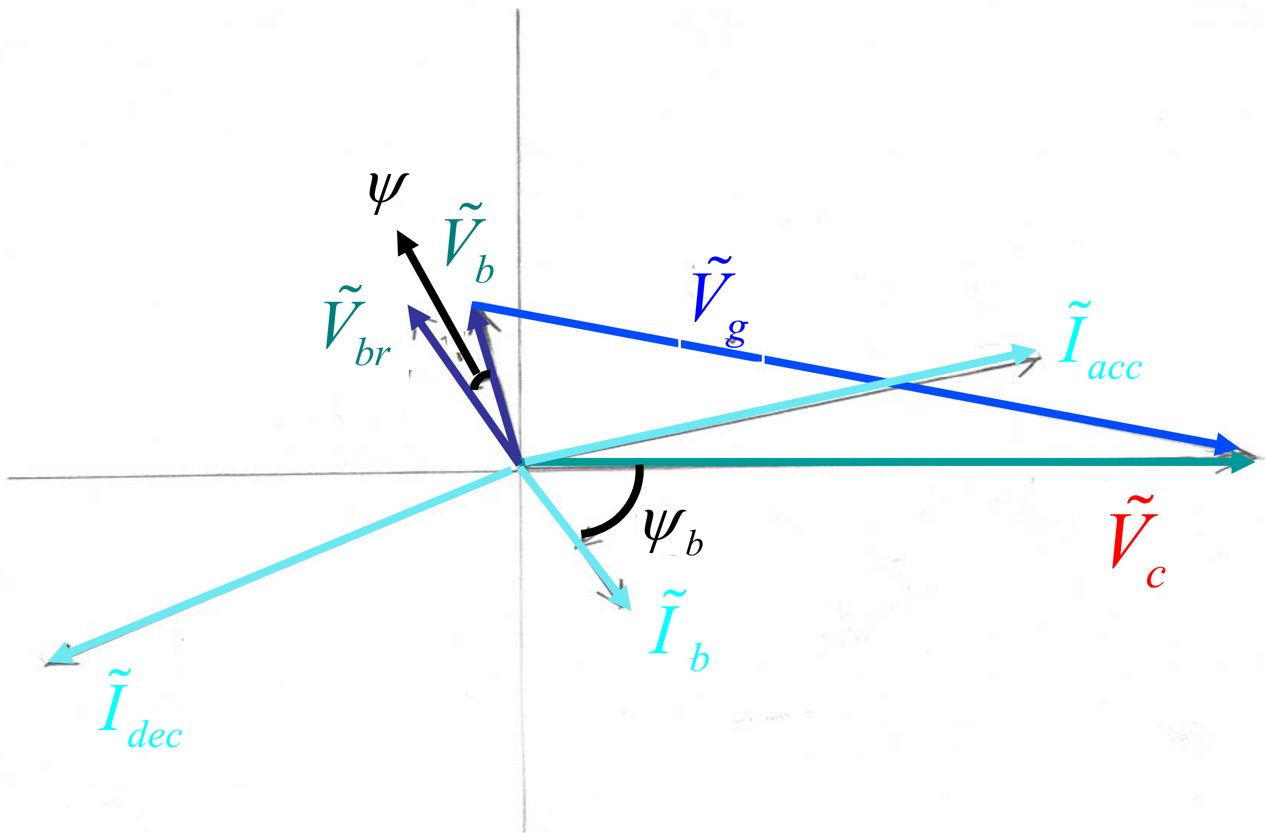
Equivalent Circuit for a Cavity with Beam (cont'd)

$$\tilde{V}_c = \tilde{V}_g + \tilde{V}_b$$

- Cavity voltage is the superposition of the generator and beam-loading voltage.

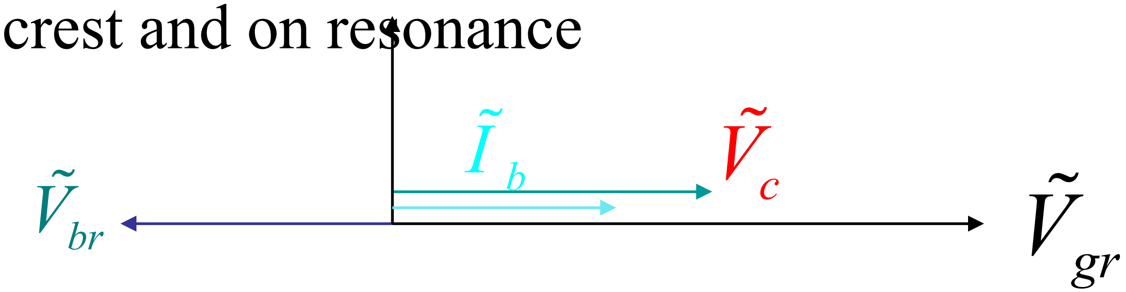
- This is the basis for the vector diagram analysis.

Example of a Phasor Diagram



On Crest and On Resonance Operation

- Typically linacs operate on resonance and on crest in order to receive maximum acceleration.
- On crest and on resonance



$$V_a = V_{gr} - V_{br}$$

$$\Rightarrow$$

where V_a is the accelerating voltage.

More Useful Equations

- We derive expressions for W, V_a, P_{diss}, P_{refl} in terms of β and the loading parameter K, defined by: $K=I_0/2\sqrt{R_a/P_g}$

From:

$$|\tilde{V}_{gr}| = \frac{2\sqrt{\beta}}{\sqrt{1+\beta}}\sqrt{P_g R_L}$$

$$|\tilde{V}_{br}| = R_L I_0$$

$$V_a = V_{gr} - V_{br}$$

$$\Rightarrow$$

$$V_a = \sqrt{P_g R_a}\left\{\frac{2\sqrt{\beta}}{\sqrt{1+\beta}}\left(1-\frac{K}{\sqrt{\beta}}\right)\right\}$$

$$W = \frac{4\beta}{(1+\beta)^2}\frac{Q_0}{\omega_0}\left(1-\frac{K}{\sqrt{\beta}}\right)^2 P_g$$

$$P_{diss} = \frac{4\beta}{(1+\beta)^2}\left(1-\frac{K}{\sqrt{\beta}}\right)^2 P_g$$

$$I_0 V_a = I_0\sqrt{R_a P_{diss}}$$

$$\eta \equiv \frac{I_0 V_a}{P_g} = \frac{2\sqrt{\beta}}{\sqrt{1+\beta}} 2K\left(1-\frac{K}{\sqrt{\beta}}\right)$$

$$P_{refl} = P_g - P_{diss} - I_0 V_a \;\Rightarrow\; P_{refl} = \frac{\left[(\beta-1)-2K\sqrt{\beta}\right]^2}{(\beta+1)^2} P_g$$

More Useful Equations (cont'd)

- For β large,

$$P_g \approx \frac{1}{4R_L}(V_a + I_0 R_L)^2$$

$$P_{refl} \approx \frac{1}{4R_L}(V_a - I_0 R_L)^2$$

- For P_{refl}=0 (condition for matching) ⇒

$$R_L = \frac{V_a^M}{I_0^M}$$

and

$$P_g \approx \frac{I_0^M V_a^M}{4}\left(\frac{V_a}{V_a^M} + \frac{I_0}{I_0^M}\right)^2$$

Example

- For V_a=20 MV/m, L=0.7 m, $Q_L=2x10^7$, $Q_0=1x10^{10}$:

Power	$I_0 = 0$	$I_0 = 100\ \mu A$	$I_0 = 1$ mA
P_g	3.65 kW	4.38 kW	14.033 kW
P_{diss}	29 W	29 W	29 W
I_0V_a	0 W	1.4 kW	14 kW
P_{refl}	3.62 kW	2.951 kW	~ 4.4 W

Off Crest and Off Resonance Operation

- Typically electron storage rings operate off crest in order to ensure stability against phase oscillations.
- As a consequence, the rf cavities must be detuned off resonance in order to minimize the reflected power and the required generator power.
- Longitudinal gymnastics may also impose off crest operation operation in recirculating linacs.
- We write the beam current and the cavity voltage as

$$\tilde{I}_b = 2I_0 e^{i\psi_b}$$

$$\tilde{V}_c = V_c e^{i\psi_c} \quad \text{and set } \psi_c = 0$$

- The generator power can then be expressed as:

$$\boxed{P_g = \frac{V_c^2}{R_L}\frac{(1+\beta)}{4\beta}\left\{\left[1+\frac{I_0 R_L}{V_c}\cos\psi_b\right]^2 + \left[\tan\Psi - \frac{I_0 R_L}{V_c}\sin\psi_b\right]^2\right\}}$$

Off Crest and Off Resonance Operation (cont'd)

- Condition for optimum tuning:

$$\tan\Psi = \frac{I_0 R_L}{V_c}\sin\psi_b$$

- Condition for optimum coupling:

$$\beta_0 = 1 + \frac{I_0 R_a}{V_c}\cos\psi_b$$

- Minimum generator power:

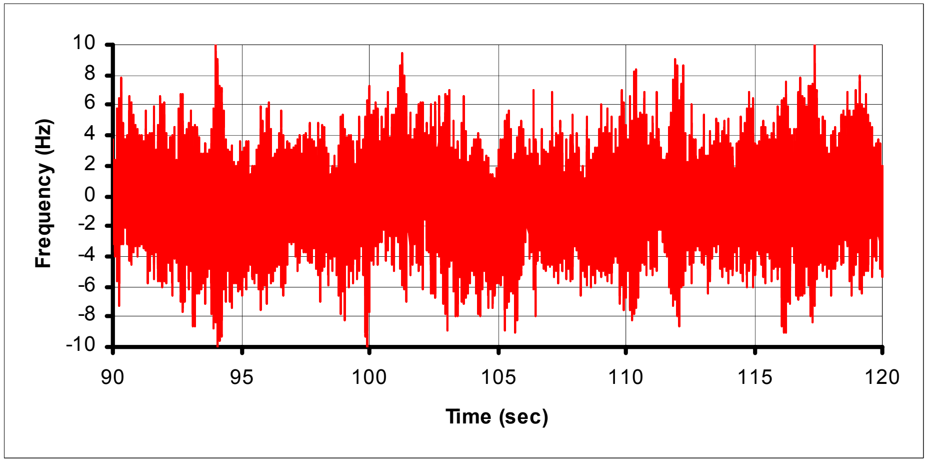
$$P_{g,\min} = \frac{V_c^2 \beta_0}{R_a}$$

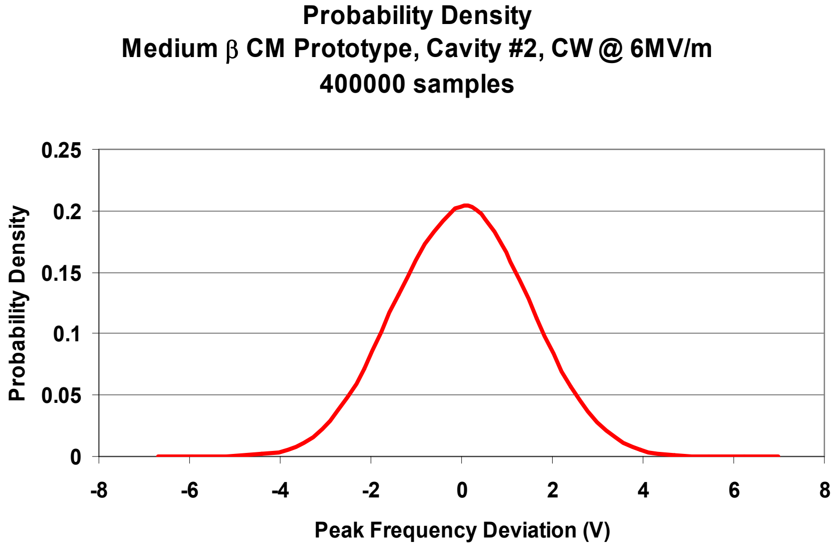
RF Cavity with Beam and Microphonics

The detuning is now: $\tan\Psi = -2Q_L \dfrac{\delta f_0 \pm \delta f_m}{f_0}$ $\qquad \tan\psi_0 = -2Q_L \dfrac{\delta f_0}{f_0}$

where δf_0 is the static detuning (controllable)

and δf_m is the random dynamic detuning (uncontrollable)





Q_{ext} Optimization under Beam Loading and Microphonics

- Beam loading and microphonics require careful optimization of the external Q of cavities.
- Derive expressions for the optimum setting of cavity parameters when operating under

  a) heavy beam loading

  b) little or no beam loading, as is the case in energy recovery linac cavities

  and in the presence of microphonics.

Q_{ext} Optimization (cont'd)

$$P_g = \frac{V_c^2}{R_L}\frac{(1+\beta)}{4\beta}\left\{\left[1+\frac{I_{tot}R_L}{V_c}\cos\psi_{tot}\right]^2+\left[\tan\Psi-\frac{I_{tot}R_L}{V_c}\sin\psi_{tot}\right]^2\right\}$$

$$\tan\Psi = -2Q_L\frac{\delta f}{f_0}$$

where δf is the total amount of cavity detuning in Hz, including static detuning and
microphonics.

- Optimization of the generator power with respect to coupling gives:

$$\beta_{opt} = \sqrt{(b+1)^2+\left[2Q_0\frac{\delta f}{f_0}+b\tan\psi_{tot}\right]^2}$$

where $b \equiv \frac{I_{tot}R_a}{V_c}\cos\psi_{tot}$

where Itot is the magnitude of the resultant beam current vector in the cavity and ψtot is the phase of the resultant beam vector with respect to the cavity voltage.

Q_{ext} Optimization (cont'd)

$$P_g = \frac{V_c^2}{R_L}\frac{(1+\beta)}{4\beta}\left\{\left[1+\frac{I_{tot}R_L}{V_c}\cos\psi_{tot}\right]^2+\left[\tan\Psi-\frac{I_{tot}R_L}{V_c}\sin\psi_{tot}\right]^2\right\}$$

$$\tan\Psi = -2Q_L\frac{\delta f_0+\delta f_m}{f_0}$$

where:

- To minimize generator power with respect to tuning:

$$\delta f_0 = -\frac{f_0}{2Q_0}b\tan\Psi$$

$$\Rightarrow \quad P_g = \frac{V_c^2}{R_L}\frac{(1+\beta)}{4\beta}\left\{(1+b+\beta)^2+\left[2Q_0\frac{\delta f_m}{f_0}\right]^2\right\}$$

Q_{ext} Optimization (cont'd)

- Condition for optimum coupling:

$$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta f_m}{f_0}\right)^2}$$

$$P_g^{opt} = \frac{V_c^2}{2R_a}\left[b+1+\sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta f_m}{f_0}\right)^2}\right]$$

and

- In the absence of beam (b=0):

$$\beta_{opt} = \sqrt{1 + \left(2Q_0 \frac{\delta f_m}{f_0}\right)^2}$$

$$P_g^{opt} = \frac{V_c^2}{2R_a}\left[1+\sqrt{1 + \left(2Q_0 \frac{\delta f_m}{f_0}\right)^2}\right]$$

and

Homework

- Assuming no microphonics, plot β_{opt} and P^g_{opt} as function of b (beam loading), b=-5 to 5, and explain the results.

- How do the results change if microphonics is present?

Example

- ERL Injector and Linac:

  δf_m=25 Hz, Q_0=1x10^{10} , f_0=1300 MHz, I_0=100 mA, V_c=20 MV/m, L=1.04 m, R_a/Q_0=1036 ohms per cavity

- ERL linac: Resultant beam current, I_{tot} = 0 mA (energy recovery)

  and β_{opt}=385 $\Rightarrow$ Q_L=2.6x10^7 $\Rightarrow$ P_g = 4 kW per cavity.

- ERL Injector: I_0=100 mA and β_{opt}= 5x10^4 ! $\Rightarrow$ Q_L= 2x10^5 $\Rightarrow$ P_g = 2.08 MW per cavity!

  Note: I_0V_a = 2.08 MW $\Rightarrow$ optimization is entirely dominated by beam loading.

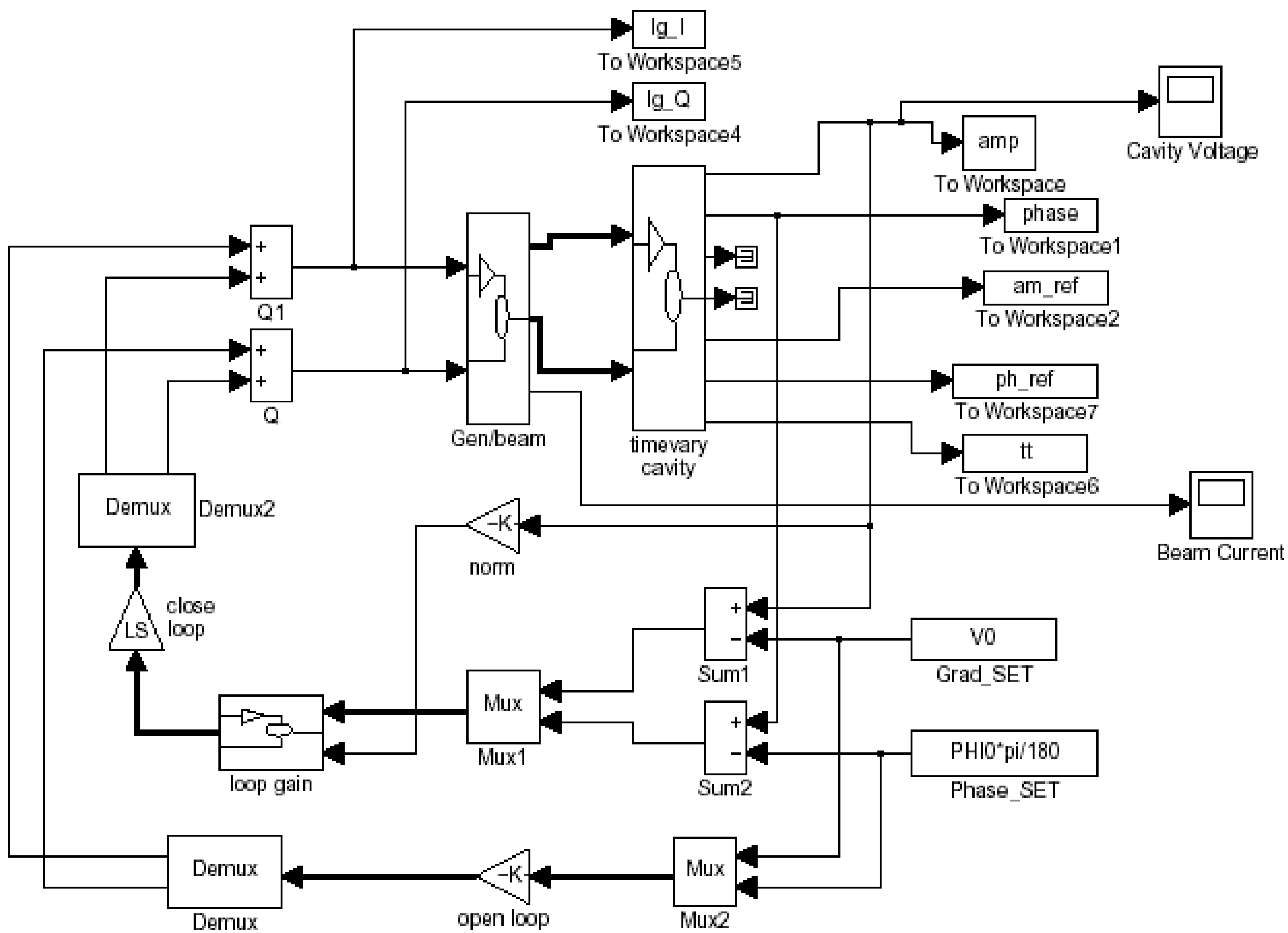
RF System Modeling

- To include amplitude and phase feedback, nonlinear effects from the klystron and be able to analyze transient response of the system, response to large parameter variations or beam current fluctuations

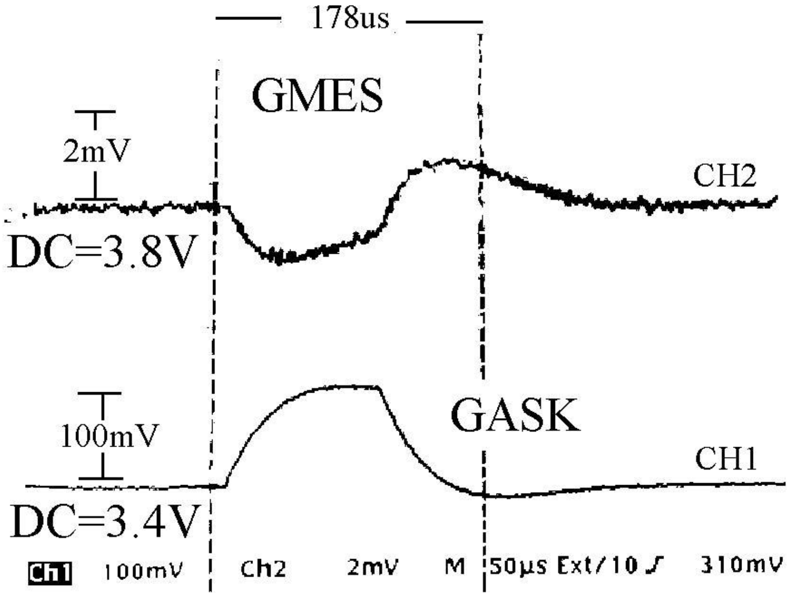
  - we developed a model of the cavity and low level controls using SIMULINK, a MATLAB-based program for simulating dynamic systems.

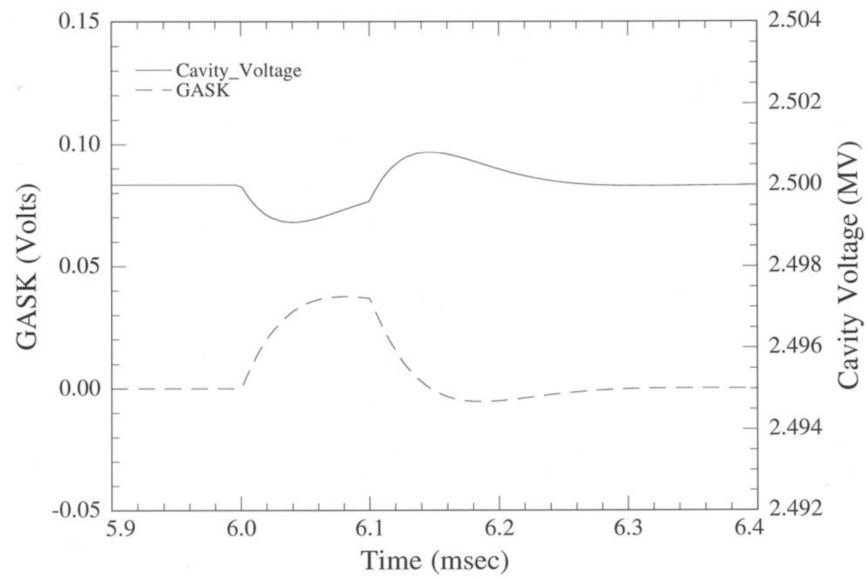
- Model describes the beam-cavity interaction, includes a realistic representation of low level controls, klystron characteristics, microphonic noise, Lorentz force detuning and coupling and excitation of mechanical resonances

RF System Model



RF Modeling: Simulations vs. Experimental Data





Measured and simulated cavity voltage and amplified gradient error signal (GASK) in one of CEBAF's cavities, when a 65 µA, 100 µsec beam pulse enters the cavity.

Conclusions

- We derived a differential equation that describes to a very good approximation the rf cavity and its interaction with beam.
- We derived useful relations among cavity's parameters and used phasor diagrams to analyze steady-state situations.
- We presented formula for the optimization of Q_{ext} under beam loading and microphonics.
- We showed an example of a Simulink model of the rf control system which can be useful when nonlinearities can not be ignored.

RF Focussing

In any RF cavity that accelerates longitudinally, because of Maxwell Equations there must be additional transverse electromagnetic fields. These fields will act to focus the beam and must be accounted properly in the beam optics, especially in the low energy regions of the accelerator. We will discuss this problem in greater depth in injector lectures. Let **A**(x,y,z) be the vector potential describing the longitudinal mode (Lorenz gauge)

$$\nabla \cdot \vec{A} = -\frac{1}{c}\frac{\partial \phi}{\partial t}$$

$$\nabla^2 \vec{A} = -\frac{\omega^2}{c^2}\vec{A} \qquad \nabla^2 \phi = -\frac{\omega^2}{c^2}\phi$$

For cylindrically symmetrical accelerating mode, functional form can only depend on r and z

$$A_z(r,z) = A_{z0}(z) + A_{z1}(z) r^2 + \ldots$$
$$\phi(r,z) = \phi_0(z) + \phi_1(z) r^2 + \ldots$$

Maxwell's Equations give recurrence formulas for succeeding approximations

$$(2n)^2 A_{zn} + \frac{d^2 A_{z,n-1}}{dz^2} = -\frac{\omega^2}{c^2} A_{z,n-1}$$

$$(2n)^2 \phi_n + \frac{d^2 \phi_{n-1}}{dz^2} = -\frac{\omega^2}{c^2} \phi_{n-1}$$

Gauge condition satisfied when

$$\frac{dA_{zn}}{dz} = -\frac{i\omega}{c}\phi_n$$

in the particular case n = 0

$$\frac{dA_{z0}}{dz} = -\frac{i\omega}{c}\phi_0$$

Electric field is

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \overleftarrow{A}}{\partial t}$$

And the potential and vector potential must satisfy

$$E_z(0,z) = -\frac{d\phi_0}{dz} - \frac{i\omega}{c} A_{z0}$$

$$\therefore \quad \frac{i\omega}{c} E_z(0,z) = \frac{d^2 A_{z0}}{dz^2} + \frac{\omega^2}{c^2} A_{z0} = -4 A_{z1}$$

So the magnetic field off axis may be expressed directly in terms of the electric field on axis

$$\therefore \quad B_\theta \approx -2rA_{z1} = \frac{i}{2} \frac{\omega r}{c} E_z(0,z)$$

And likewise for the radial electric field (see also $\nabla \cdot \vec{E} = 0$)

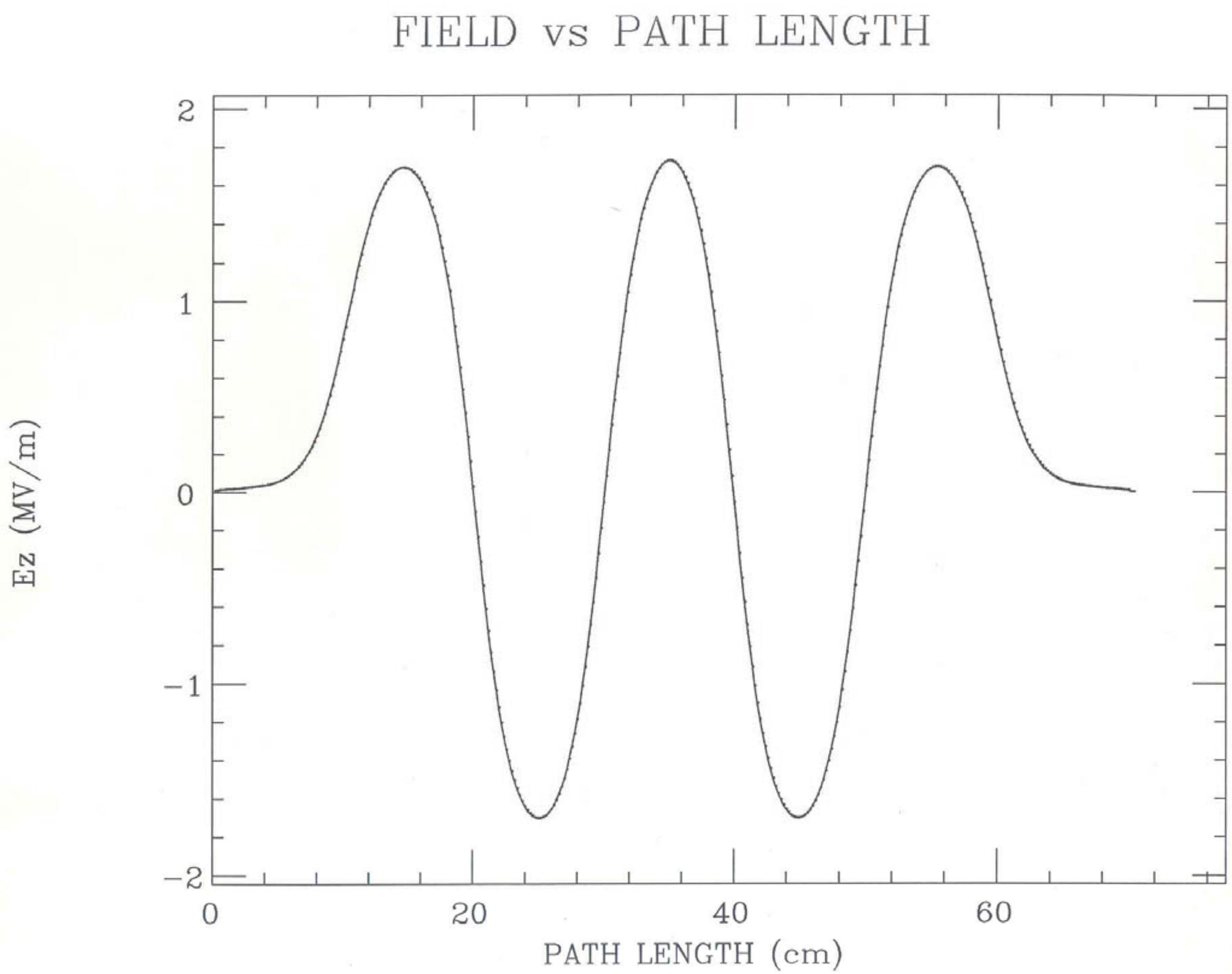
$$\therefore \quad E_r \approx -2r\phi_1(z) = -\frac{r}{2}\frac{dE_z(0,z)}{dz}$$

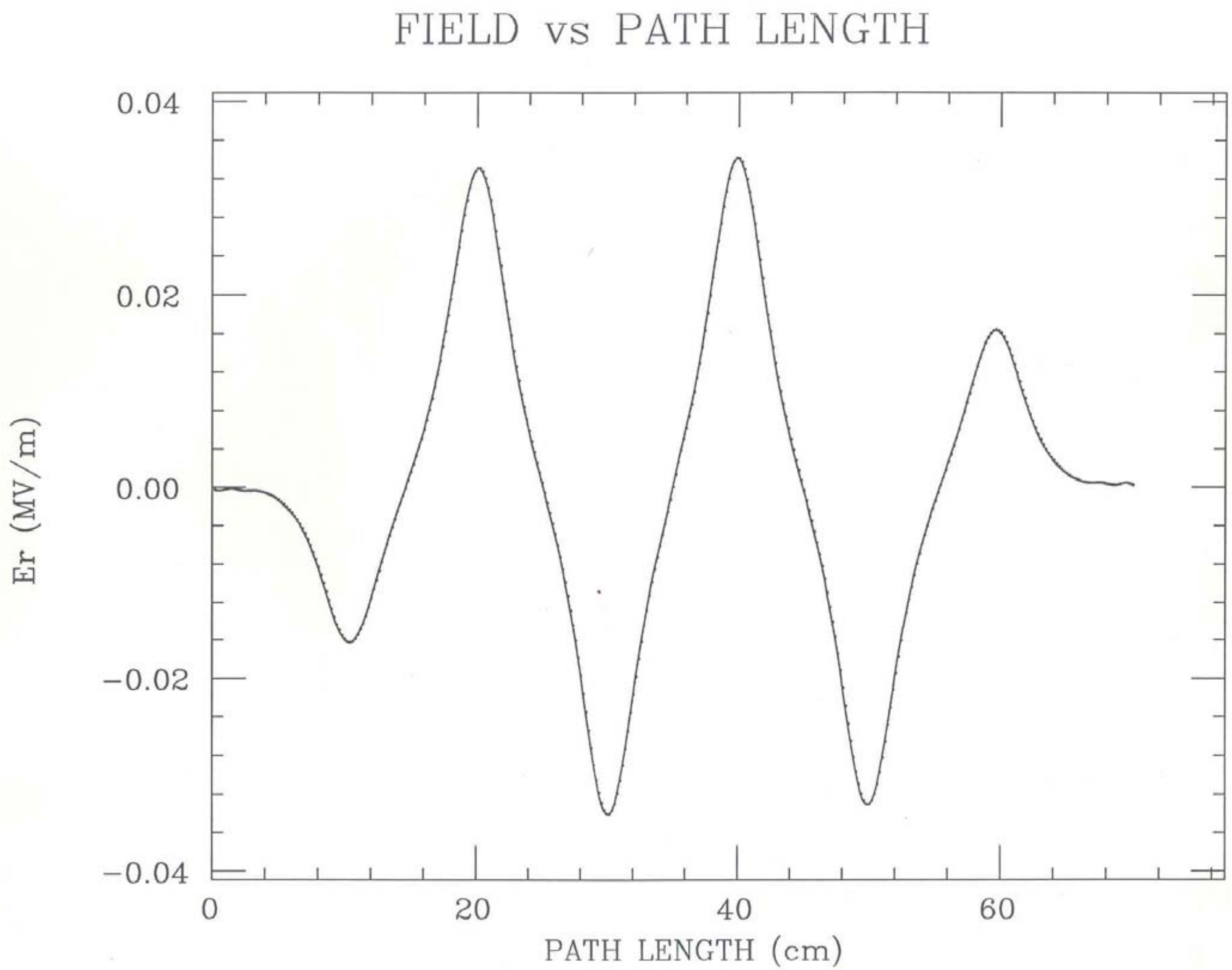
Explicitly, for the time dependence cos(ωt + δ)

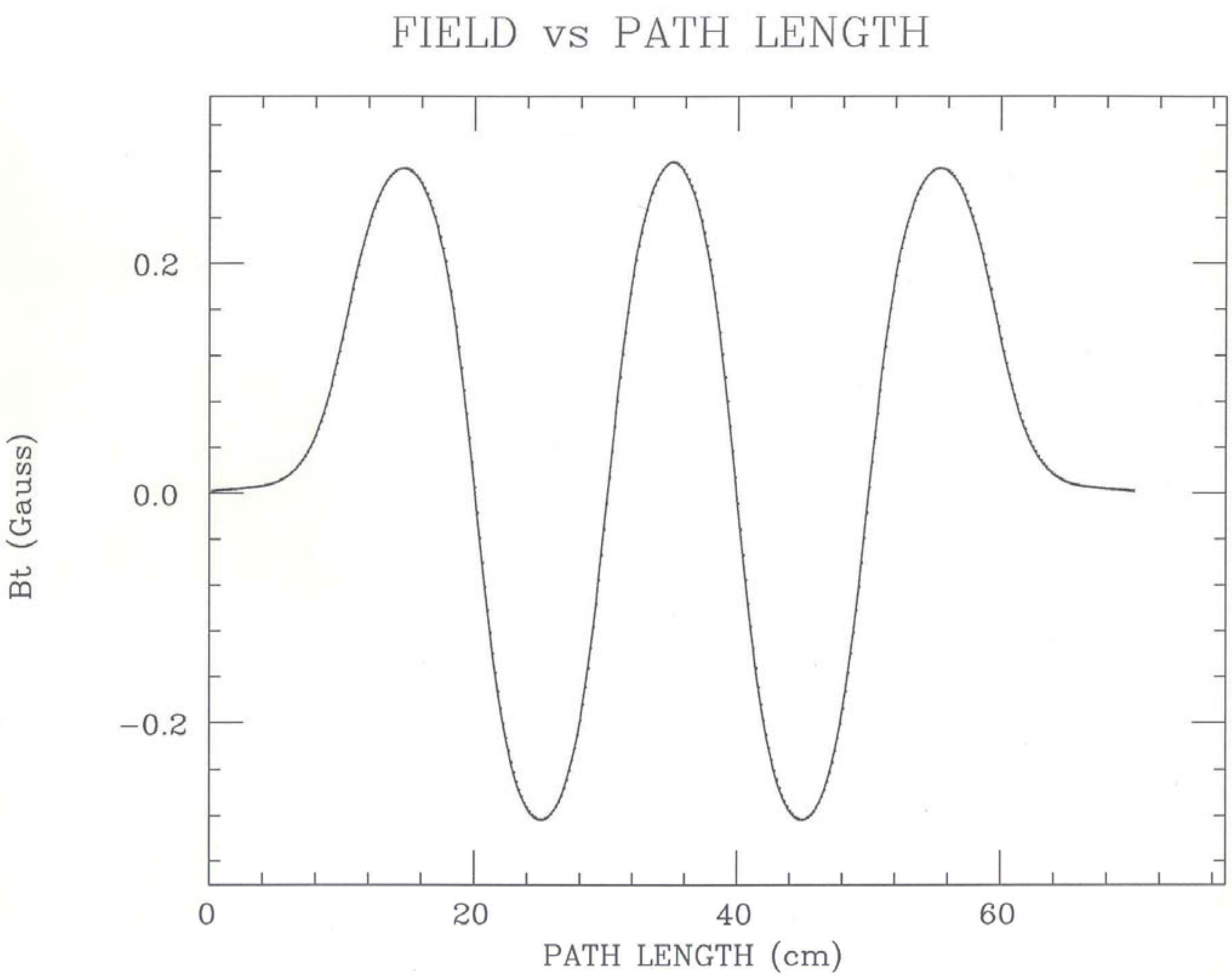
$$E_z(r,z,t) \approx E_z(0,z)\cos(\omega t + \delta)$$

$$E_r(r,z,t) \approx -\frac{r}{2}\frac{dE_z(0,z)}{dz}\cos(\omega t + \delta)$$

$$B_\theta(r,z,t) \approx -\frac{\omega r}{2c}E_z(0,z)\sin(\omega t + \delta)$$


FIELD vs PATH LENGTH
Ez (MV/m)
PATH LENGTH (cm)
2
1
0
−1
−2
0
20
40
60


FIELD vs PATH LENGTH
0.04
0.02
0.00
−0.02
−0.04
Er (MV/m)
0
20
40
60
PATH LENGTH (cm)


FIELD vs PATH LENGTH
0.2
0.0
−0.2
Bt (Gauss)
0
20
40
60
PATH LENGTH (cm)

Motion of a particle in this EM field

$$\frac{d\left(\gamma m\vec{\mathrm{V}}\right)}{dt} = -e\left(\vec{E} + \frac{\vec{V}}{c}\times\vec{B}\right)$$

$$\gamma(z)\beta_x(z) = \gamma(-\infty)\beta_x(-\infty) + \int_{-\infty}^{z}\left[\begin{array}{l} -\dfrac{x(z')}{2}\dfrac{dG(z')}{dz'}\cos(\omega t(z')+\delta) \\ +\dfrac{\omega\beta_z(z')x(z')}{2c}G(z')\sin(\omega t(z')+\delta)\end{array}\right]\frac{dz'}{\beta_z(z')}$$

The normalized gradient is

$$G(z) = \frac{eE_z(z,0)}{mc^2}$$

and the other quantities are calculated with the integral equations

$$\gamma(z) = \gamma(-\infty) + \int_{-\infty}^{z} G(z')\cos(\omega t(z') + \delta)dz'$$

$$\gamma(z)\beta_z(z) = \gamma(-\infty)\beta_z(-\infty) + \int_{-\infty}^{z} \frac{G(z')}{\beta_z(z')}\cos(\omega t(z') + \delta)dz'$$

$$t(z) = \lim_{z_0 \to -\infty} \frac{z_0}{\beta_z(-\infty)c} + \int_{-\infty}^{z} \frac{dz'}{\beta_z(z')c}$$

These equations may be integrated numerically using the cylindrically symmetric CEBAF field model to form the Douglas model of the cavity focussing. In the high energy limit the expressions simplify.

$$x(z) = x(a) + \int_a^z \frac{\gamma(z')\beta_x(z')}{\gamma(z')\beta_z(z')} dz'$$

$$\approx x(a) + \frac{\beta_x(-\infty)}{\beta_z(-\infty)}(z-a) - \int_a^z \frac{x(z')}{2} \frac{G(z')}{\gamma(z')\beta_z^2(z')} \cos(\omega t(z') + \delta) dz'$$

Transfer Matrix

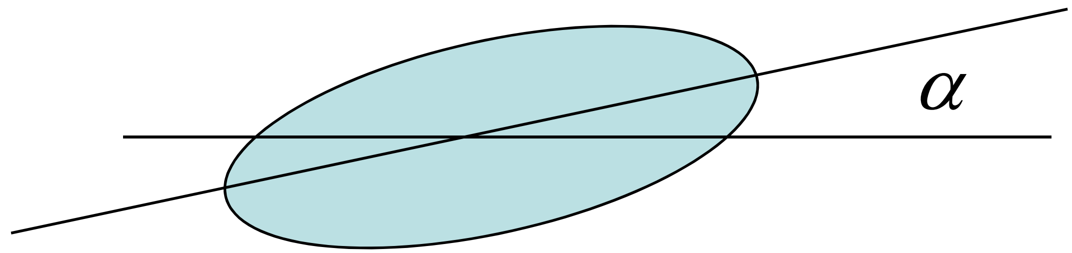
For position-momentum transfer matrix

$$T = \begin{pmatrix} 1 - \dfrac{E_G}{2E} & \dfrac{L}{\gamma} \\ -\dfrac{I}{4\gamma} & 1 + \dfrac{E_G}{2E} \end{pmatrix}$$

$$I = \cos^2(\delta)\int_{-\infty}^{\infty} G^2(z)\cos^2(\omega z / c)dz$$

$$+\sin^2(\delta)\int_{-\infty}^{\infty} G^2(z)\sin^2(\omega z / c)dz$$

Kick Generated by mis-alignment



$$\Delta\gamma\beta = \frac{E_G \alpha}{2E}$$

Damping and Antidamping

By symmetry, if electron traverses the cavity exactly on axis, there is no transverse deflection of the particle, but there is an energy increase. By conservation of transverse momentum, there must be a decrease of the phase space area. For linacs NEVER use the word “adiabatic”

$$\frac{d\left(\gamma m \vec{\mathrm{V}}_{\mathrm{transverse}}\right)}{dt} = 0$$

$$\gamma\left(z\right)\beta_x\left(z\right) = \gamma\left(-\infty\right)\beta_x\left(-\infty\right)$$

Conservation law applied to angles

$$\beta_x, \beta_y << \beta_z \approx 1$$

$$\theta_x = \beta_x / \beta_z \simeq \beta_x \quad \theta_y = \beta_y / \beta_z \simeq \beta_y$$

$$\theta_x(z) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)}\theta_x(-\infty)$$

$$\theta_y(z) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)}\theta_y(-\infty)$$

Phase space area transformation

$$dx \wedge d\theta_x(z) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)} dx \wedge d\theta_x(-\infty)$$

$$dy \wedge d\theta_y(z) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)} dy \wedge d\theta_y(-\infty)$$

Therefore, if the beam is accelerating, the phase space area after the cavity is less than that before the cavity and if the beam is decelerating the phase space area is greater than the area before the cavity. The determinate of the transformation carrying the phase space through the cavity has determinate equal to

$$\mathrm{Det}\left(M_{cavity}\right) = \frac{\gamma(-\infty)\beta_z(-\infty)}{\gamma(z)\beta_z(z)}$$

By concatenation of the transfer matrices of all the accelerating or decelerating cavities in the recirculated linac, and by the fact that the determinate of the product of two matrices is the product of the determinates, the phase space area at each location in the linac is

$$dx \wedge d\theta_x(z) = \frac{\gamma(0)\beta_z(0)}{\gamma(z)\beta_z(z)} dx \wedge d\theta_x(0)$$

$$dy \wedge d\theta_y(z) = \frac{\gamma(0)\beta_z(0)}{\gamma(z)\beta_z(z)} dy \wedge d\theta_y(0)$$

Same type of argument shows that things like orbit fluctuations are damped/amplified by acceleration/deceleration.

Transfer Matrix Non-Unimodular

$$M_{tot} = M_1 \cdot M_2$$

$$P(M) \equiv \frac{M}{\det M}$$

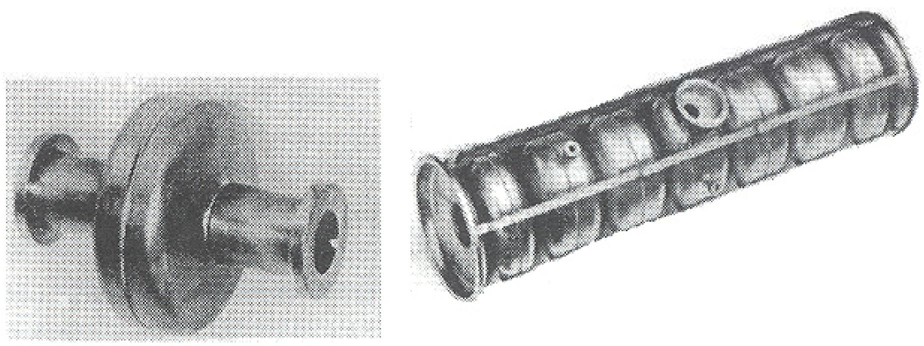
$P(M)$ unimodular!

$$P(M_{tot}) = \frac{M_{tot}}{\det M_{tot}} = \frac{M_1}{\det M_1} \frac{M_2}{\det M_2} = P(M_1) \cdot P(M_2)$$

$\therefore$ can separately track the "unimodular part" (as before!) and normalize by accumulated determinate

Stanford Superconducting Accelerator

- HEPL at Stanford University was the pioneer laboratory in exploration of srf for accelerator applications. In 1965 they accelerated electrons in a lead-plated resonator
- In 1977 HEPL completed first Superconducting Accelerator (SCA) providing 50 MV in 27m linac at 1.3 GHz.



(a) (b)

(a) Single-c ... Hz
(b) HEPL 7

MUSL-2 at University of Illinois

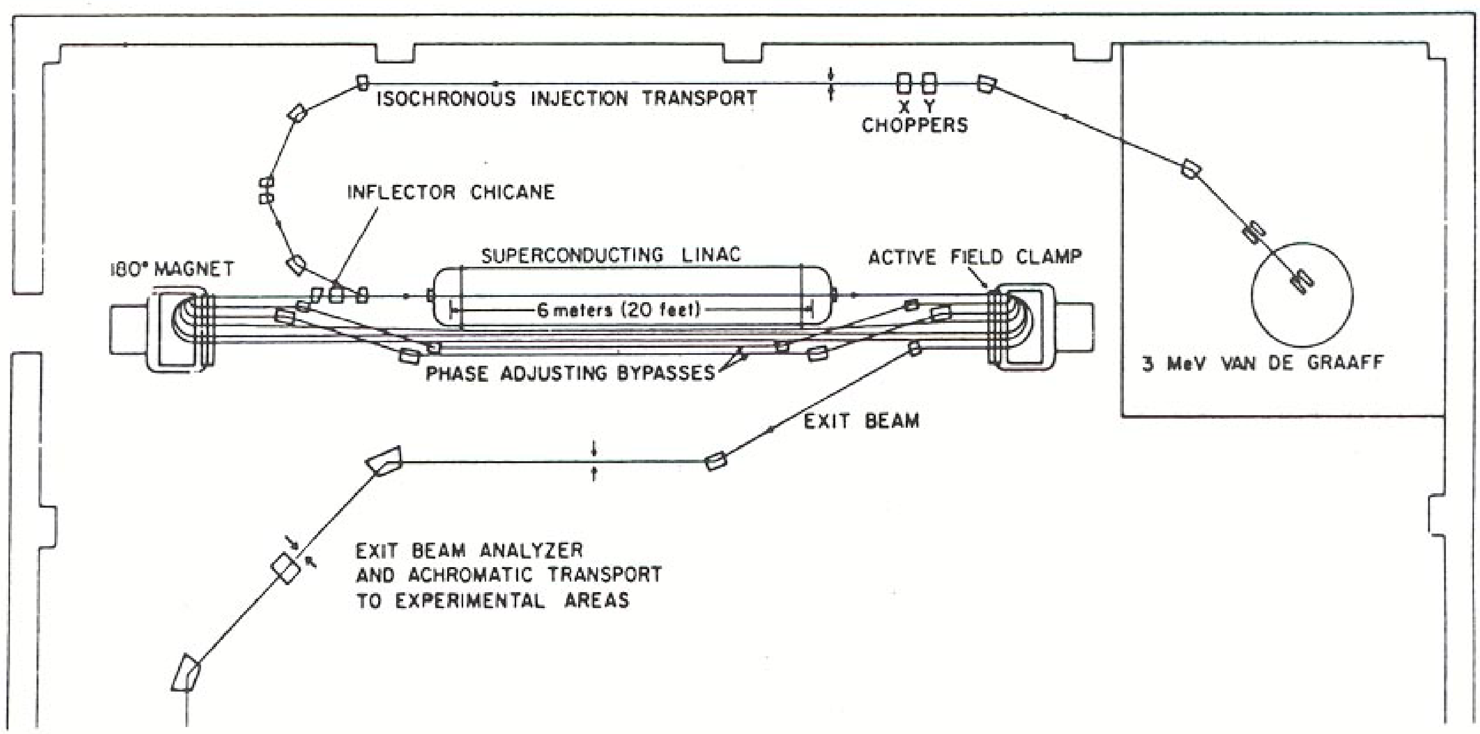
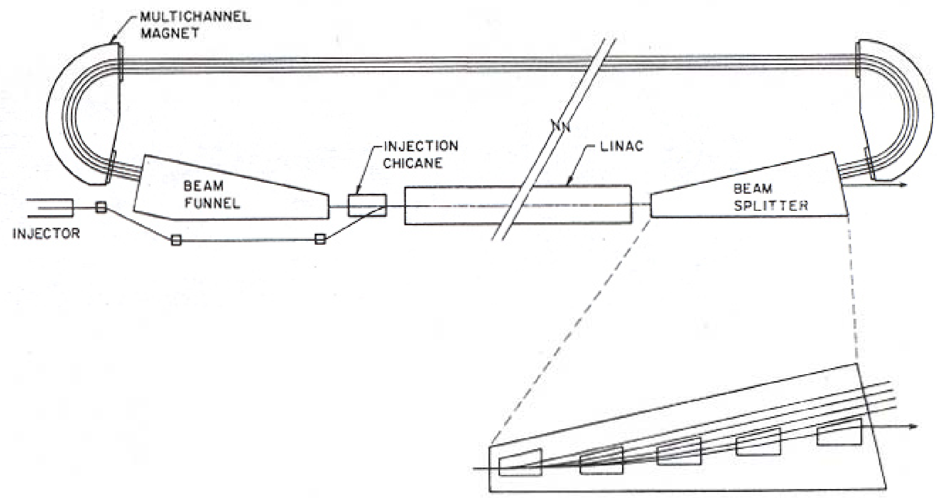


Figure 5.3 The second Illinois superconducting race-trace microtron, MUSL-2 (Axel *et al.*, 1977; © 1977 IEEE)

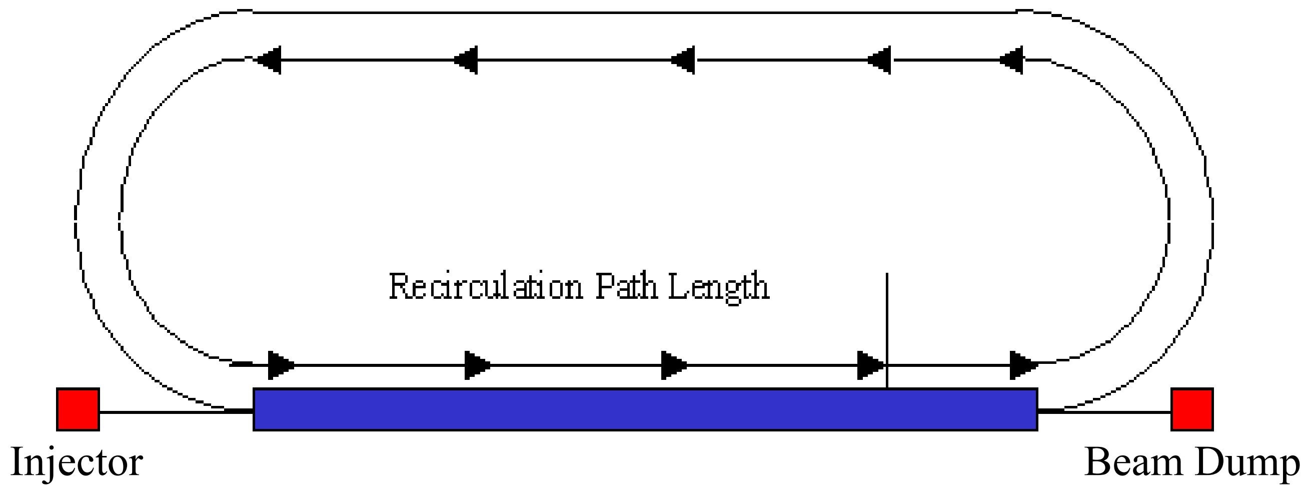
The Stanford–HEPL “Recyclotron”

- Main recirculation magnets incorporate four channels (tracks) in which the uniform fields are independently tailored to the momenta of the separate orbits.
  – Use a constant magnet gap with staggered coil windings which produce an
  appropriately stepped field profile.



Beam Energy Recovery in an RF Linac

$$\frac{d\gamma}{dt} = \frac{e\vec{E} \cdot \vec{\mathrm{v}}}{mc^2}$$

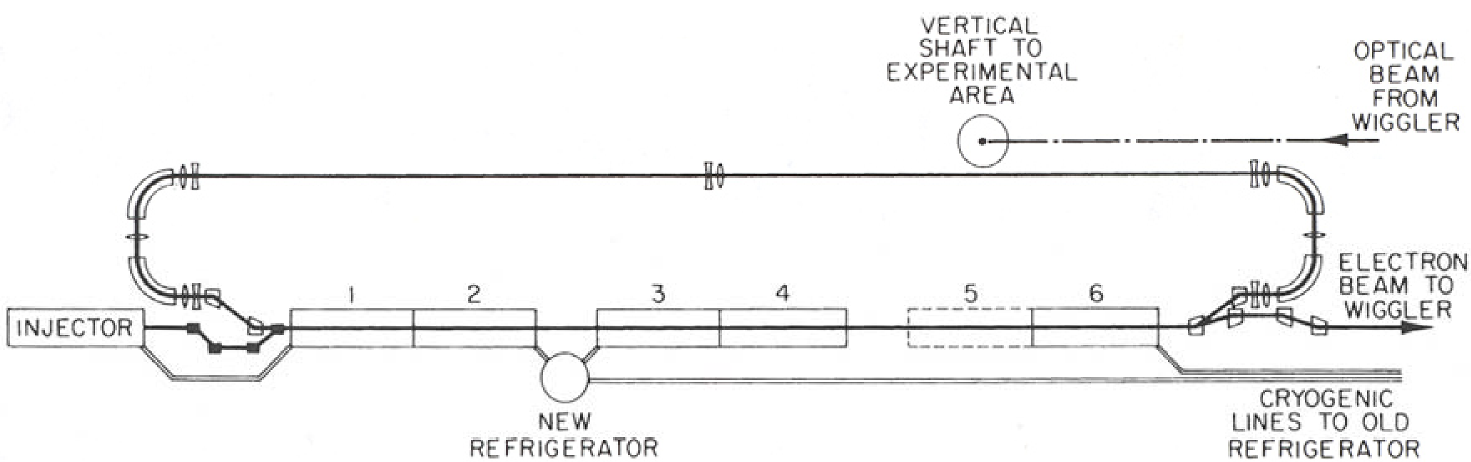


Recirculation path length in standard configuration front-to-back recirculated linac. For energy recovery choose it to be $(n + 1/2)\lambda_{RF}$. Then

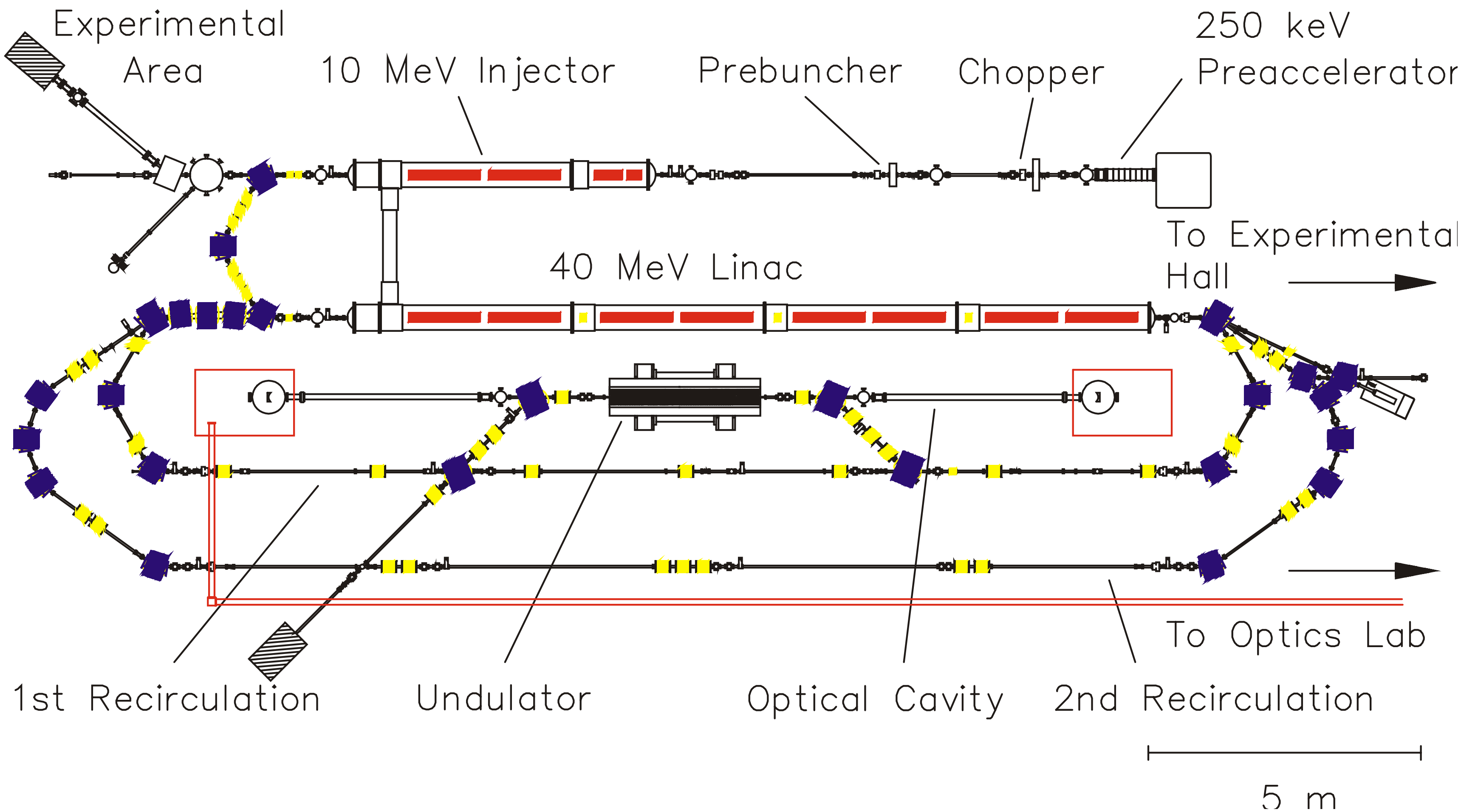
$$E_{\text{Injector}} \approx E_{\text{Beam Dump}}$$

SCA/FEL Energy Recovery Experiment

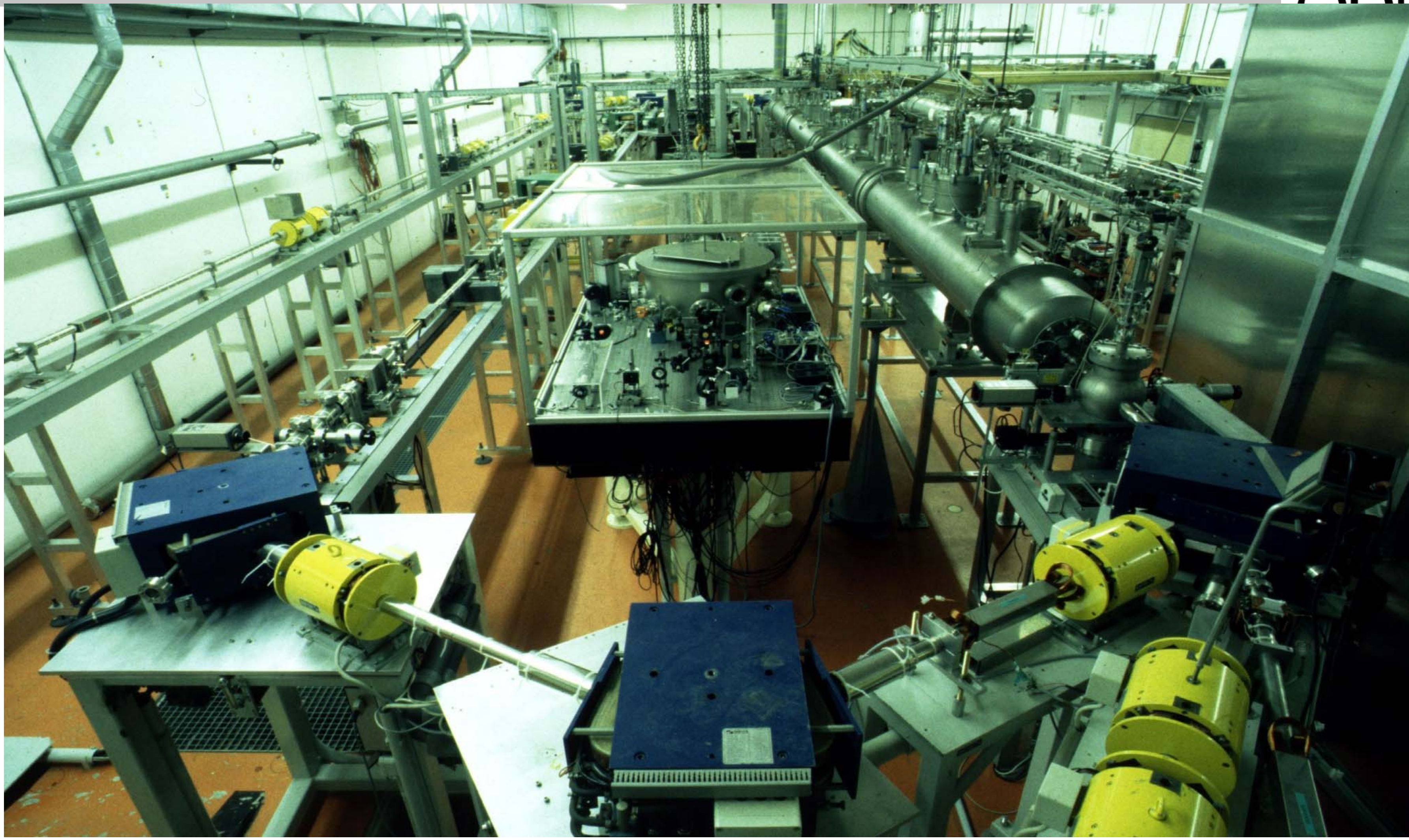
- Same-cell energy recovery was first demonstrated in a superconducting linac at the Stanford SCA/FEL in July 1986 NIMA 259, 1 (1987)
- Beam was injected at 5 MeV into a 45 MeV linac (up to 95 MeV in 2 passes), 150 μA average current (12.5 pC per bunch at 11.8 MHz)
- The previous "Recyclotron" beam recirculation system could be not used to produce the peak current required for lasing and was replaced by a doubly achromatic single-turn recirculation line.
- Nearly all the energy was recovered. No FEL inside the recirculation loop.



Layout of S-DALINAC (Darmstadt)



S-DALINAC



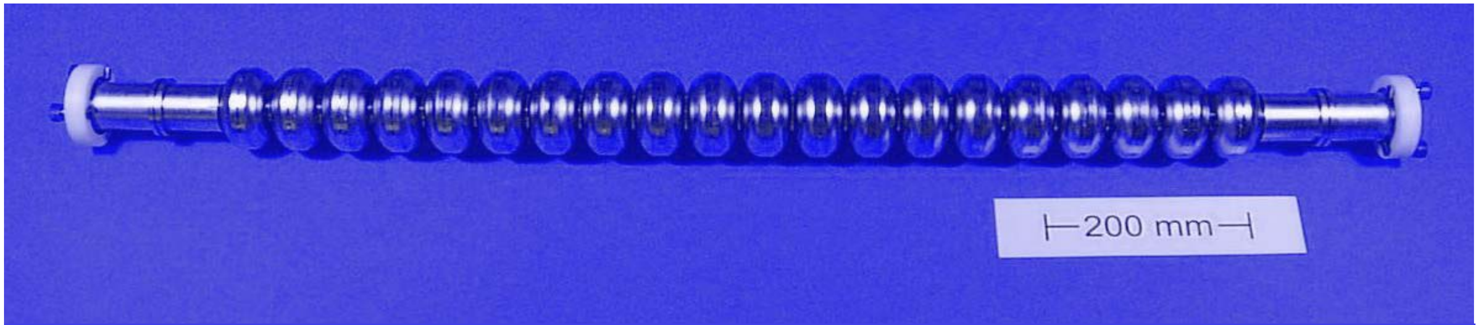
S-DALINAC Beam Parameters

Experiments	Energy (MeV)	Current (μA)	Mode	Time (h)
(γ,γ')	2.5 – 10	50	3 GHz, cw	6400
LEC, PXR	3 – 10	0.001 - 10	3 GHz, cw	2100
HEC, PXR	35 – 87	0.1	3 GHz, cw	800
(e,e'), (e,e'x)	22 – 120[1)]	5	3 GHz, cw	7800
FEL	30 – 38	2.7 A_{peak}	10 MHz, cw	2900
				$\sum$ 20000

1) Dutycycle 33%

Resolution: ΔE_{FWHM} = 50 keV @ 85 MeV, $\Delta E/E = \pm 3\cdot 10^{-4}$

Superconducting 20-Cell Cavity



Material:	Niobium (RRR=280)
Frequency:	3 GHz
Temperature:	2 K
Accelerating Field:	5 MV/m
Q_0/Q_L:	$3\cdot10^9$ / $3\cdot10^7$
$\Delta f/\Delta l$:	500 Hz/µm

The CEBAF at Jefferson Lab

- "Radical" innovations (had not been done before on the scale of CEBAF):
  - choice of Superconducting Radio Frequency (SRF) technology
  - use of multipass beam recirculation
- Until LEP II came into operation, CEBAF was the world's largest implementation of SRF technology.

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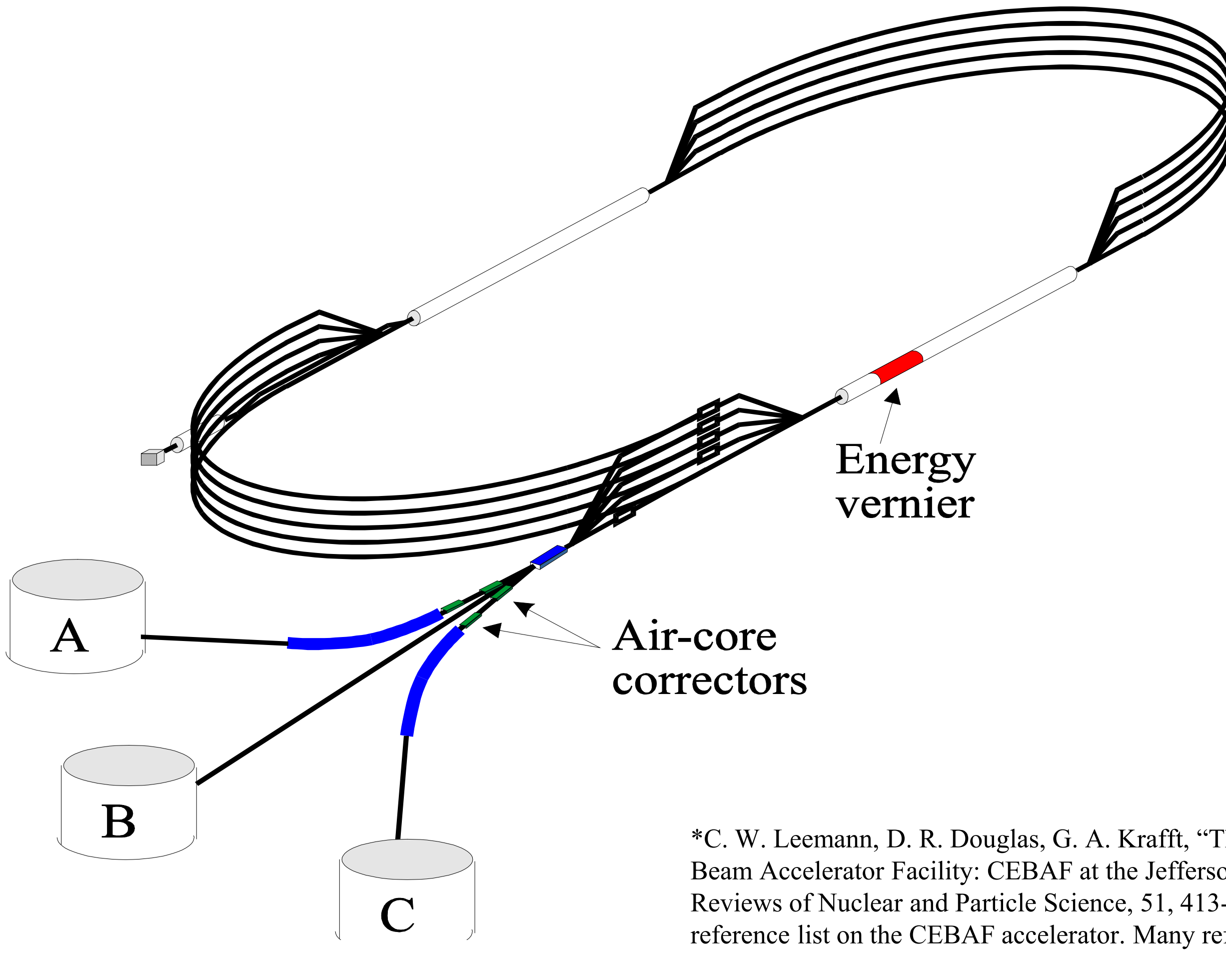
CEBAF Accelerator Layout*

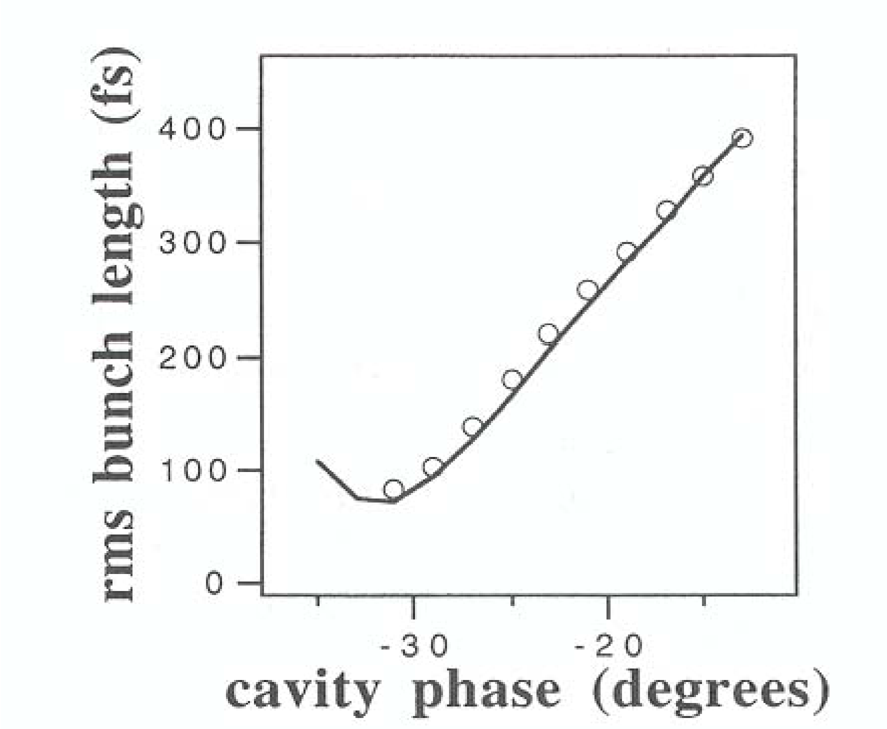


*C. W. Leemann, D. R. Douglas, G. A. Krafft, “The Continuous Electron Beam Accelerator Facility: CEBAF at the Jefferson Laboratory”, Annual Reviews of Nuclear and Particle Science, 51, 413-50 (2001) has a long reference list on the CEBAF accelerator. Many references on Energy Recovered Linacs may be found in a recent ICFA Beam Dynamics Newsletter, #26, Dec. 2001: http://icfa-usa/archive/newsletter/icfa_bd_nl_26.pdf

CEBAF Beam Parameters

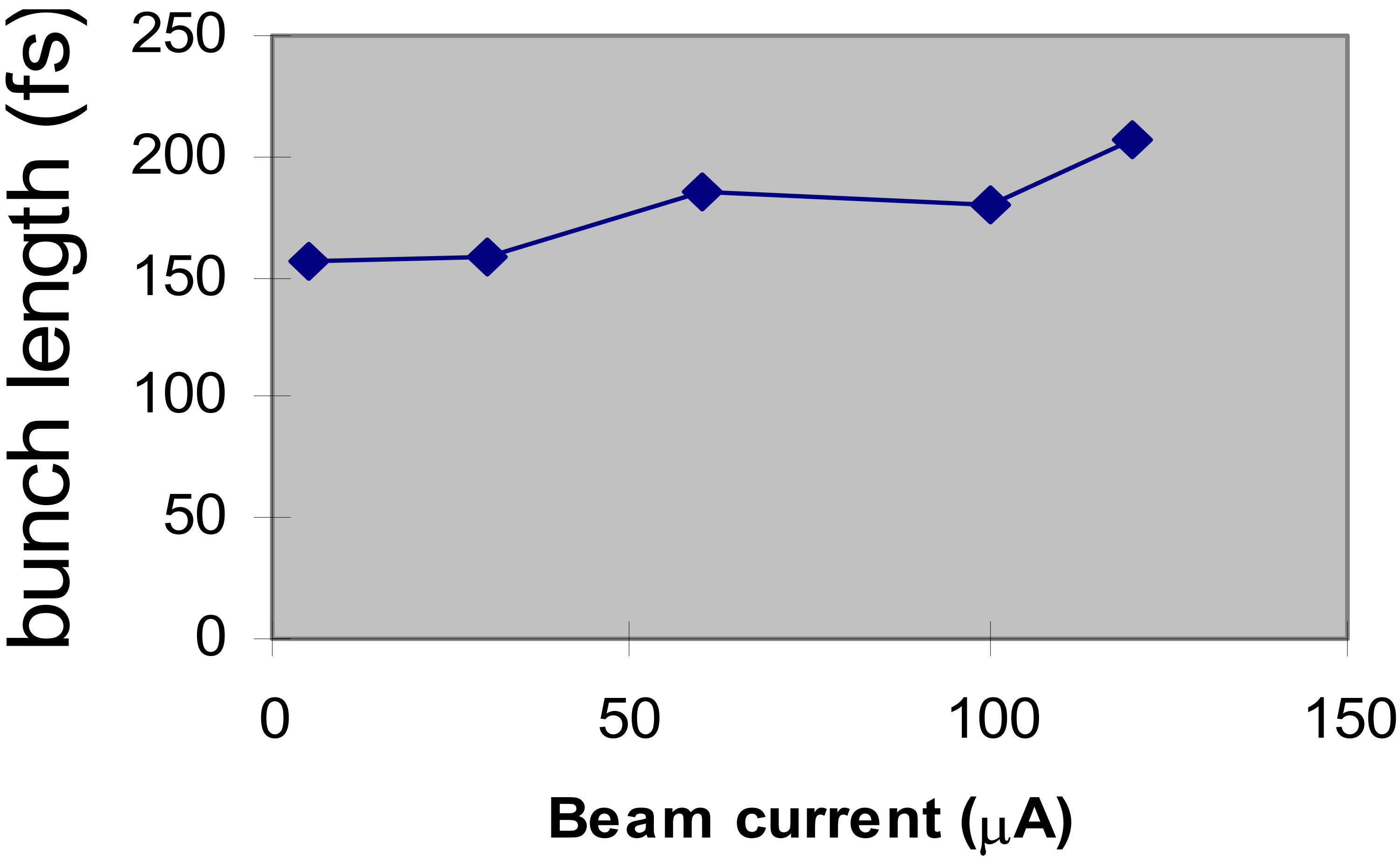
Beam energy	6 GeV
Beam current	A 100 μA, B 10-200 nA, C 100 μA
Normalized rms emittance	1 mm mrad
Repetition rate	500 MHz/Hall
Charge per bunch	< 0.2 pC
Extracted energy spread	< 10^{-4}
Beam sizes (transverse)	< 100 microns
Beam size (longitudinal)	<100 microns (330 fsec)
Beam angle spread	< $0.1/\gamma$

Short Bunches in CEBAF



Wang, Krafft, and Sinclair, Phys. Rev. E, 2283 (1998)

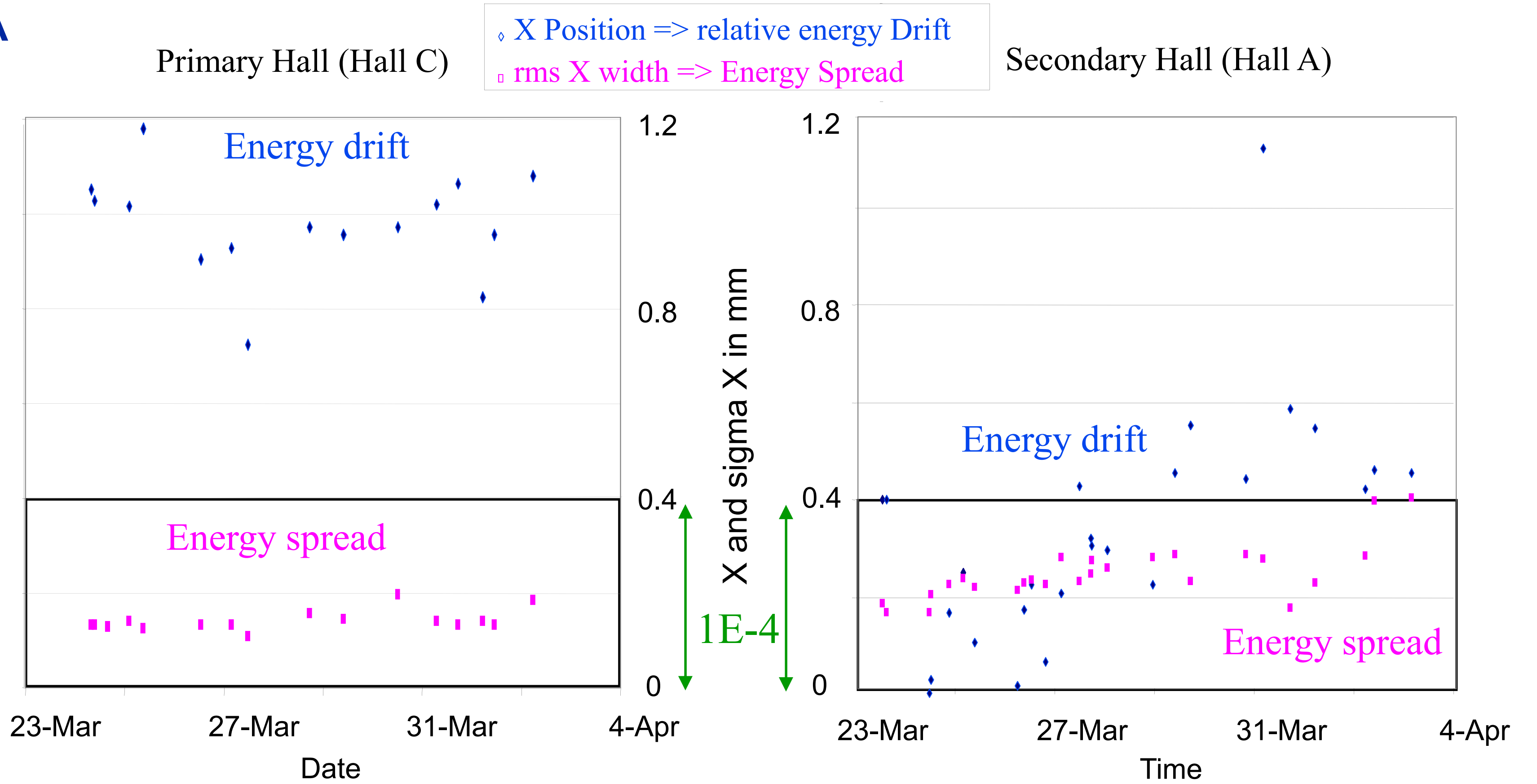
Short Bunch Configuration



Kazimi, Sinclair, and Krafft, *Proc. 2000 LINAC Conf.*, 125 (2000)

dp/p data: 2-Week Sample Record

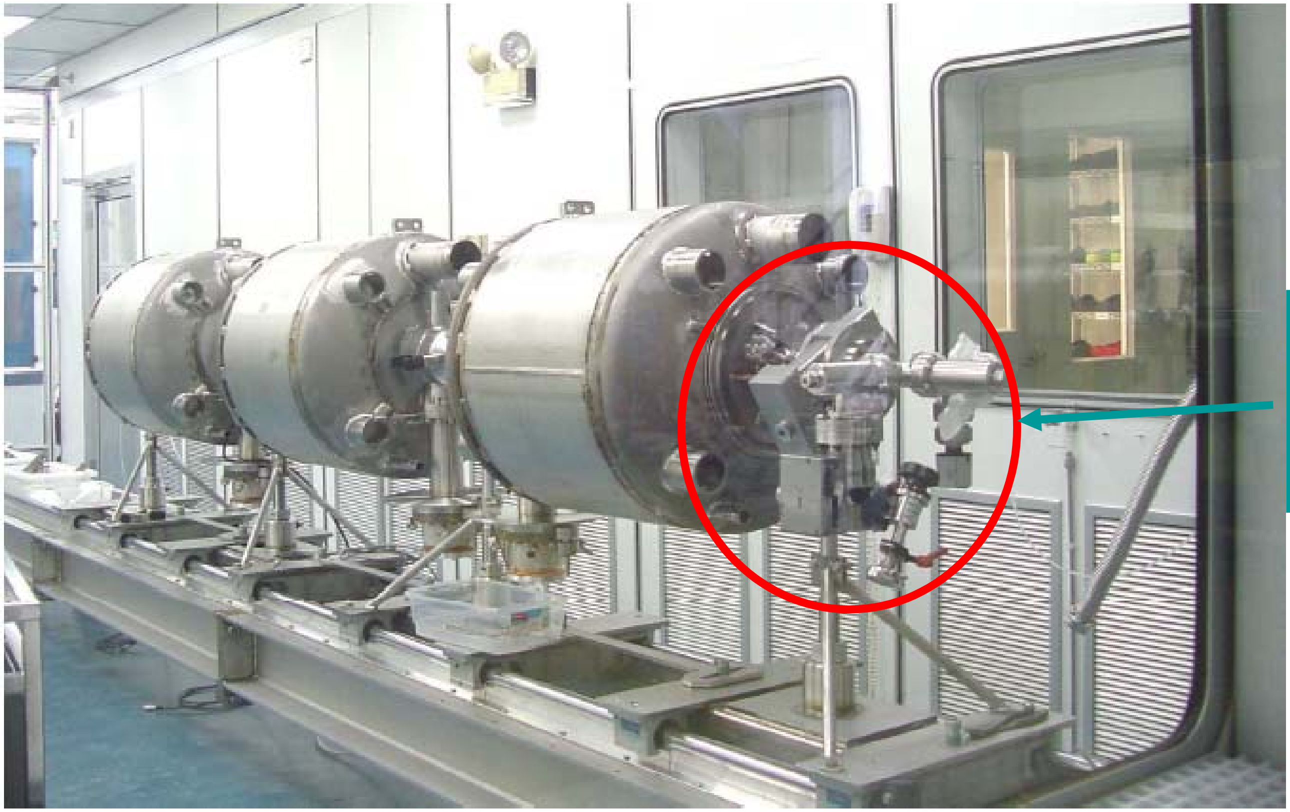
Energy Spread less than 50 ppm in Hall C, 100 ppm in Hall A



Courtesy: Jean-Claude Denard

SNS Medium Beta Modules

SNS Medium Beta Cavity String: three 6-cell 805 MHz cavities



Pump-out assembly

Energy Recovered Linacs

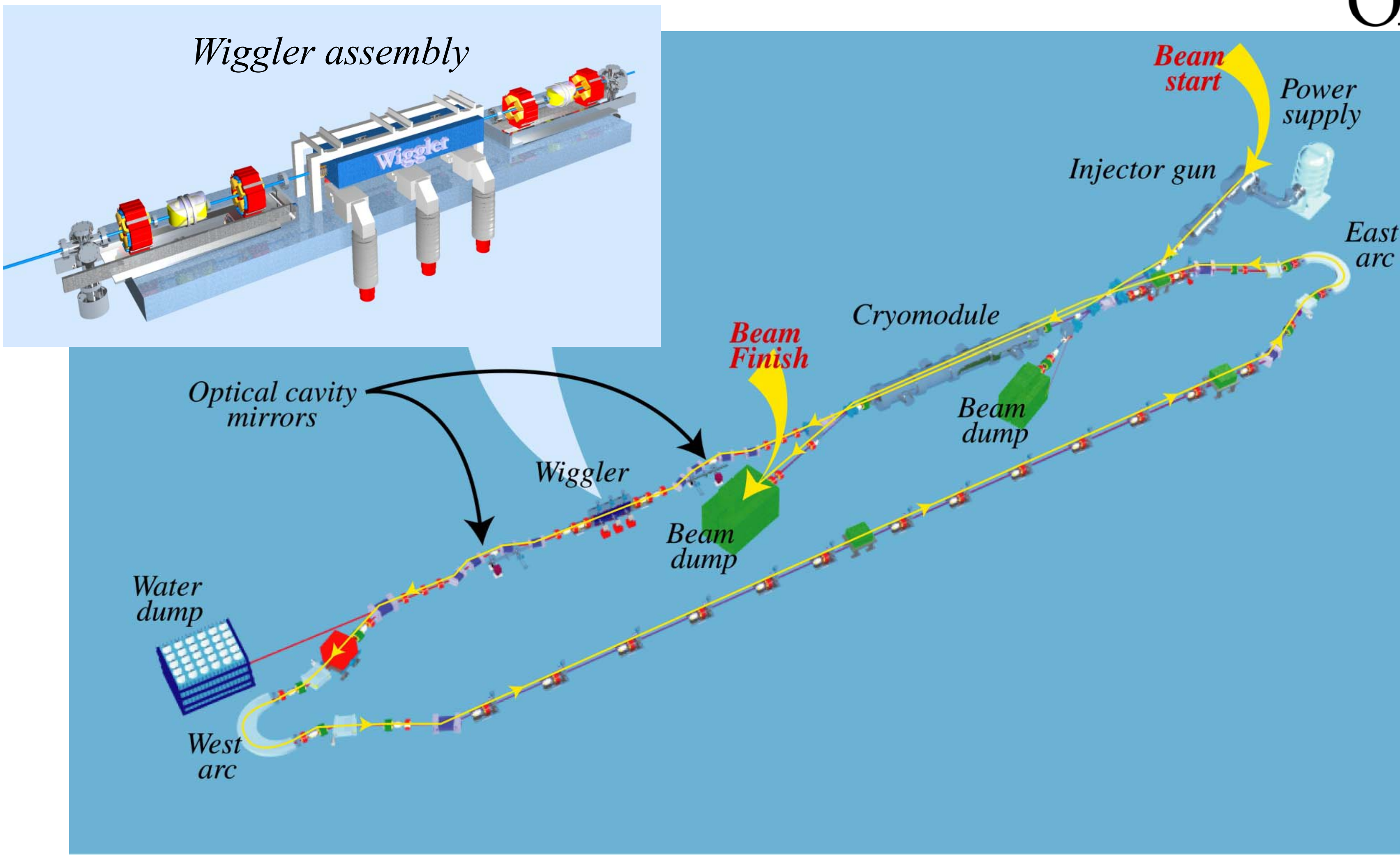
- The concept of energy recovery first appears in literature by Maury Tigner, as a suggestion for alternate HEP colliders*

- There have been several energy recovery experiments to date, the first one in a superconducting linac at the Stanford SCA/FEL**

- Same-cell energy recovery with cw beam current up to 10 mA and energy up to 150 MeV has been demonstrated at the Jefferson Lab 10 kW FEL. Energy recovery is used routinely for the operation of the FEL as a user facility

* Maury Tigner, Nuovo Cimento 37 (1965)

Jefferson Lab IR DEMO FEL



Neil, G. R., *et. al, Physical Review Letters,* 84, 622 (2000)

FEL Accelerator Parameters

Parameter	Designed	Measured
Kinetic Energy	48 MeV	48.0 MeV
Average current	5 mA	4.8 mA
Bunch charge	60 pC	Up to 135 pC
Bunch length (rms)	<1 ps	0.4±0.1 ps
Peak current	22 A	Up to 60 A
Trans. Emittance (rms)	<8.7 mm-mr	7.5±1.5 mm-mr
Long. Emittance (rms)	33 keV-deg	26±7 keV-deg
Pulse repetition frequency (PRF)	18.7 MHz, x2	18.7 MHz, x0.25, x0.5, x2, and x4

FEL Accelerator Parameters

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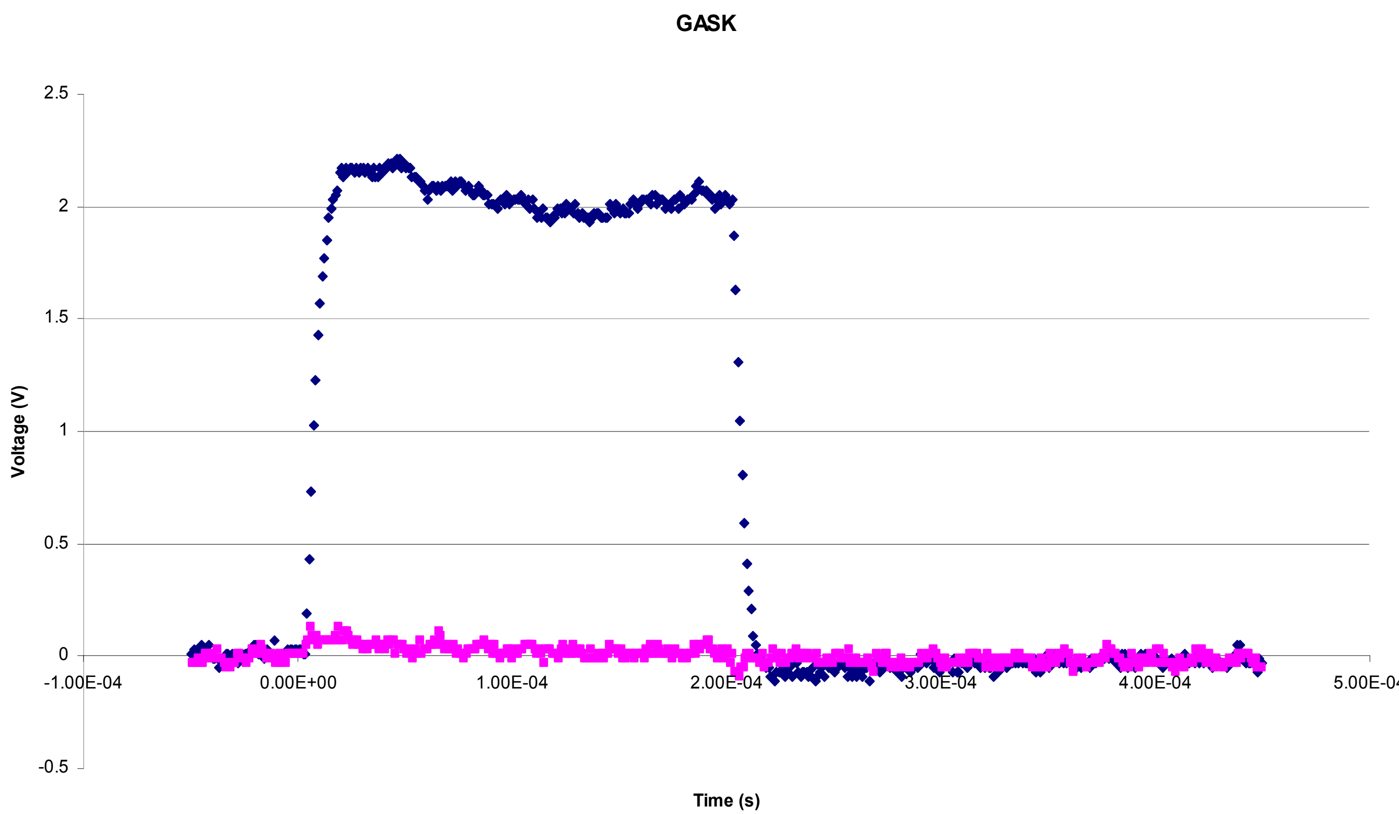
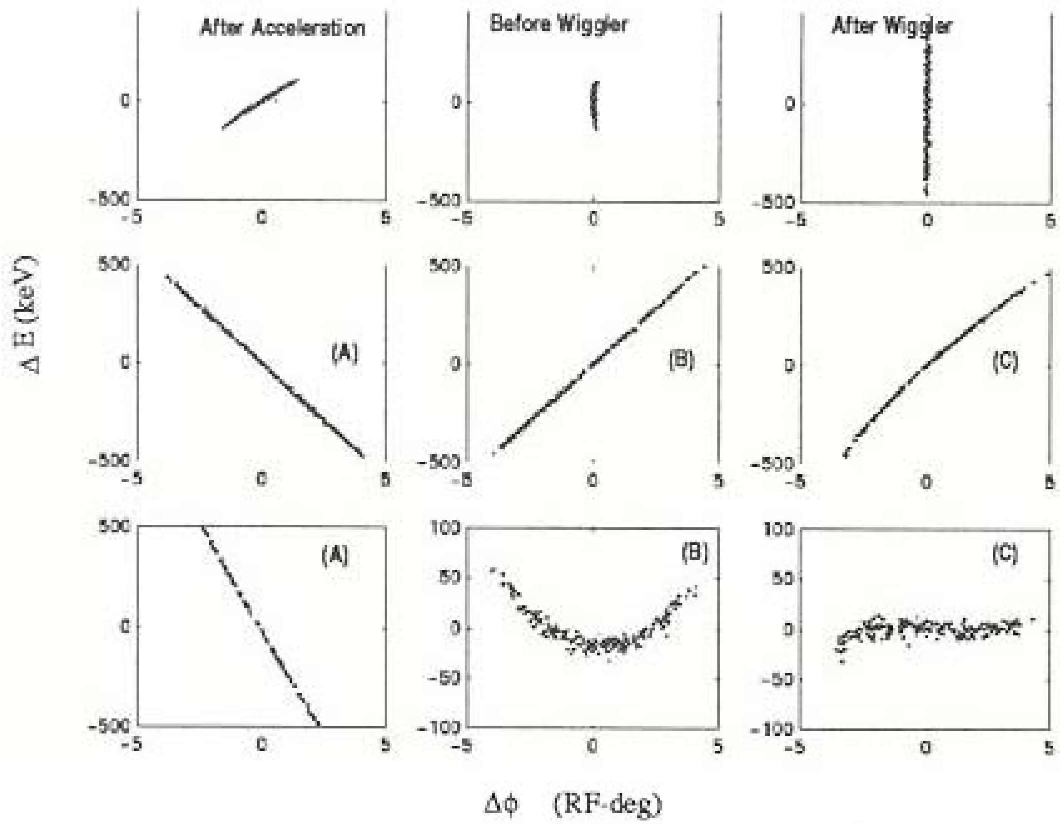
ENERGY RECOVERY WORKS

Gradient modulator drive signal in a linac cavity measured without energy recovery (signal level around 2 V) and with energy recovery (signal level around 0).



Courtesy: Lia Merminga

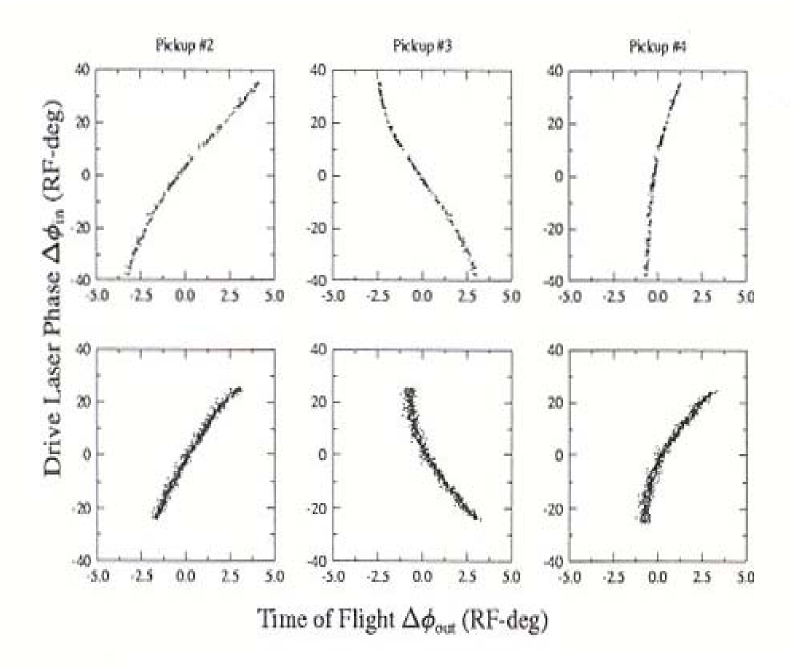
Longitudinal Phase Space Manipulations



Simulation calculations of longitudinal dynamics of JLAB FEL

Piot, Douglas, and Krafft, *Phys. Rev. ST-AB,* 6, 0030702 (2003)

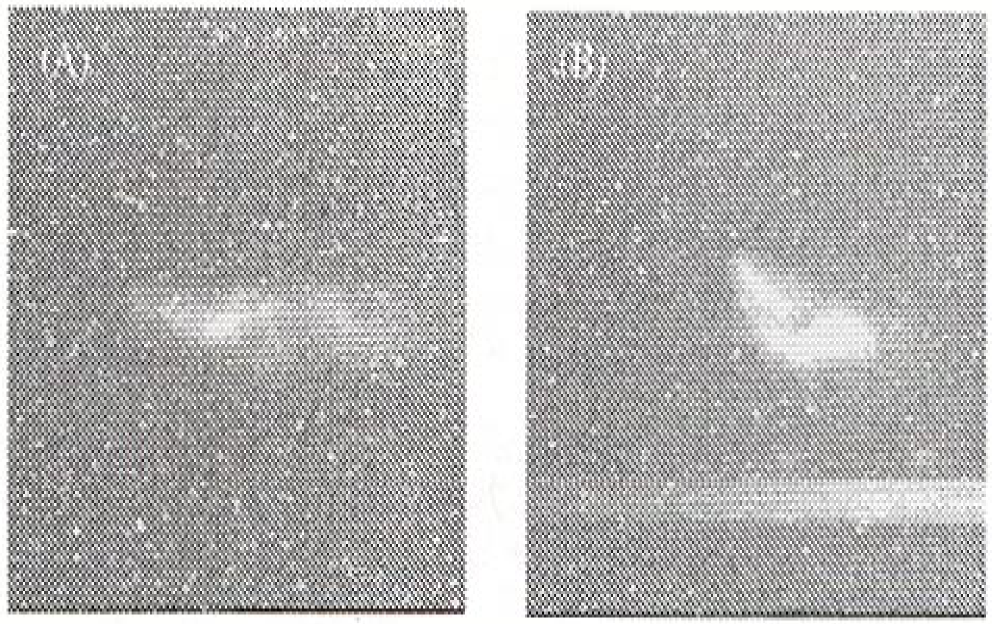
Phase Transfer Function Measurements



Experiment		
# 2	0.1172	0.0008
# 3	-0.0801	0.0016
# 4	0.0911	0.0006
Simulation		
# 2	0.1070	0.0007
# 3	-0.0834	0.0003
# 4	0.0256	0.0004

Krafft, G. A., *et. al, ERL2005 Workshop Proc. in NIMA*

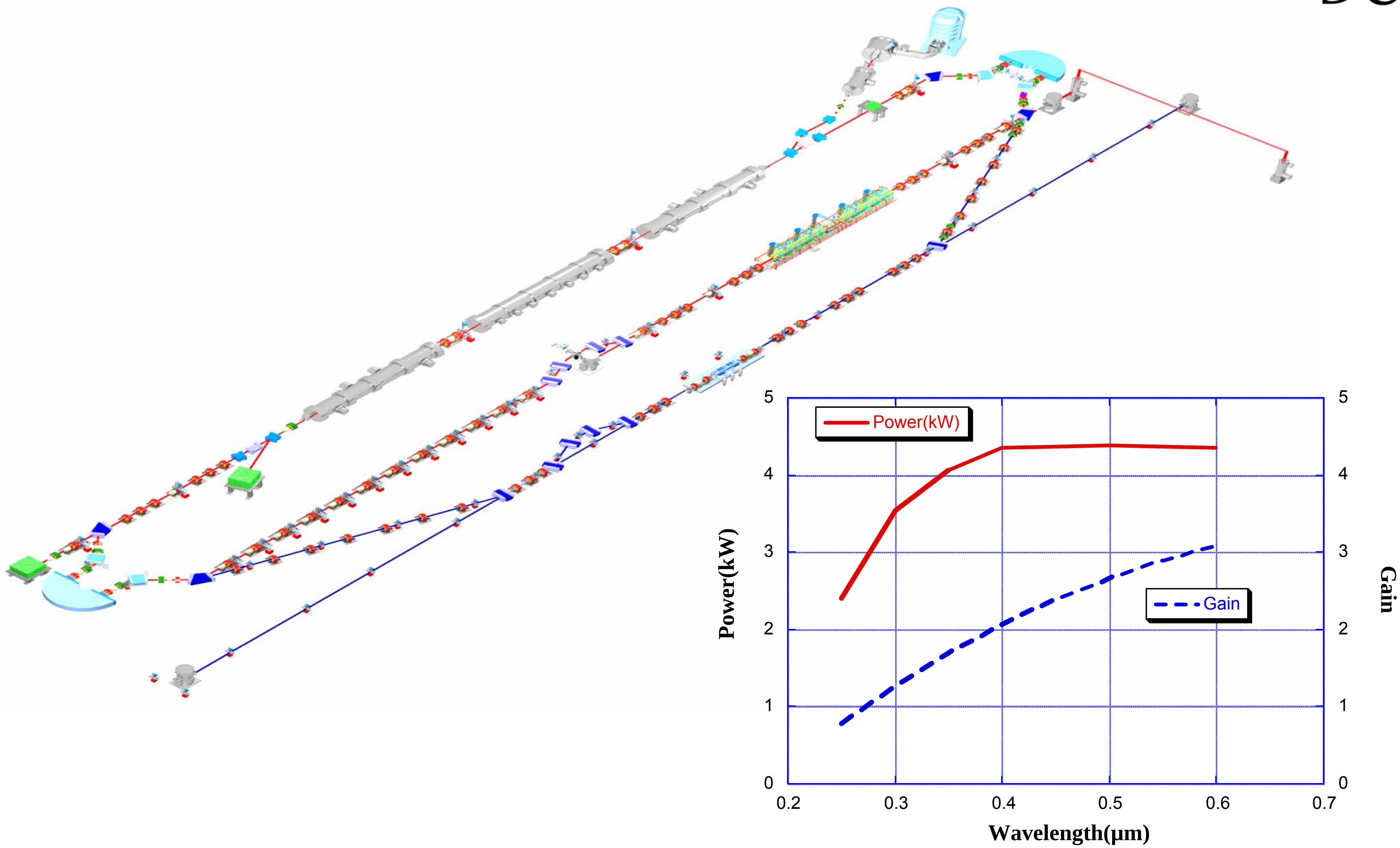
Longitudinal Nonlinearities Corrected by Sextupoles



Sextupoles Off Nominal Settings

Basic Idea is to use sextupoles to get T_{566} in the bending arc to compensate any curvature in the phase space.

IR FEL Upgrade



IR FEL 10 kW Upgrade Parameters

Parameter	Design Value
Kinetic Energy	160 MeV
Average Current	10 mA
Bunch Charge	135 pC
Bunch Length	<300 fsec
Transverse Emittance	10 mm mrad
Longitudinal Emittance	30 keV deg
Repetition Rate	75 MHz

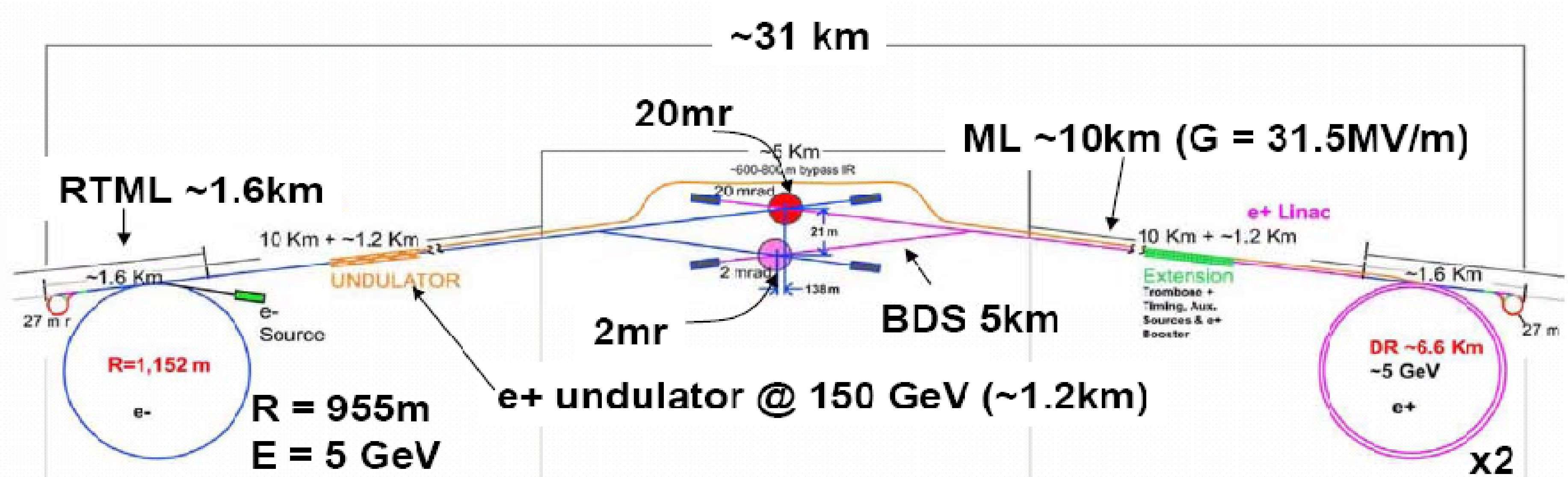
Parameters for the ILC

World Wide Study establishes parameters:

- e^+ »« e^-
- E_{cm} adjustable from 200 – 500 GeV
- $\int \mathscr{L} dt$ = 500 fb^{-1} in 4 years
- Energy stability and precision better than 0.1%
- Electron polarization of at least 80% (positron polarization, if possible)
- upgradeable to 1 TeV

From Barry Barish

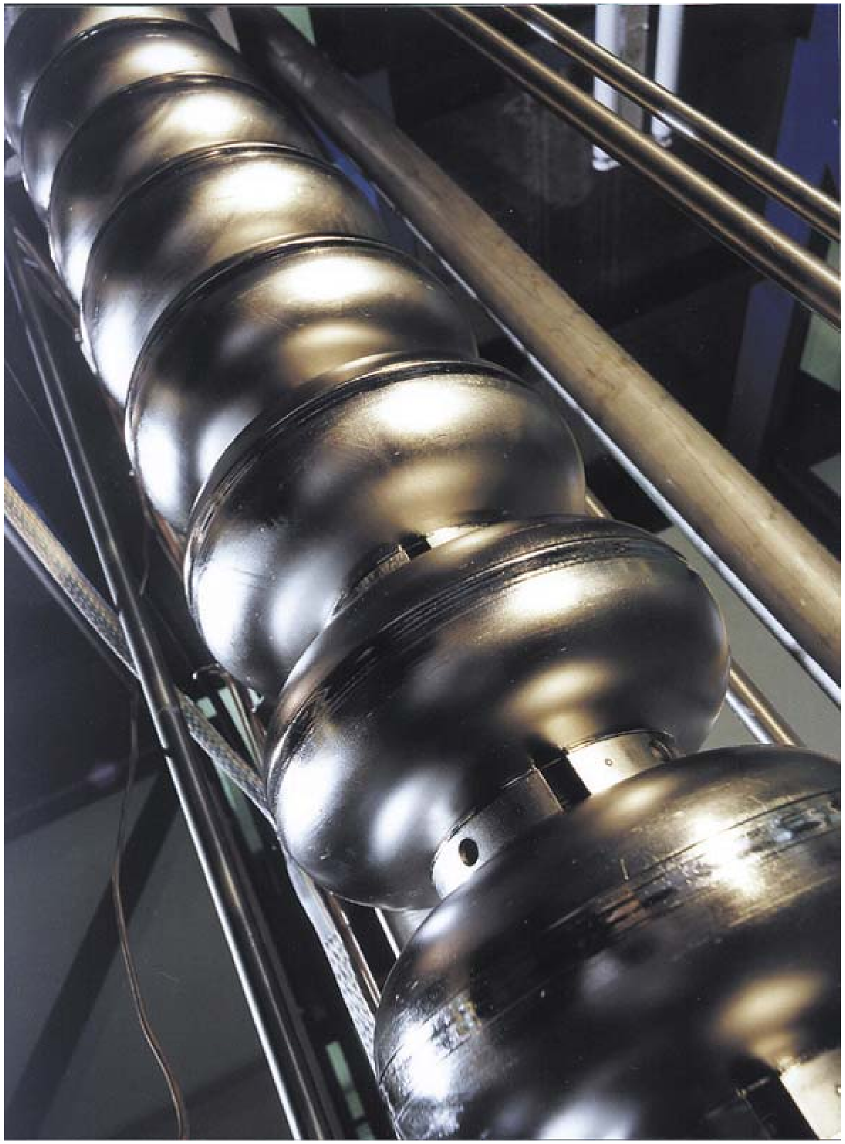
The Baseline Machine



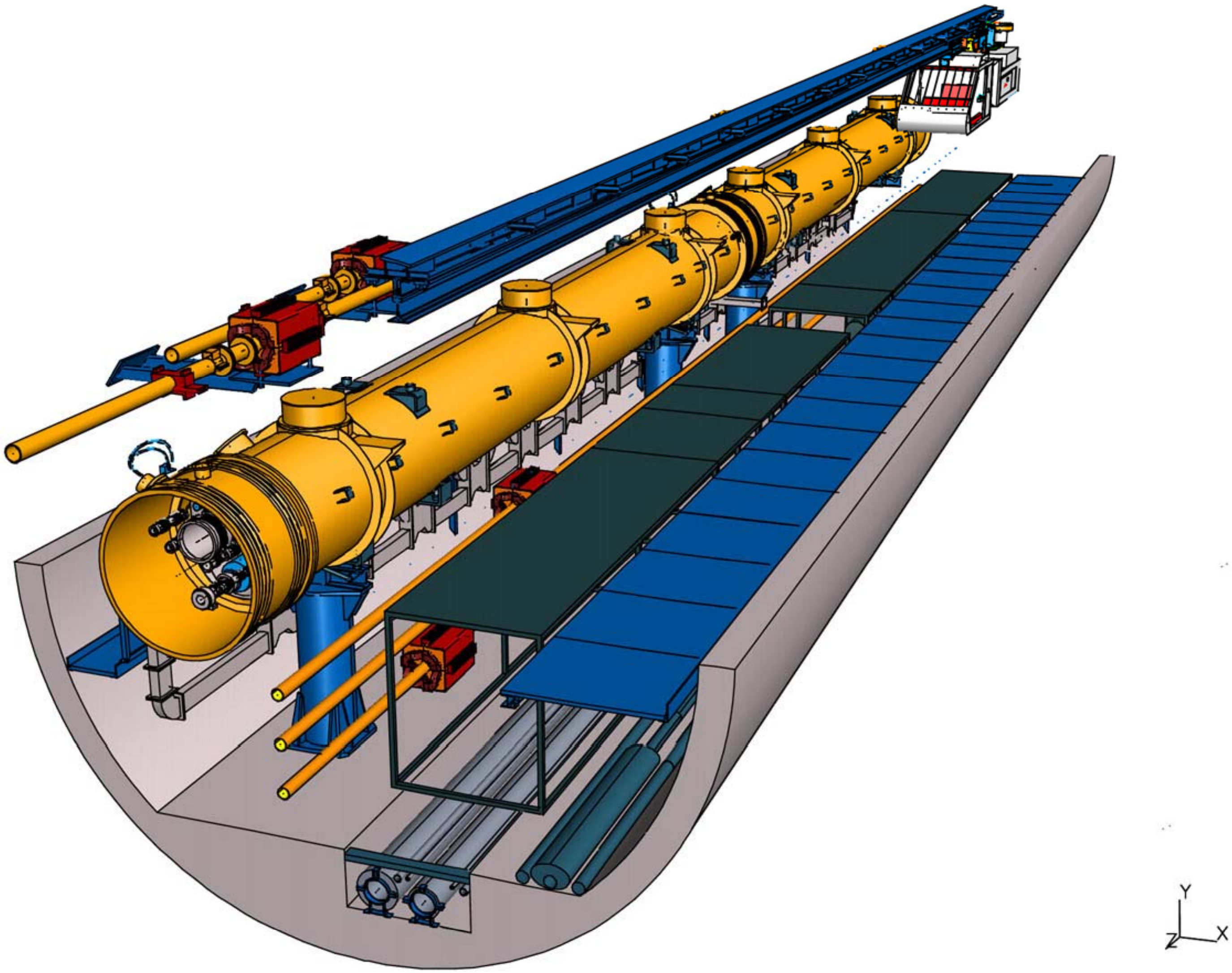
not to scale

From Barry Barish

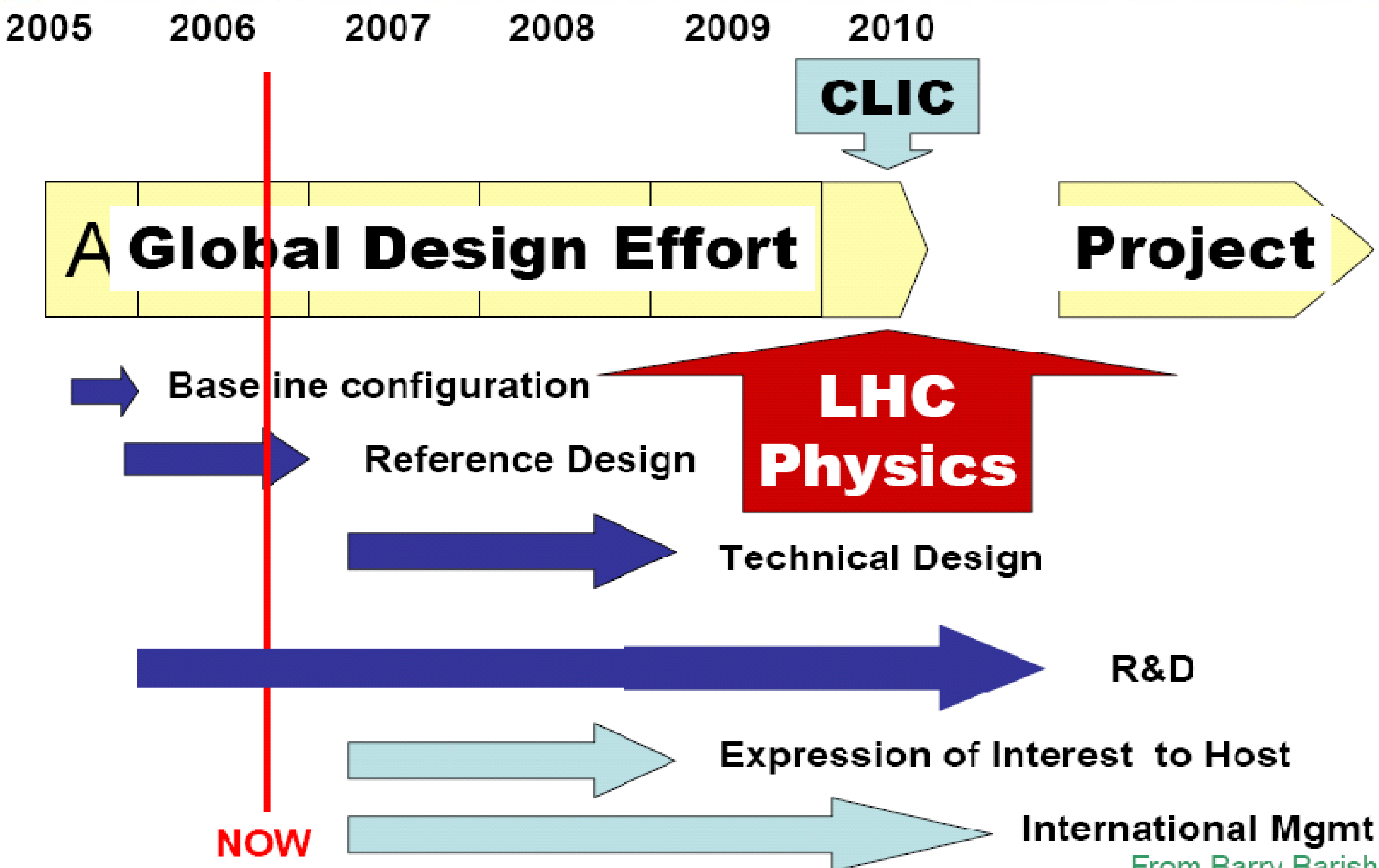
ILC (TESLA) 9-Cell Cavity



ILC Superconducting Linac

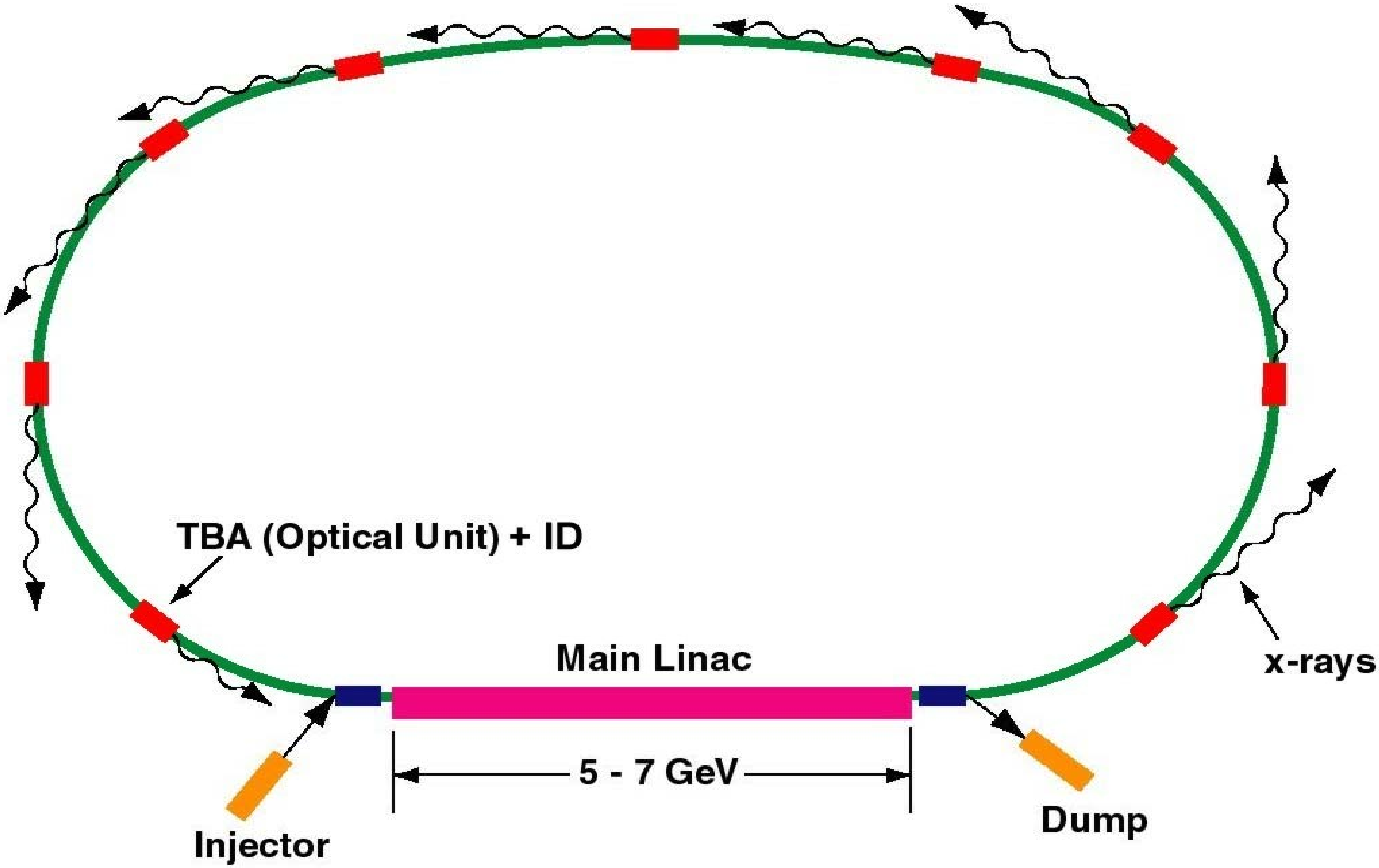


The Project Schedule

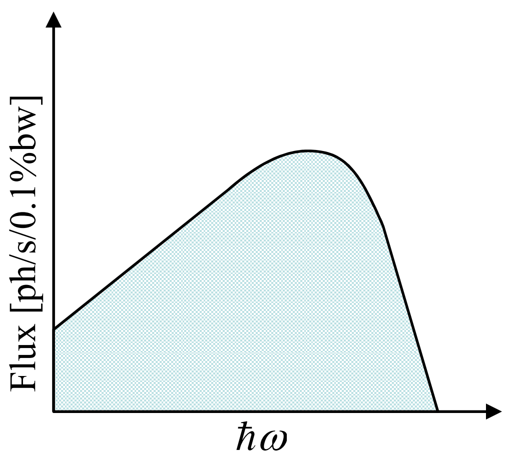


From Barry Barish

ERL X-ray Source Conceptual Layout

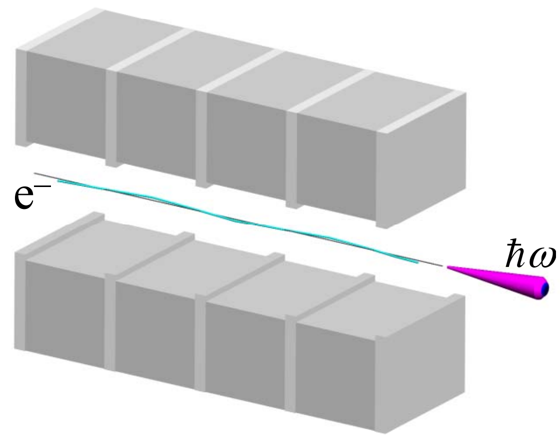


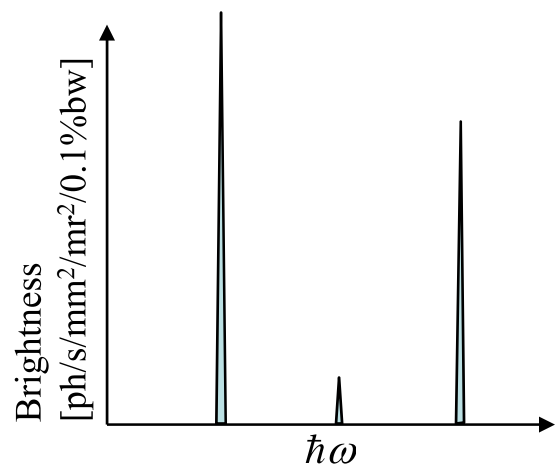
Bend



white source

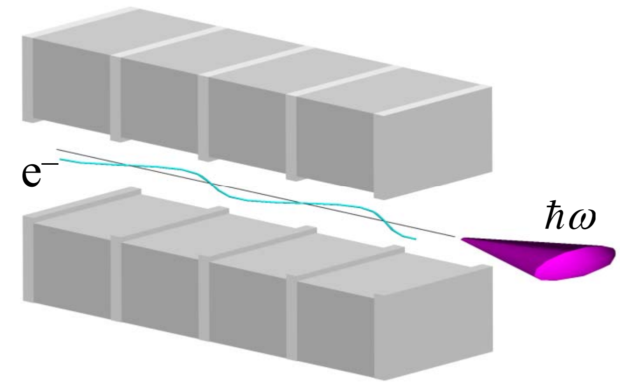
Undulater

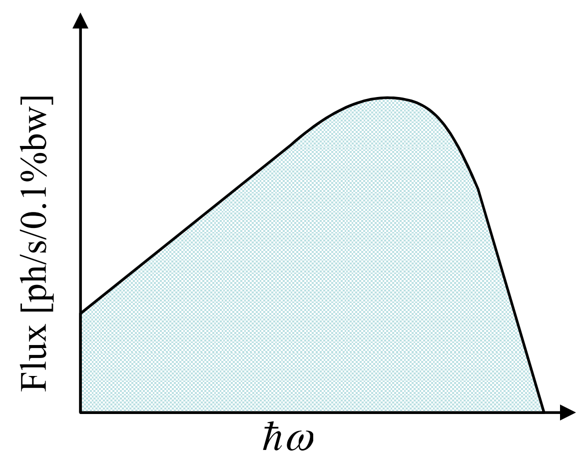




partially coherent source

Wiggler





powerful white source

Why ERLs for X-rays?

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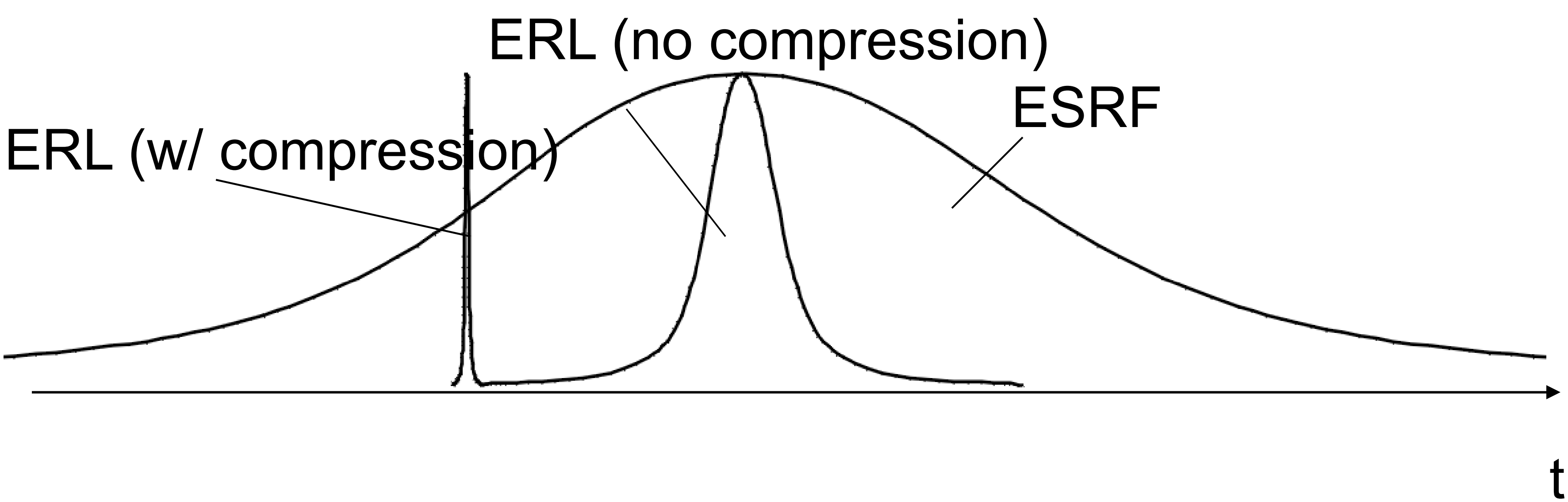
ODU

ESRF 6 GeV @ 200 mA

ε_x = 4 nm mrad
ε_y = 0.02 nm mrad
B ~ 10^{20} ph/s/mm^2/mrad^2/0.1%BW
L_{ID} = 5 m

ERL 5 GeV @ 10-100 mA

$\varepsilon_x = \varepsilon_y \rightarrow$ 0.01 nm mrad
B ~ 10^{23} ph/s/mm^2/mrad^2/0.1%BW
L_{ID} = 25 m



Brilliance Scaling and Optimization

- For 8 keV photons, 25 m undulator, and 1 micron normalized emittance, X-ray source brilliance

$$B \propto \frac{I}{\varepsilon^2} = \frac{fQ}{\varepsilon_{th}^2 + AQ^p}$$

- For any power law dependence on charge-per-bunch, Q, the optimum is

$$AQ^p \approx \varepsilon_{th}^2 / (p-1)$$

- If the “space charge/wake” generated emittance exceeds the thermal emittance ε_{th} from whatever source, you’ve already lost the game!
- BEST BRILLIANCE AT LOW CHARGES, once a given design and bunch length is chosen! Therefore, higher RF frequencies preferred
- Unfortunately, best flux at high charge

ERL Phase II Sample Parameters

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ODU

Parameter	Value	Unit
Beam Energy	5-7	GeV
Average Current	100 / 10	mA
Fundamental frequency	1.3	GHz
Charge per bunch	77 / 8	pC
Injection Energy	10	MeV
Normalized emittance	2 / 0.2*	µm
Energy spread	0.02-0.3*	%
Bunch length in IDs	0.1-2*	ps
Total radiated power	400	kW

* rms values

ERL X-ray Source Average Brilliance and Flux

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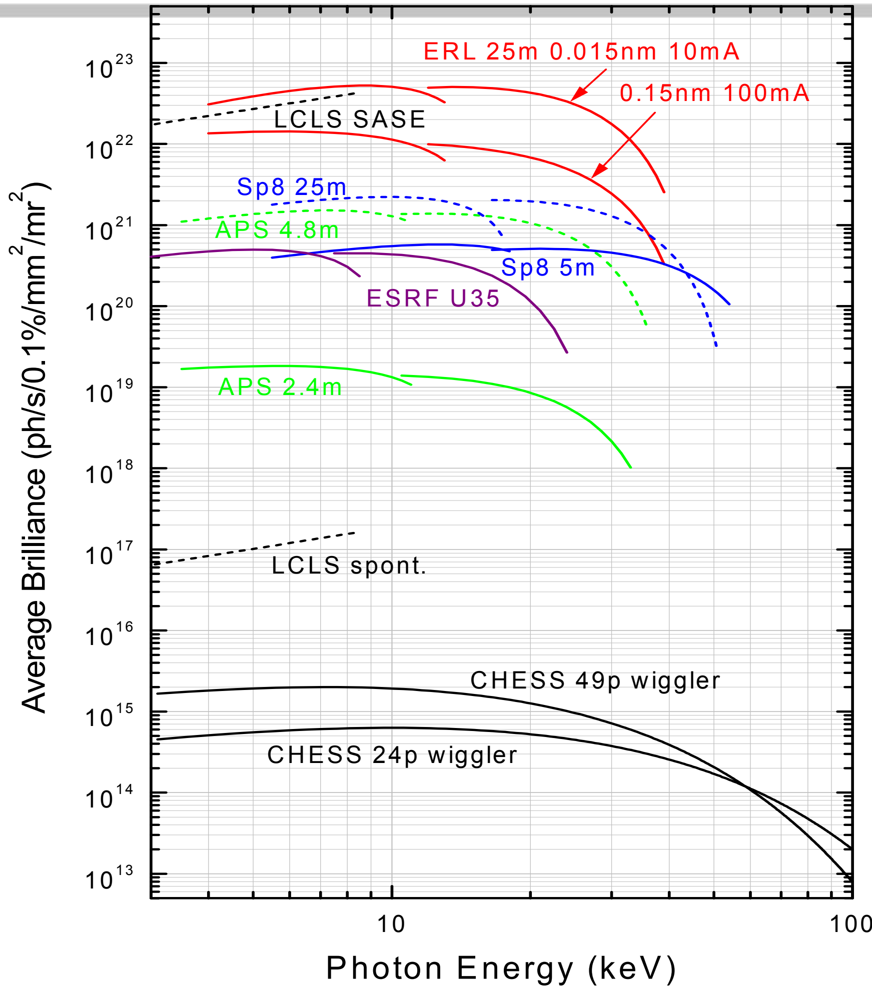
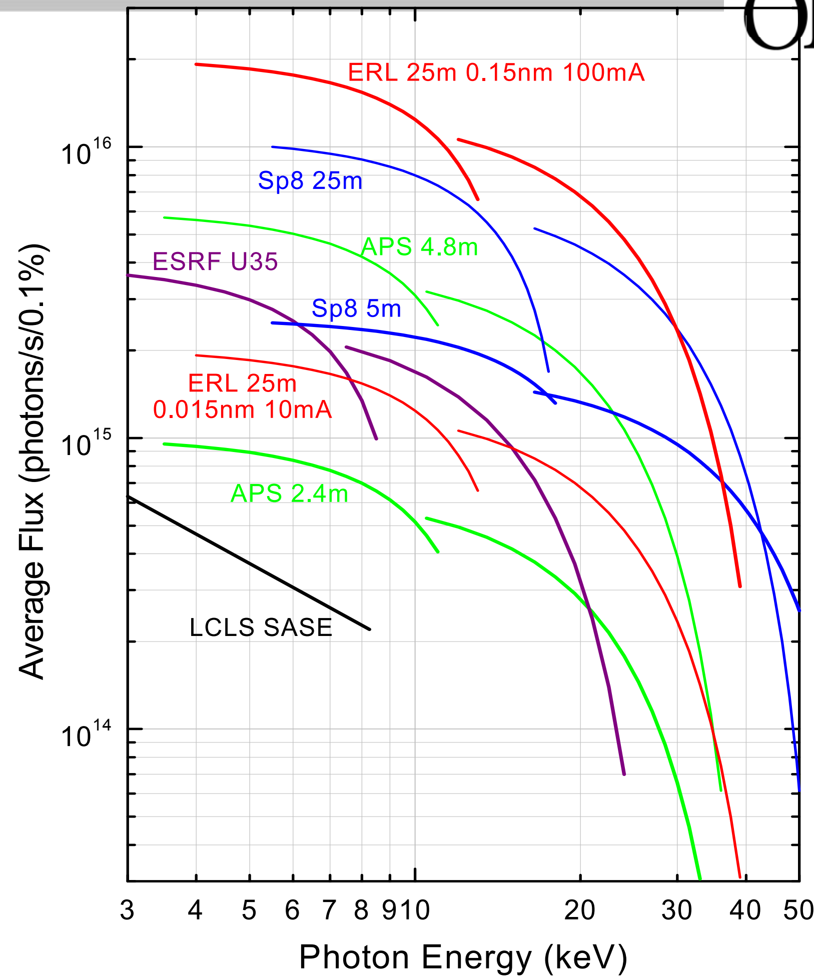
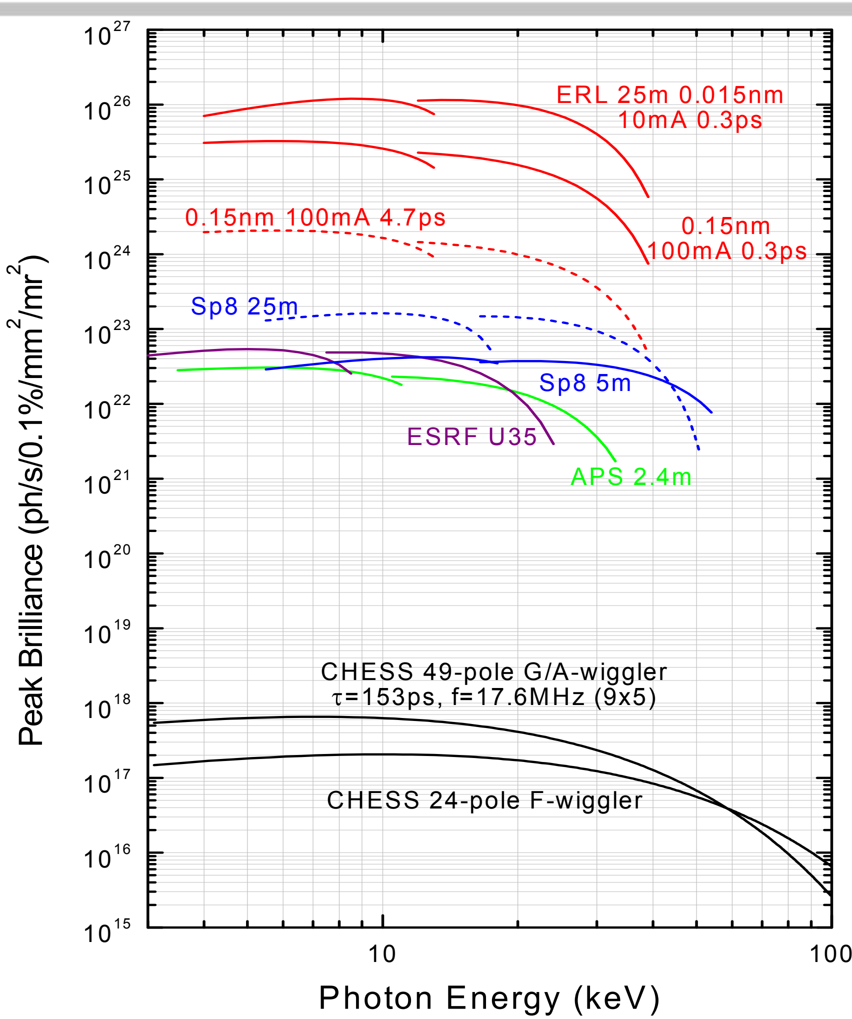
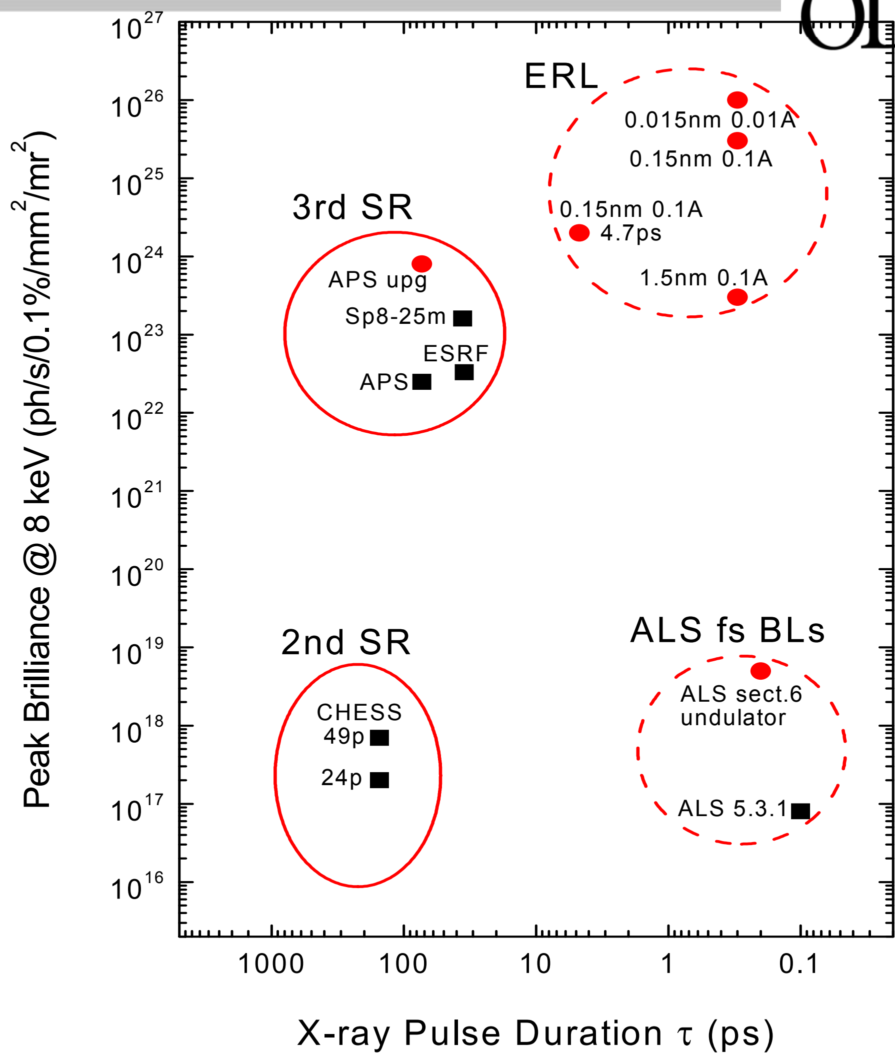




Courtesy: Qun Shen, CHESS Technical Memo 01-002, Cornell University

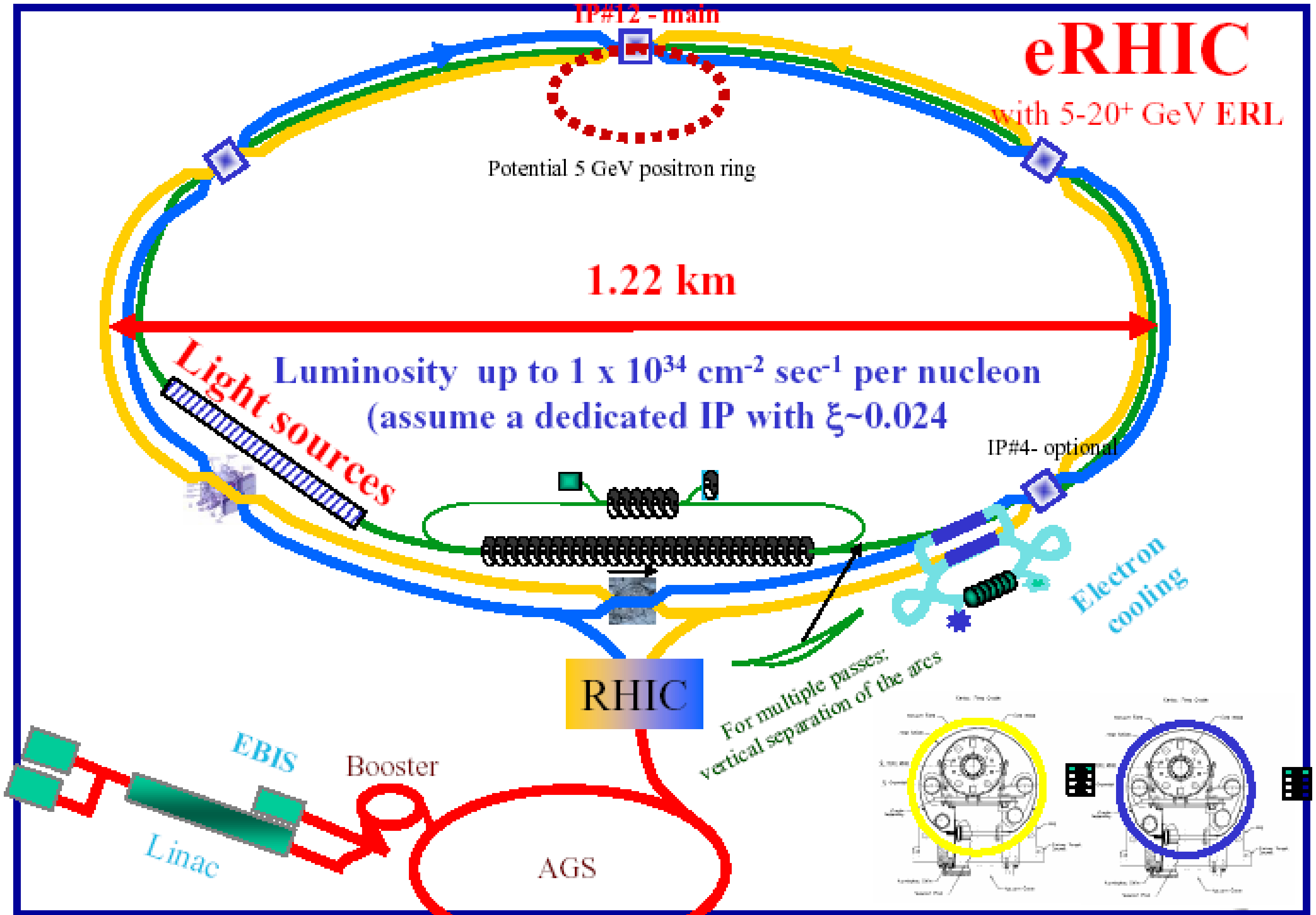
ERL Peak Brilliance and Ultra-Short Pulses





Courtesy: Q. Shen, I. Bazarov

eRHIC



ERL'2005

I. Ben-Zvi & V. Litvinenko, March 19, 2005

BROOKHAVEN NATIONAL LABORATORY

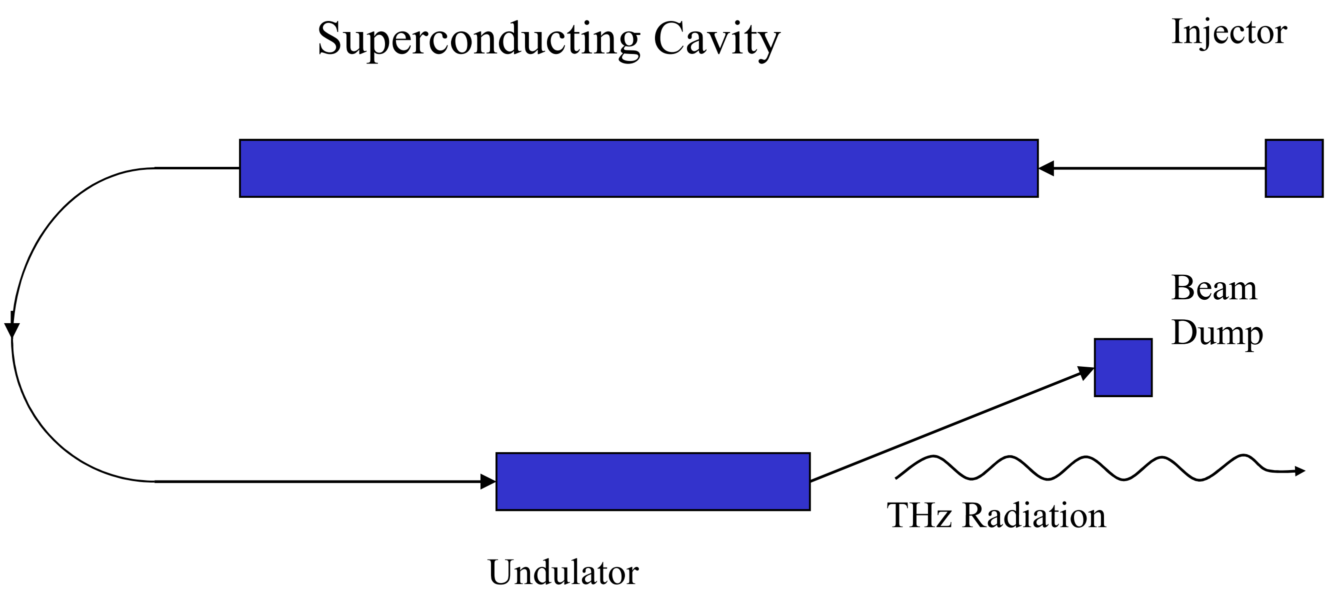
Coherent Synchrotron Radiation

- Coherent synchrotron radiation (CSR) is electromagnetic energy radiated from the “bunch-as-a-whole” at wavelengths longer than the bunch length.
- The radiation from the individual electrons occurs at the same phase. Coherent superposition increases the output power level. Goes as bunch charge squared. Not unique to synchrotron radiation (e.g., transition and undulator radiations)
- Nodvick and Saxon (in more modern notation)

$$\frac{d^2E}{d\omega d\Omega}(\omega) = N_e \left.\frac{d^2E}{d\omega d\Omega}\right|_{1e} \left(1 + (N_e - 1)\left|S(\omega)\right|^2\right)$$

- First observed in electron linacs (Nakazato, *et al.*)
- Not observed in rings originally because the damped bunch

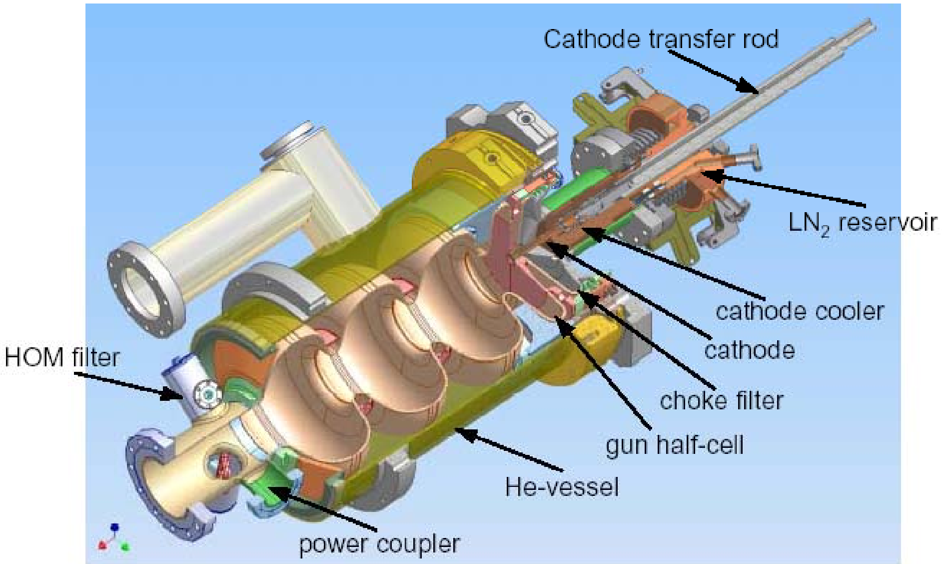
Compact THz Source

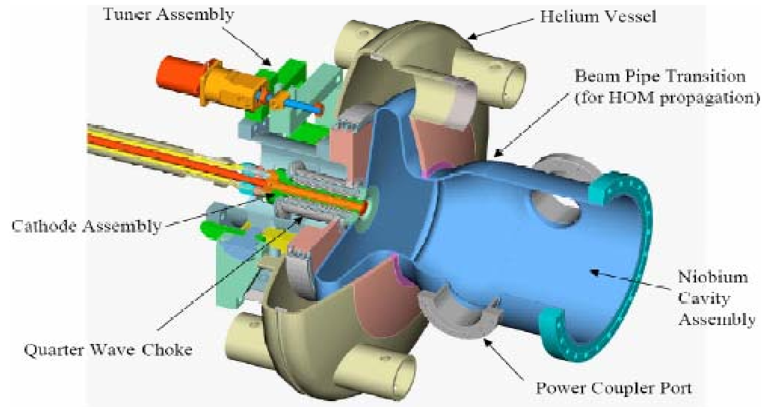


U. S. Patent 6,753,662 assigned to Jefferson Lab

SRF photoinjectors

- High CW RF fields possible
- Significant R&D required





Rossendorf proof of principle experiment:
1.3 GHz, 10 MeV
77 pC at 13 MHz and 1 nC at < 1 MHz

BNL/AES/JLAB development:
1.3 GHz ½-cell Nb cavity at 2K
Test diamond amplified cathode

AES/BNL development:
703.75 MHz ½-cell Nb photoinjector

Source Parameters

TABLE I. THz source accelerator parameters.

Quantity	Value	Unit
Beam energy	3.1–9.9	MeV
Average beam current	100	μA
Charge per beam bunch	12	pC
Bunch repetition rate	8.3	MHz
Normalized rms beam emittance	5	mm mrad
Longitudinal rms emittance	10	keV degrees
rms bunch length at wiggler	300 (90)	fsec (μm)

TABLE II. THz source undulator and calculated optical parameters.

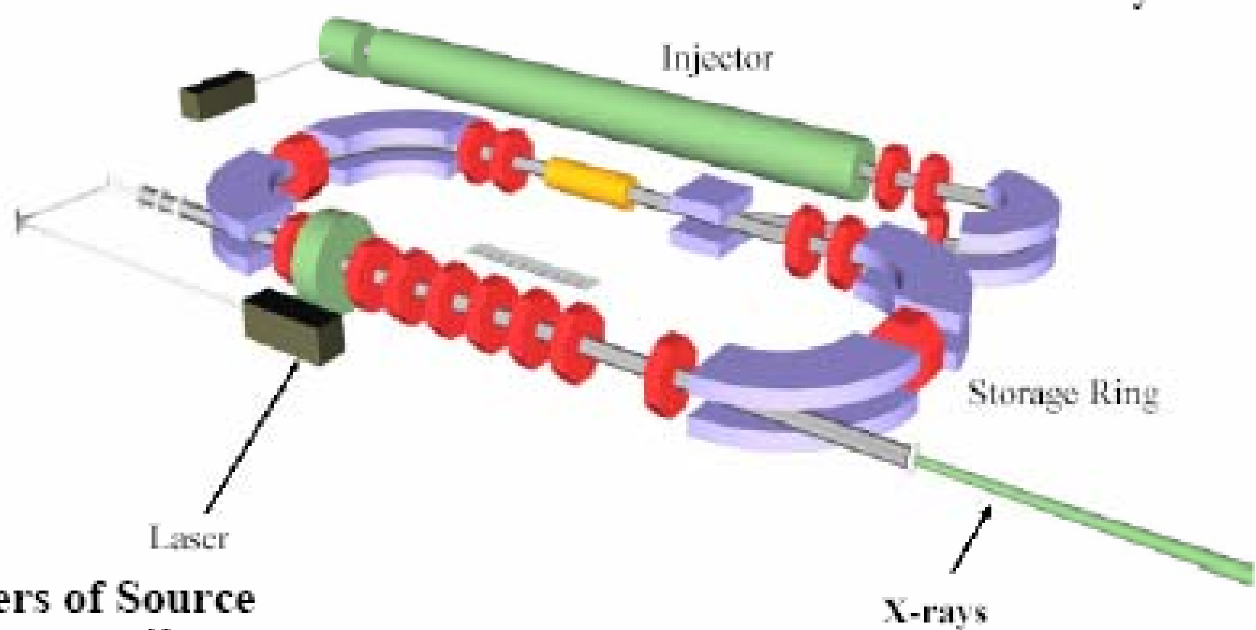
Quantity	Value	Unit
Undulator		
Period length	5	cm
Period number	3, 25	
Field strength, $K = eB\lambda_0/2\pi mc^2$	1	
Wavelength at 5.7 MeV	0.3	mm
Fundamental optical power	0.7, 5.9	W
Fundamental flux	0.9×10^{18}, 7.3×10^{18}	photon/s in 0.1% bandwidth
Fundamental brilliance	1.6×10^{13}, 2.8×10^{14}	photon/(s mm^2 mrad^2) in 0.1% bandwidth
Optical pulse length ($N\lambda$)	0.9, 7.5	mm

Lyncean Technologies Compact Source Concept

A Conceptual Picture of the CLS

(The 30 cm ruler in the middle is shown for scale.)

Courtesy of Ron Ruth

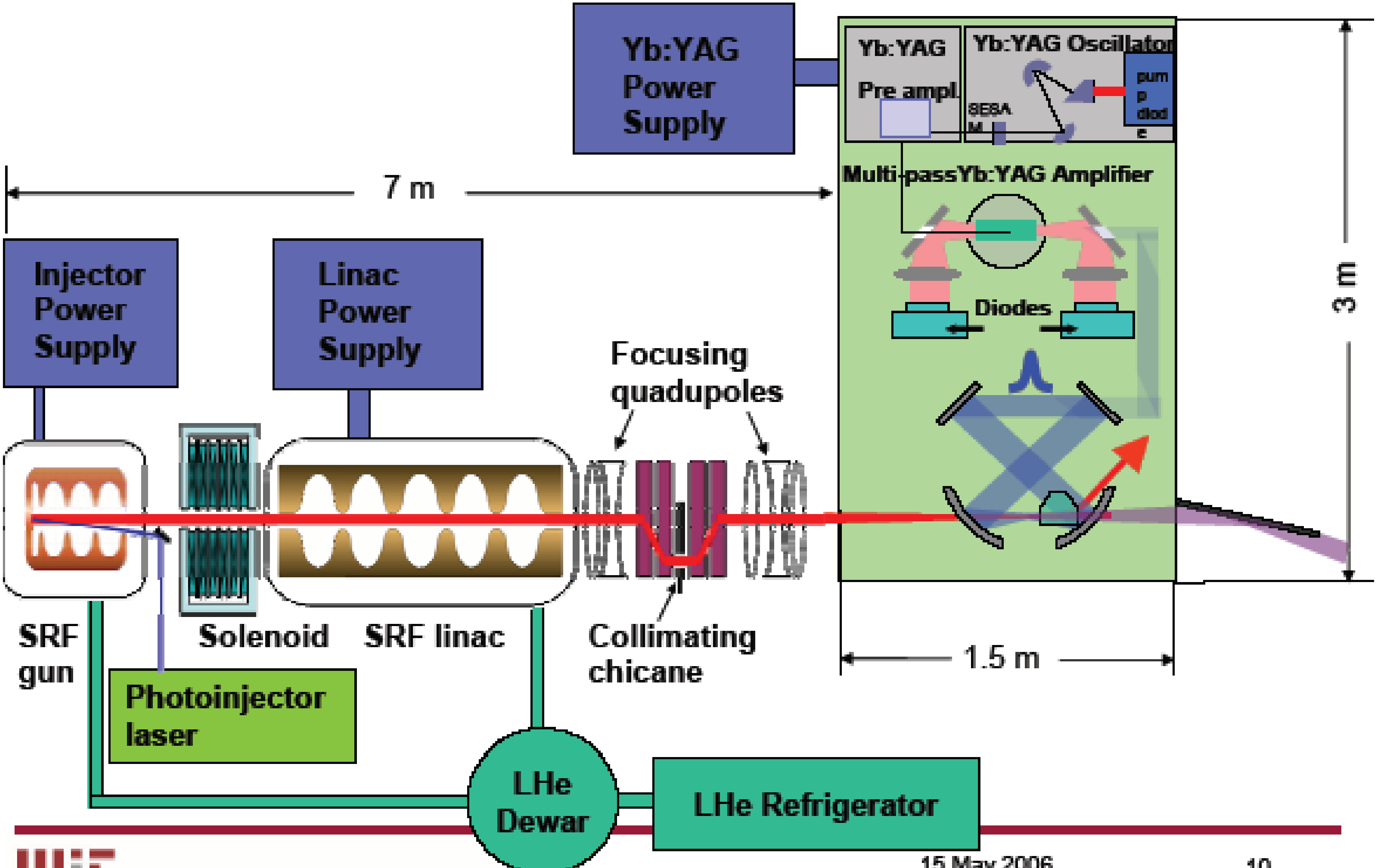


Parameters of Source

Average flux	10^{12} photons/sec
Source size	100 microns

"This is not a good time now for us to present results because we are in the middle of tune up"—5/11

MIT Inverse Compton Source Concept



Advantages of an SC Linac: Low Emittance, Short Pulses, High Rep Rate

- Like conventional synchrotron beams, the figures of merit for Inverse Compton Sources will be flux and brilliance (brightness).
- High performance will depend on achieving low electron emittance, short pulses, and high time-average currents (and excellent laser properties as well).
  - **Low Emittance:** Normalized electron emittance may approach 0.3 μm. With electron energies of 25 MeV ($\gamma = 50$) the electron beam emittance would be 6 nm—comparable to APS (3 nm)!
  - **Short Pulses:** Pulse durations below 1 ps will enable full advantage to be taken from the low emittance beams
  - **High Current:** Superconducting linac-driven Inverse Compton Sources will employ photo-cathode guns operating at 10 MHz with 0.1 nano-coulomb charge. Currents of 1 milli-amp are generated.
- SC linacs outperform storage rings, and are more reliable.
- They are the next generation ICS after the Lyncean machine