TAAD1
Homework Set 1
Due 9/24/2012

1) The potential energy for the linear harmonic oscillator is $V(x) = m\omega^2 x^2 / 2$, where m is the particle mass and ω is the (angular) frequency of the oscillator.
(a) Trace through the steps in the lectures (define $L = T - V$, compute the canonical momentum $\partial L / \partial \dot{x}$, perform the Legendre Transformation) and show that the Hamiltonian function is

$$H(x, p) = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}.$$

(b) Show Hamilton's equations of motion lead to

$$\frac{d^2 x}{dt^2} + \omega^2 x = 0,$$

whose solutions are simple oscillations of angular frequency ω.

2) Show that applying the Euler-Lagrange equations to the (non-relativistic) Lagrangian for a particle of charge q in an electromagnetic field in Cartesian coordinates,

$$L(\dot{\vec{x}}, \vec{x}, t) = \frac{m\dot{\vec{x}} \cdot \dot{\vec{x}}}{2} - q\phi(\vec{x}, t) + q\dot{\vec{x}} \cdot \vec{A}(\vec{x}, t),$$

leads to the Lorentz electromagnetic force law ($\vec{p} = m\dot{\vec{x}}$)

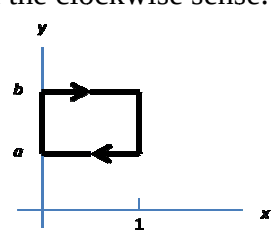
$$\frac{d\vec{p}}{dt} = m\frac{d\dot{\vec{x}}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right).$$

[Hint: for a function that depends on time through the coordinates and an explicit time dependence $f(\vec{x}(t), t)$, $df / dt = \frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial y}\dot{y} + \frac{\partial f}{\partial z}\dot{z} + \frac{\partial f}{\partial t}$ by the chain rule for derivatives. Also note lowercase $\vec{p}$ is NOT the canonical momentum, but is the usual mechanical momentum.]

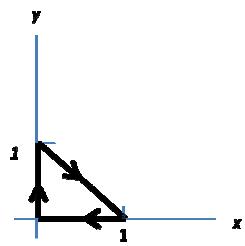
3) (a-c) Verify in the simple cases below that the area enclosed inside the curve in the x - y plane is

$$Area = \oint_C y dx,$$

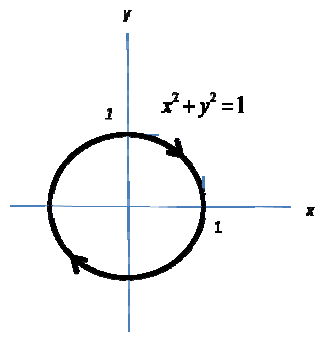
where the curve is oriented in the clockwise sense.



(a)



(b)



(c)

(d) For one of the cases above (you choose!) change the parameterization of the curve and demonstrate that you still obtain the same answer.