Electrons Sources (Part 1) Homework Solution

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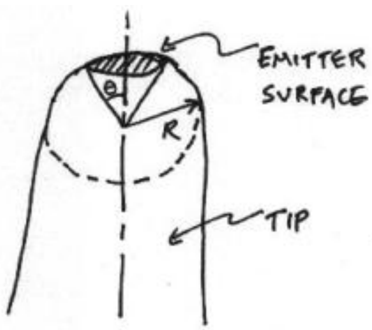
Electron sources require energy in various forms (applied field, temperature, light quanta) in order to liberate electrons. In each case below calculate the requested parameter in order to produce an average current of 1 milliamp. Here the average current is *I*[mA].

1. What applied electric field is required for a tungsten tip with work function 4.5eV and radius 1 micron to emit by field emission? Assume the emitting tip may be locally approximated as the opening angle of a sphere θ=30° (hint: An infinitesimal surface element is spherical coordinates is $dA = R^2 \sin\theta\, d\theta\, d\varphi$). Here area is A[cm^2], electric field is E[V/cm] and work function is ϕ[eV].

$$I = A \times 1.54 \times 10^{-6} \frac{E^2}{\phi} \exp\left[-6.83 \times 10^9 \frac{\phi^{3/2}}{E}\right]$$



$$\theta = 30° = \pi/6$$

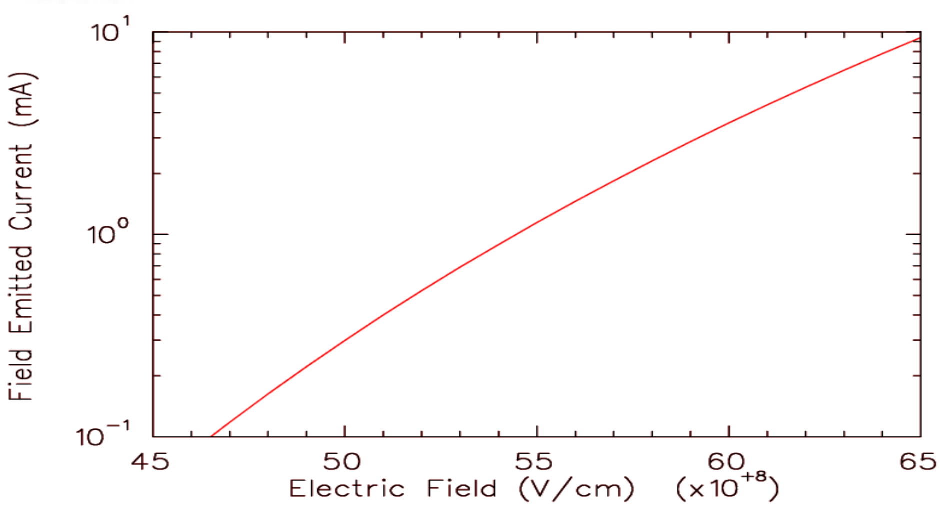
$$R = 1\ \mu m = 10^{-4}\ cm = 10^{-6}\ m$$

$$A_{EMIT} = \int dA = \int R^2 \sin\theta\, d\theta\, d\varphi$$

$$= R^2 \int_0^{2\pi} d\varphi \int_0^{\pi/6} \sin\theta\, d\theta$$

$$= 1.1\, R^2 = 1.1 \times 10^{-8}\ cm^2$$

$$I[mA] = (1.1 \times 10^{-8}\ cm^2)\, \frac{1.54 \times 10^{-6}\, E^2}{4.5\ eV} \cdot \exp\left[-\frac{6.83 \times 10^9}{E} \cdot 4.5^{3/2}\right]$$



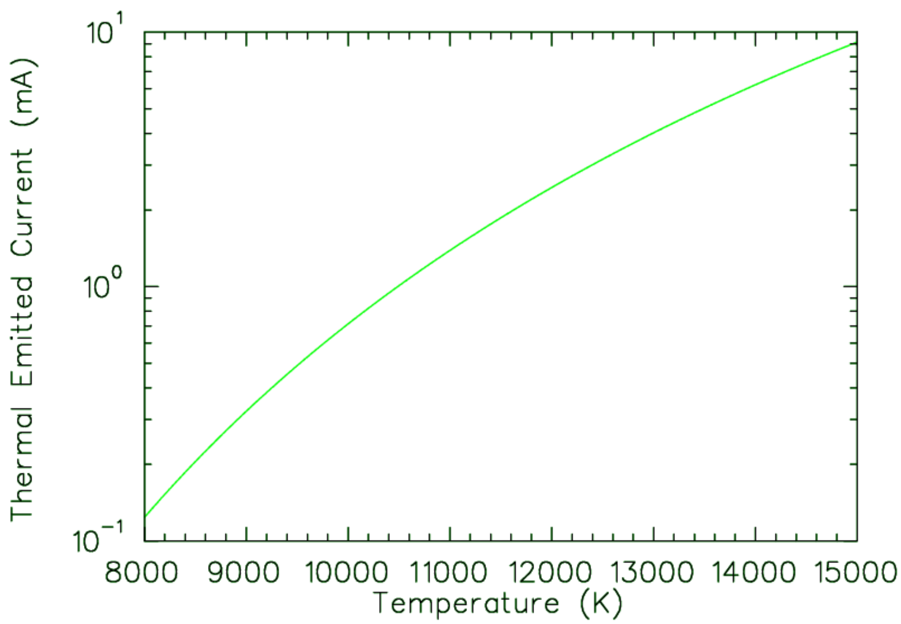
Numerically solve the problem to find that about 5.5E9 V/cm are required. Recall the tip radius is 1 micron (1E-4cm) so in very rough approximate one would need a power supply of order 100kV.

2. What temperature is required in order to achieve by thermionic emission the same average current from the same tip area described above? Here the area is A[m^2], the temperature is T[K], the work function is ϕ[eV], and Boltzmann's constant k_B is 8.62x10^{-5} eV/K.

$$I = A \cdot 1.2 \times 10^6 \cdot T^2 \cdot \exp\left[-\frac{\phi}{k_B T}\right]$$

$$A_{EMIT} = 1.1\times10^{-8}\ cm^2 = 1.1\times10^{-12}\ m^2$$

$$I[mA] = (1.1\times10^{-12}\ m^2)\ 1.2\times10^6 \cdot T^2 \cdot \exp\left[-4.5eV/(8.62\times10^{-5}\ \tfrac{eV}{K} \cdot T)\right]$$



Numerically solve the problem to determine that 10,400K temperature is required. But wait, the melting point of tungsten is 3,695K so clearly such a small wouldn't work.

Imagining that you are able to push tungsten just up to the melting point you would need at least 8.3E-8m^2 to produce 1mA of electrons, roughly speaking a disk of radius 0.16mm. This is still rather small, but certainly not the 1 micron radius of the field emitter tip.

3. You now choose to liberate electrons via photoemission from your tungsten tip. You have two lasers available, but at different wavelengths 233nm and 780nm. Assuming tungsten has a quantum efficiency (QE) of 0.05% how much power is required (from which laser?) in order to liberate the necessary current? A helpful quantity is Planck's constant multiplied by the speed of light, numerically 1240 eV-nm. Here laser wavelength is λ_{laser}[nm] and laser power is P_{laser}[W].

$$QE = \frac{124}{\lambda_{laser}} \cdot \frac{I}{P_{laser}}$$

TWO Lasers :

$$\lambda = 233\,nm \Rightarrow E_\gamma = \frac{hc}{\lambda} = \frac{1240\,eVnm}{233\,nm} = 5.32\,eV$$

$$\lambda = 780\,nm \Rightarrow E_\gamma = \frac{hc}{\lambda} = \frac{1240\,eVnm}{780\,nm} = 1.59\,eV$$

BUT $1.59\,eV < 4.5\,eV < 5.32\,eV$

$\Rightarrow$ MUST USE $\lambda = 233\,nm$

$$P[W] = \frac{124\, I[mA]}{\lambda[nm] \cdot QE[\%]} = \frac{124 \cdot 1}{233 \cdot (0.05)} = 10.6\,W$$

Hopefully you have a nice lab, because such a powerful ultraviolet laser is quite costly!

4. Suppose at your first job you are asked to provide 1 milliampere of highly spin-polarized electrons from a strained superlattice GaAs/GaAsP photocathode. You have previously determined your photocathode has a quantum efficiency of 1.5% at the wavelength (780nm) of your laser. Assuming only 80% of your laser's light will reach the photocathode, after passing through assorted optical elements, how powerful must your laser be?

$$P[W] = \frac{124\ I\ [mA]}{\lambda[nm] \cdot QE\,[\%]} = \frac{124 \cdot 1}{780 \cdot 1.5} = 0.106\ W$$

$$P = 0.8\ P_{LASER} \implies P_{LASER} = 132\ mW$$

5. *Extra Challenge* – As used in the last two problems, determine how the constant '124' is arrived upon for photoemission?

$$\text{QUANTUM EFFICIENCY} = QE \equiv \frac{\#\ e^-\ \text{OUT}}{\#\ \gamma\ \text{IN}} = \frac{N_{e^-}}{N_\gamma}$$

ON AVERAGE $\quad N_e = I[A]/e^- \qquad (e^- = 1.6\times10^{-19}\,C)$

$$N_\gamma = \frac{\text{POWER}}{\text{PHOTON ENERGY}} = \frac{P[W]}{E_\gamma[J]}$$

$$E_\gamma = \frac{1240\ eV\cdot nm}{\lambda[nm]} \cdot \frac{1.6\times10^{-19}\,J}{eV}$$

$$\therefore\ QE = \frac{I[A]/1.6\times10^{-19}\,C}{P[W]/\left(\frac{1240\ eV\cdot nm}{\lambda}\cdot\frac{1.6\times10^{-19}\,J}{eV}\right)} = \frac{1240\ I[A]}{\lambda[nm]\cdot P[W]}$$

OR $\quad 10^{-2}\ QE[\%] = \dfrac{1240\ 10^{-3}\ I[mA]}{\lambda[nm]\cdot P[W]}$

$$QE[\%] = \frac{124\ I[mA]}{\lambda[nm]\cdot P[W]}$$