Physics 696
Topics in Advanced Accelerator Design I
Monday, November 19 2012

Synchrotrons, Phase Stability,
Momentum Compaction, Transition Energy
And How To Measure The a Lower Bound on the Speed of Light With 200 Monks

Todd Satogata

Jefferson Lab and Old Dominion University

Happy Birthday to Jean-Antoine Nollet, Georg Hermann Quincke, and Arthur R. von Hippel!

Happy Gettysburg Address Day, Alligator Wrestling Day, and Have a Bad Day Day!

(Possibly correlated)

Jean-Antoine Nollet

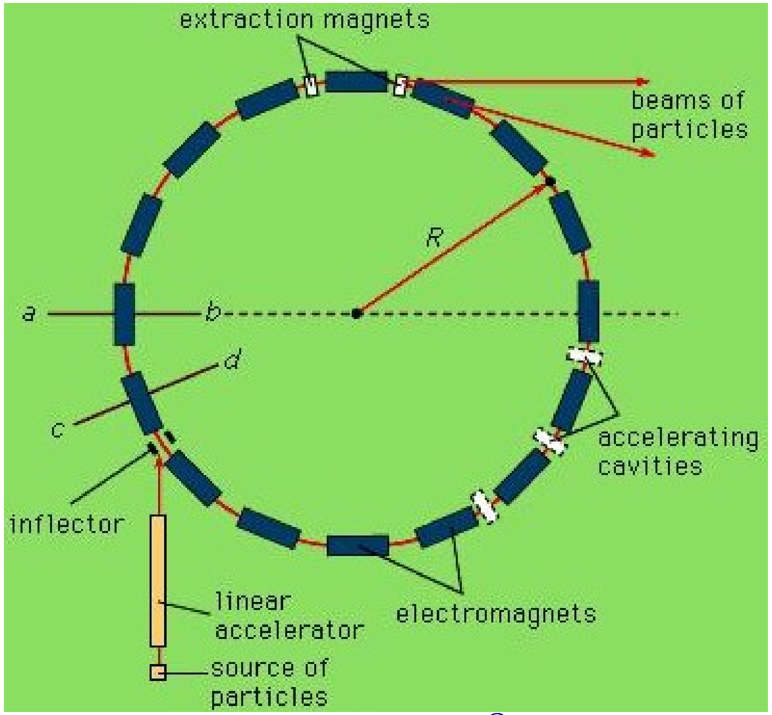
In 1746 he gathered about **two hundred monks** into a circle over a mile in circumference, with pieces of iron wire connecting them. He then discharged a battery of Leyden jars through the human chain and observed that **each man reacted at substantially the same time to the electric shock**, showing that the speed of electricity's propagation was very high.

The Monkotron

- Nollet had
  - lots of charged particles
  - moving in a confined 2km ring (!)
  - at very high velocities
  - accelerated by high voltage

- Nollet didn't have
  - controlled magnets
  - controlled acceleration
  - proper instrumentation
  - many friends after this experiment

The Synchrotron



- The best of both worlds (1944)
  - Cyclotron accelerating system (RF gaps)
    - (Not inductive betatron acceleration)
  - Variable Betatron magnetic bending field
    - (Not constant cyclotron bending field)
- "Synch"-rotron
  - Particle bend radius is close to **constant**

$$B\rho = \frac{p}{q} \quad \Rightarrow \quad \rho = \frac{1}{q}\left(\frac{p}{B}\right)$$

    - B field changes with particle momentum p
  - Circumference is also close to **constant**
    - Revolution frequency and RF frequency also changes with particle velocity β and particle momentum p

$$f_{\mathrm{rev}} = \frac{\beta c}{2\pi R}$$

$$f_{\mathrm{rf}} = 2\pi h f_{\mathrm{rev}} = \frac{h\beta c}{R}$$

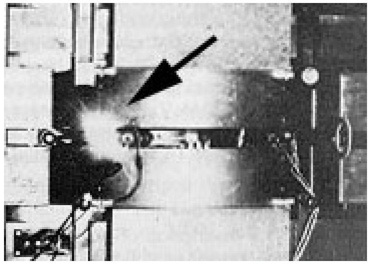
$$h \equiv \text{harmonic number}$$

Why is this such a big deal?

- The **big deal** is that both existing technologies scaled very badly with particle energy
  - **Betatrons**: central induction magnet area (flux) scales quadratically with accelerator radius (energy); beam size also scales badly
  - **Cyclotrons**: main magnet scales quadratically with energy radius (energy); problems with relativistic hadrons
  - (High gradient linacs weren't quite developed yet)
- Large, high-energy accelerator cost was **completely** dominated by scaling of **large** magnets
  - The synchrotron permitted the **decoupling** of peak accelerator energy and magnetic field apertures
  - Higher energies required more magnets (linear scaling) but not larger aperture magnets (quadratic scaling, or worse)

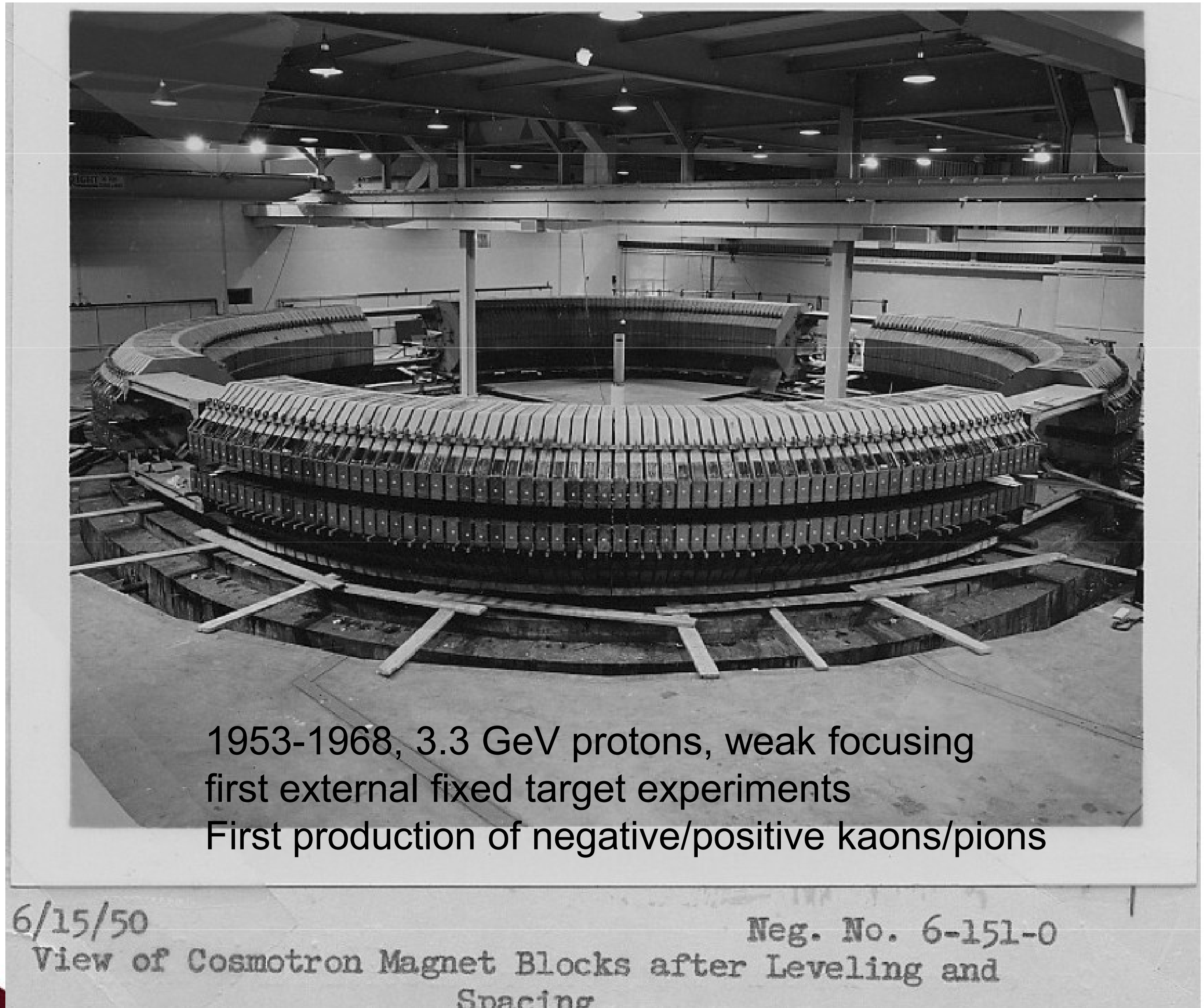
Synchrotron and Phase Stability

- The synchrotron depends on our old friend, longitudinal phase stability
  - We'll review this concept after some history
- Historical context
  - Phase stability: V. Veksler (Russia, 1944) and E.M. McMillan (Los Alamos/LBL, 1945) (1951 Nobel w/G. Seaborg, first transuranic element)
  - First synchrotrons were electron accelerators (~1947)
    - Eliminate bulky core induction magnet of betatrons
    - Easily ultrarelativistic ➔ f_{rev}, f_{RF} nearly constant
    - Energy E~pc so ρ constant ➔ Energy/B = constant
    - 50 MeV betatron (GE, Schenectady) ➔ 70 MeV synchrotron
      - First observation of **synchrotron radiation**
    - Cornell electron synchrotron (1.3 GeV, 1954)
  - Proton synchrotrons came soon after (1950)



Accelerators: Machines of Nuclear Physics (Wilson/Littauer 1960)

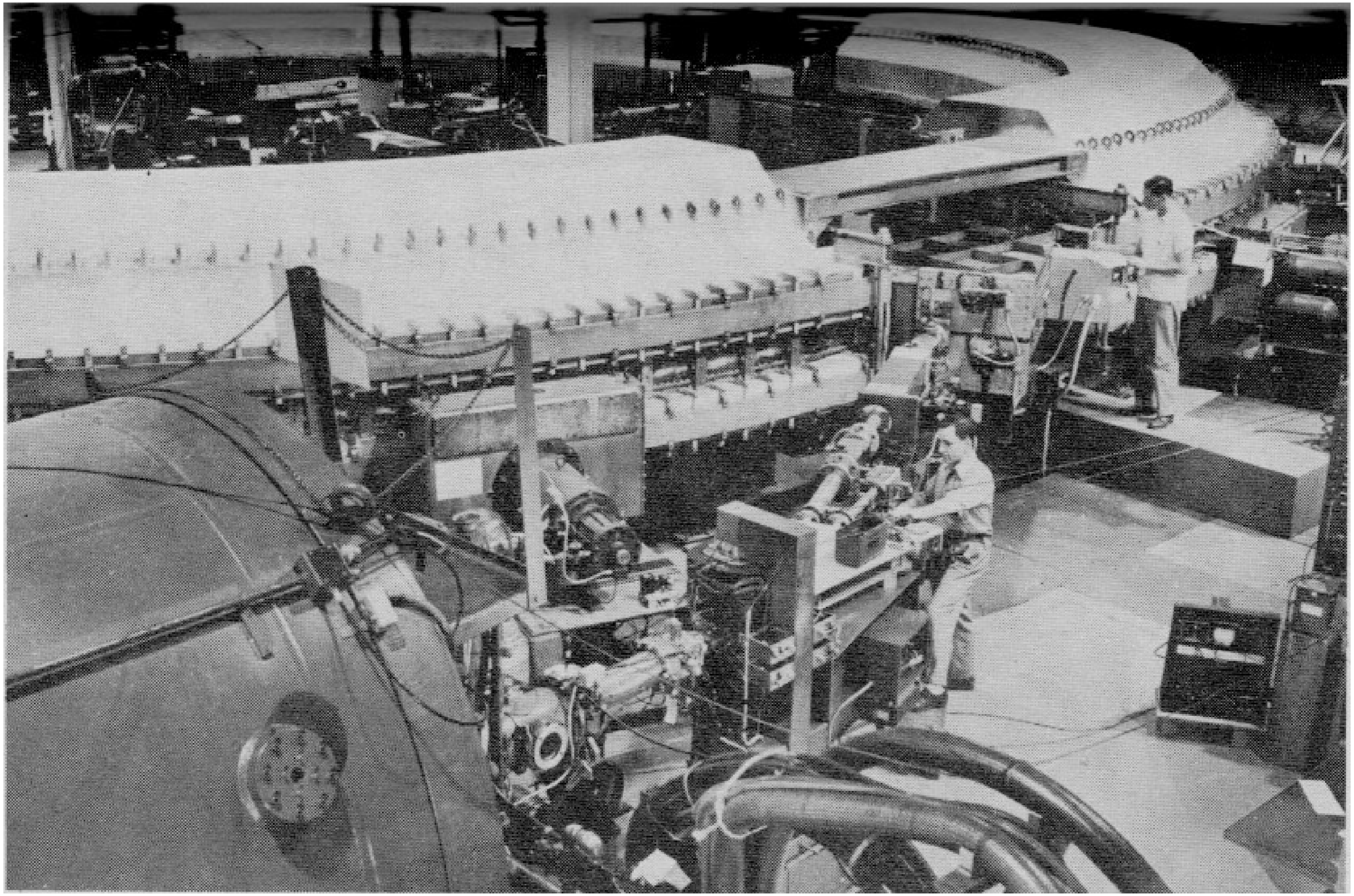
BNL Cosmotron (1953)



1953-1968, 3.3 GeV protons, weak focusing
first external fixed target experiments
First production of negative/positive kaons/pions

National Academy of Sciences, Biographical Memoir of M. Stanley Livingston by Ernest D. Courant

BNL Cosmotron (1953)



Accelerators: Machines of Nuclear Physics (Wilson/Littauer 1960)

Wanna Buy A Used Cosmotron Lamination?

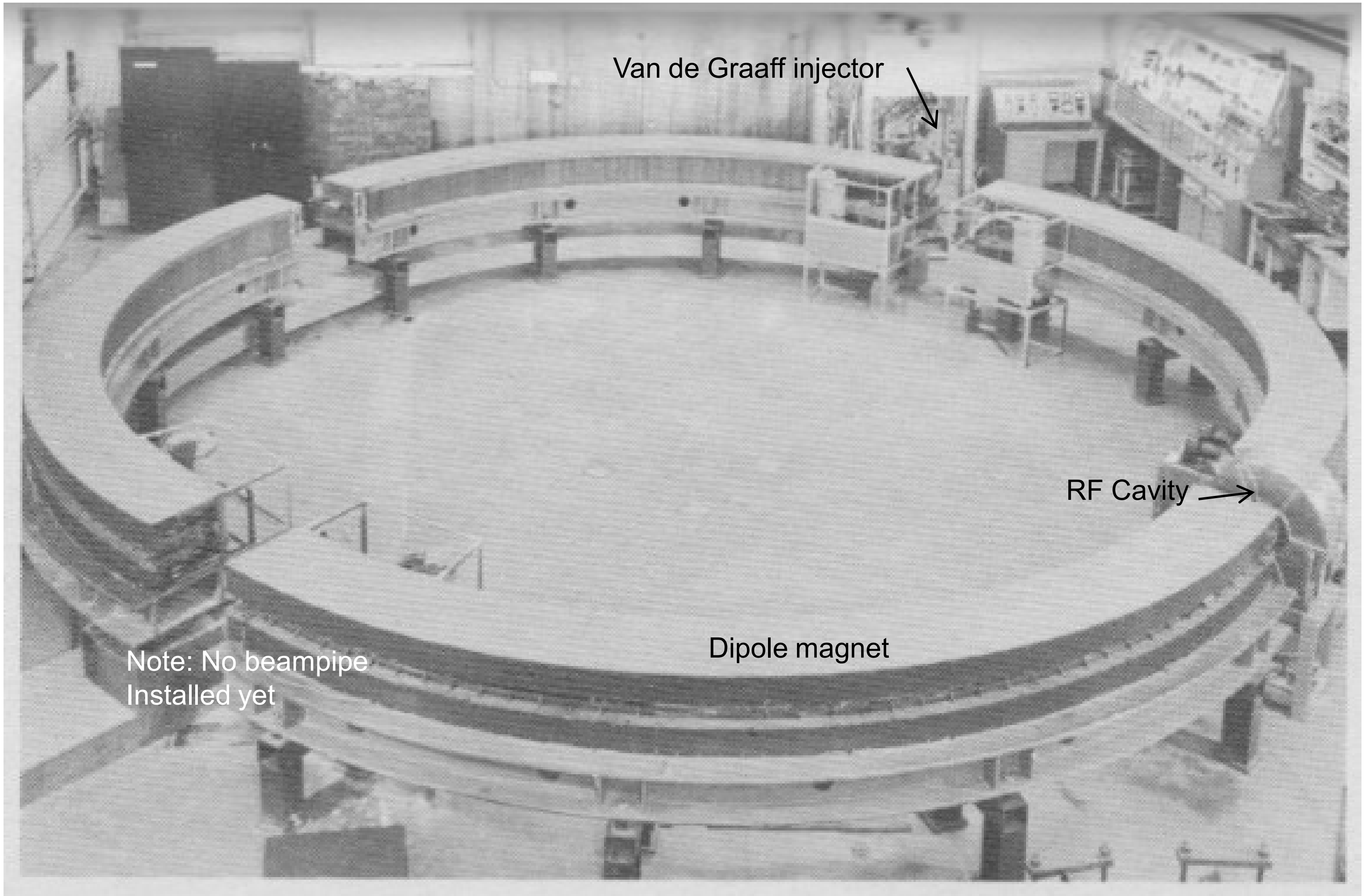
LBL Bevatron (1954)

Ed McMillan and Ed Lofgren

- Last and largest weak-focusing proton synchrotron
- Beam aperture about 4' square!, beam energy to 6.2 GeV
- Discovered antiproton 1955, 1959 Nobel for Segre/Chamberlain

(Became Bevelac, decommissioned 1993, demolished recently)

Cornell Electron Synchrotron (1954)



- 1.3 “BeV” (GeV) with van de Graaff injector
  - First strong focusing synchrotron, 16 tons of magnets, 4 cm beam pipe

Accelerators: Machines of Nuclear Physics (Wilson/Littauer 1960)

Back to Physics: Transition Energy

- Relativistic particle motion in a periodic accelerator (like a synchrotron) creates some weird effects
  - For particles moving around with frequency ω in circumference C

$$\omega = \frac{2\pi\beta_r c}{C} \quad \Rightarrow \quad \frac{d\omega}{\omega} = \frac{d\beta_r}{\beta_r} - \frac{dC}{C} = \left(\frac{1}{\gamma_r^2} - \frac{1}{\gamma_{\mathrm{tr}}^2}\right)\frac{dp}{p_0}$$

$$\text{momentum compaction } \alpha_P \equiv \frac{dC}{C}/\delta = \frac{p_0}{C}\frac{dC}{dp} \qquad \text{transition gamma } \gamma_{\mathrm{tr}} \equiv \frac{1}{\sqrt{\alpha_P}}$$

  - At "transition", $\gamma_r = \gamma_{\mathrm{tr}}$ and **particle revolution frequency does not depend on its momentum**

    Reminiscent of a cyclotron but now we're strong focusing and at constant radius!

electron ring	At $\gamma_r > \gamma_{\mathrm{tr}}$ higher momentum gives **lower** revolution frequency
electron linac	At $\gamma_r < \gamma_{\mathrm{tr}}$ higher momentum gives **higher** revolution frequency

Hadron synchrotrons can accelerate through transition!

Momentum Compaction Example

- Consider a satellite in circular orbit
- A classical gravity/centripetal force problem

$$F = \frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\frac{GM}{R} = v^2 \qquad \omega = 2\pi f = 2\pi\left(\frac{v}{2\pi R}\right) = \frac{v}{R}$$

$$\text{momentum compaction } \alpha_P = \frac{dC}{C}\Big/\frac{dp}{p} = \frac{dR}{R}\Big/\frac{dv}{v} = \frac{v}{R}\Big/\frac{dv}{dR}$$

$$2v\,dv = -\frac{GM}{R^2}dR \quad \Rightarrow \quad \frac{dv}{dR} = -\frac{v}{2R}$$

$$\boxed{\text{momentum compaction } \alpha_P = -2}$$

$$\text{raising momentum } \textbf{lowers} \text{ orbital radius, raises } \omega$$

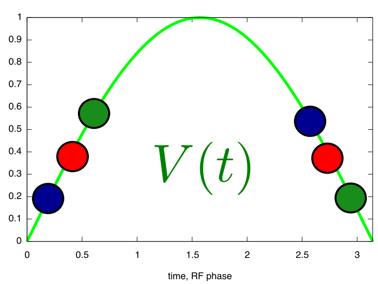
RF Fields

- We need to accomplish two things
  - Add longitudinal energy to the beam to keep p_0 constant
  - Add longitudinal focusing
- RF is also used in accelerating systems to not just balance losses from synchrotron radiation, but
  - Accelerate the beam as a whole: $E_s \neq 0$
  - Keep the beam bunched (focusing, **phase stability)**: $\frac{dE_s}{ds} \neq 0$
- Use sinusoidally varying RF voltage in **short** RF cavities
  - Run at **harmonic number** h of revolution frequency, $\omega_{\mathrm{rf}} = h\omega_{\mathrm{rev}}$

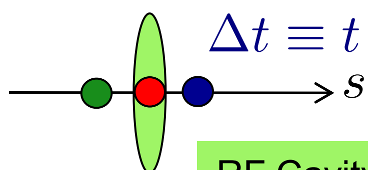
$$\vec{E}(s,t) = \hat{s}E(s,t) = \hat{s}V\sin(\omega_{\mathrm{rf}}t + \phi_s) \sum_{n=-\infty}^{\infty} \delta(s - nL)$$

energy gain/turn

$$\boxed{\Delta U = qV\sin(\omega_{\mathrm{rf}}\Delta t + \phi_s)}$$



$$\Delta t \equiv t - t_{\mathrm{synchronous}}$$

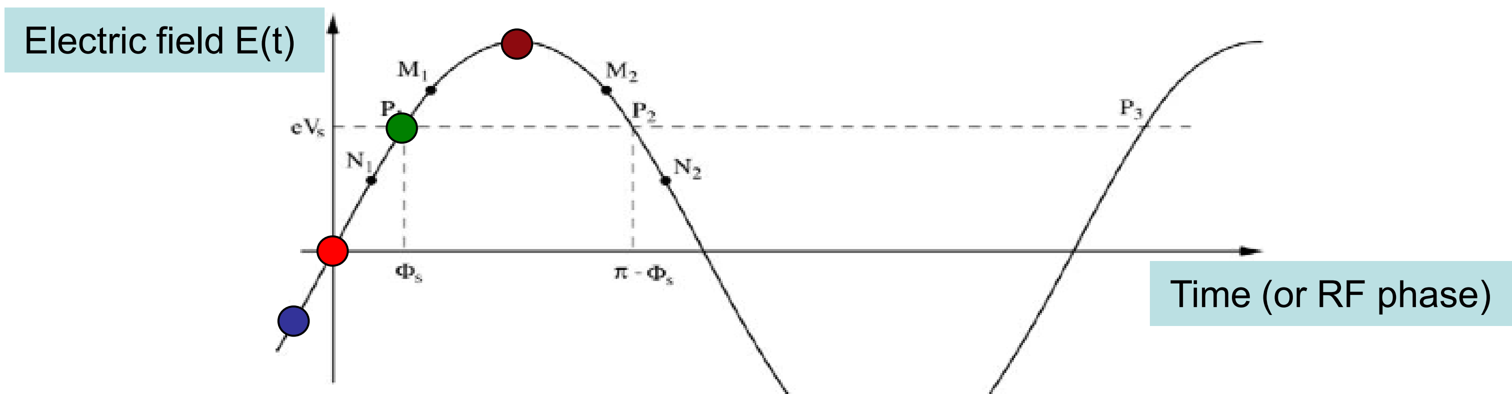


RF Cavity

Earlier in time is earlier in phase!

Synchronous Particle

- We'll be using periodic electric "RF fields"
  - Commonly in the MHz to GHz frequency range
    - Manageable wavelengths of EM waves: e.g. 100 MHz/c = 33 cm
  - Design trajectory now includes longitudinal location and time
    - **Time is equivalent to phase of arrival** in our oscillating RF field
    - The **design particle arrives** at an RF phase defined as the **synchronous phase** ϕ_s at synchronous electric field value E_s



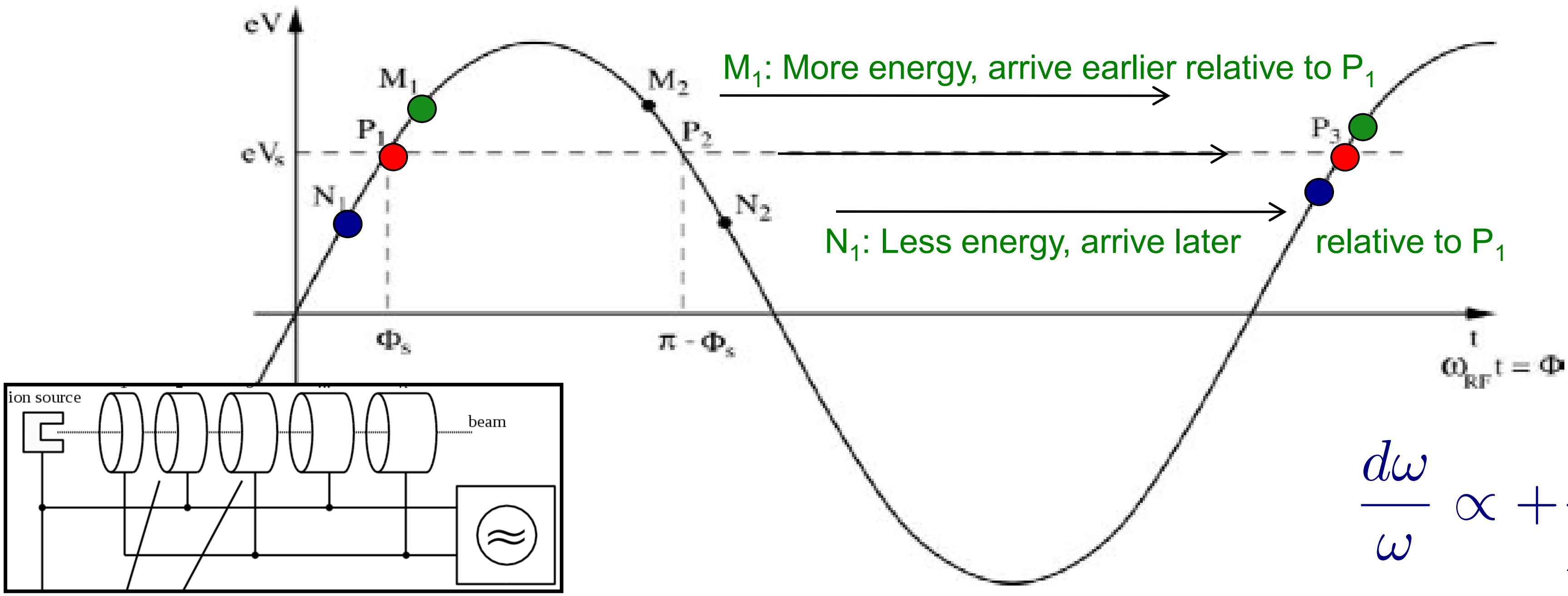
$E_s = 0 \quad \rightarrow \quad$ no design acceleration

E_s max $\quad \rightarrow \quad$ design acceleration

$E_s < 0 \quad \rightarrow \quad$ design deceleration

E_s max $\quad \rightarrow \quad$ max design acceleration

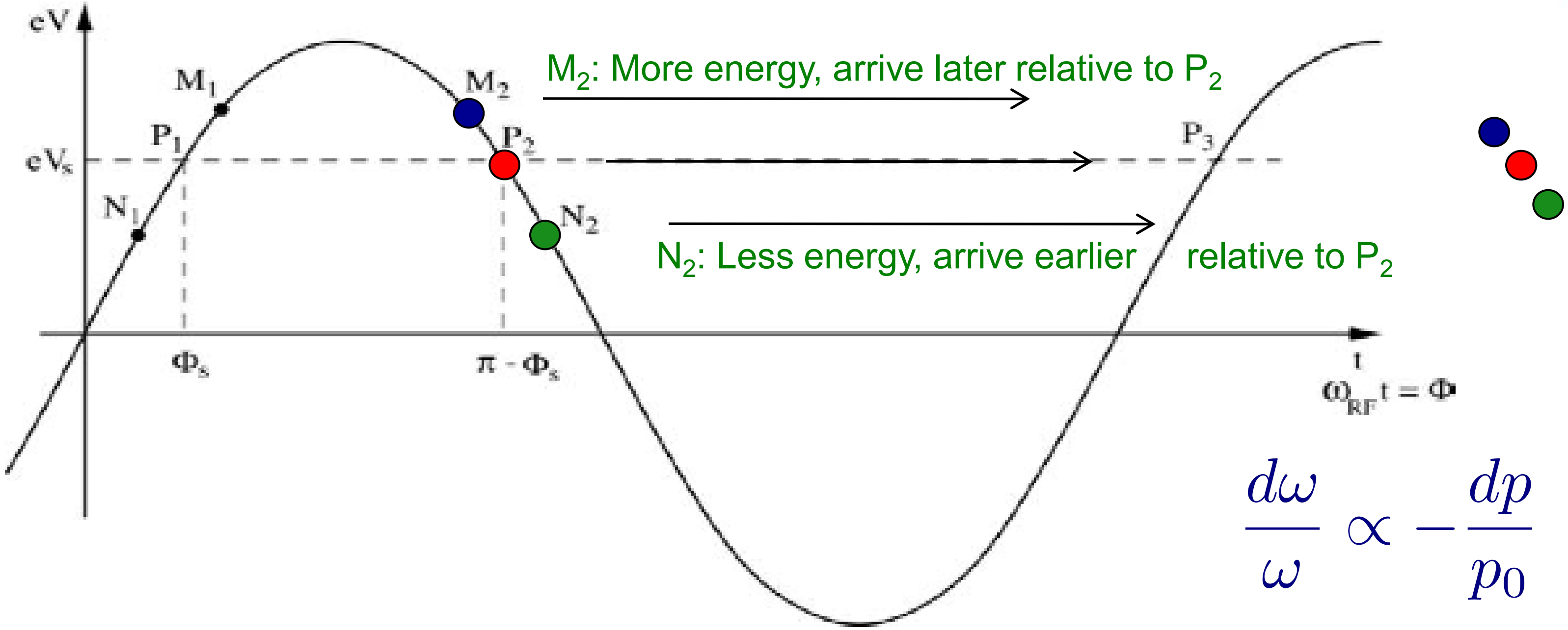
Phase Stability in a Linac (or Below Transition)



$$\frac{d\omega}{\omega} \propto +\frac{dp}{p_0}$$

- Consider a synchrotron low energy ("below transition")
  - **By design** synchronous phase Φ_s gains just enough energy to balance radiation losses and hit same phase Φ_s next time around
  - P_1 are our design particles: they "ride the wave" exactly in phase
- If increased energy means increased frequency ("below transition", e.g. linac)
  - M_1,N_1 will move towards P_1 (local stability) => **phase stability**
  - M_2,N_2 will move away from P_2 (local instability)

Phase Stability in an Electron Synchrotron



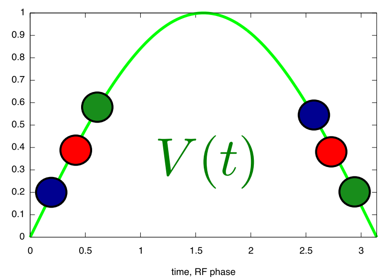
$$\frac{d\omega}{\omega} \propto -\frac{dp}{p_0}$$

- If increased energy means decreased frequency ("above transition")
  - P_2 are our design particles: they "ride the wave" exactly in phase
  - M_1,N_1 will move away from P_1 (local instability)
  - M_2,N_2 will move towards P_2 (local stability) => **phase stability**
  - All synchrotron light sources run in this regime ($\gamma_r \gg 1$)
  - Note ϕ_s is given by maximum RF voltage and required energy gain per turn

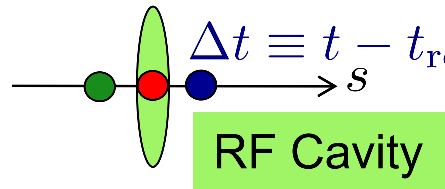
Synchrotron Oscillations

$$\vec{E}(s,t) = \hat{s}E(s,t) = \hat{s}V\sin(\omega_{\mathrm{rf}}t + \phi_s)\sum_{n=-\infty}^{\infty}\delta(s - nL)$$

$$\Delta U = qV\sin(\omega_{\mathrm{rf}}\Delta t + \phi_s)$$



$$\Delta t \equiv t - t_{\mathrm{reference}}$$



- The electric force is sinusoidal so we expect particle motion to look something like a pendulum
  - Define coordinate **synchrotron phase** of a particle $\boxed{\varphi \equiv \phi - \phi_s}$
  - We can go through tedious relativistic mathematics (book pages 144-146) to find a biased pendulum equation

$$\boxed{\ddot{\varphi} + \frac{h\omega_{\mathrm{ref}}^2\eta_{\mathrm{tr}}qV}{2\pi\beta_r^2 U_{\mathrm{ref}}}[\sin(\phi_s + \varphi) - \sin(\phi_s)] = 0}$$

where

$$\omega_{\mathrm{rf}} = h\omega_{\mathrm{ref}} \qquad \eta_{\mathrm{tr}} \equiv \left(\frac{1}{\gamma_r^2} - \frac{1}{\gamma_{\mathrm{tr}}^2}\right)$$

ω_{ref} : revolution frequency of synchronous particle

Linearized Synchrotron Oscillations

$$\ddot{\varphi} + \frac{h\omega_{\mathrm{ref}}^2 \eta_{\mathrm{tr}} qV}{2\pi\beta_r^2 U_{\mathrm{ref}}} [\sin(\phi_s + \varphi) - \sin(\phi_s)] = 0$$

- If these synchrotron phase oscillations are small, this motion looks more like (surprise!) a simple harmonic oscillator

$$\sin(\phi_s + \varphi) \approx \varphi \cos(\phi_s) + \sin(\phi_s)$$

$$\ddot{\varphi} + \Omega_s^2 \varphi = 0$$

$$\Omega_s \equiv \omega_{\mathrm{ref}} \sqrt{\frac{h\eta_{\mathrm{tr}} \cos(\phi_s)}{2\pi\beta_r^2\gamma_r} \frac{qV}{mc^2}}$$

synchrotron frequency

$$Q_s \equiv \frac{\Omega_s}{\omega_{\mathrm{ref}}} = \sqrt{\frac{h\eta_{\mathrm{tr}} \cos(\phi_s)}{2\pi\beta_r^2\gamma_r} \frac{qV}{mc^2}}$$

synchrotron tune

Note that $\eta_{\mathrm{tr}} \cos(\phi_s) > 0$ is required for phase stability.

Example: ALS synchrotron frequency on order of few 10^{-3}

$(\varphi, \dot{\varphi} \equiv d\varphi/dt)$ are natural phase space coordinates

Large Synchrotron Oscillations

- Sometimes particles achieve large momentum offset δ and therefore get a large phase offset φ relative to design
  - For example, particle-particle scattering (IBS or Touschek)
  - Then our longitudinal motion equation becomes

$$\ddot{\varphi} + \frac{\Omega_s^2}{\cos\phi_s}\left[\sin(\varphi + \phi_s) - \sin(\phi_s)\right] = 0 \qquad \varphi \equiv \phi - \phi_s$$

$$\frac{d(\dot{\phi}^2)}{dt} = 2\ddot{\phi}\frac{d\phi}{dt} \quad \Rightarrow \quad d\left(\dot{\phi}^2\right) = \frac{2\Omega_s^2}{\cos\phi_s}(-\sin\phi\, d\phi) + 2\Omega_s^2 \tan\phi_s\, d\phi$$

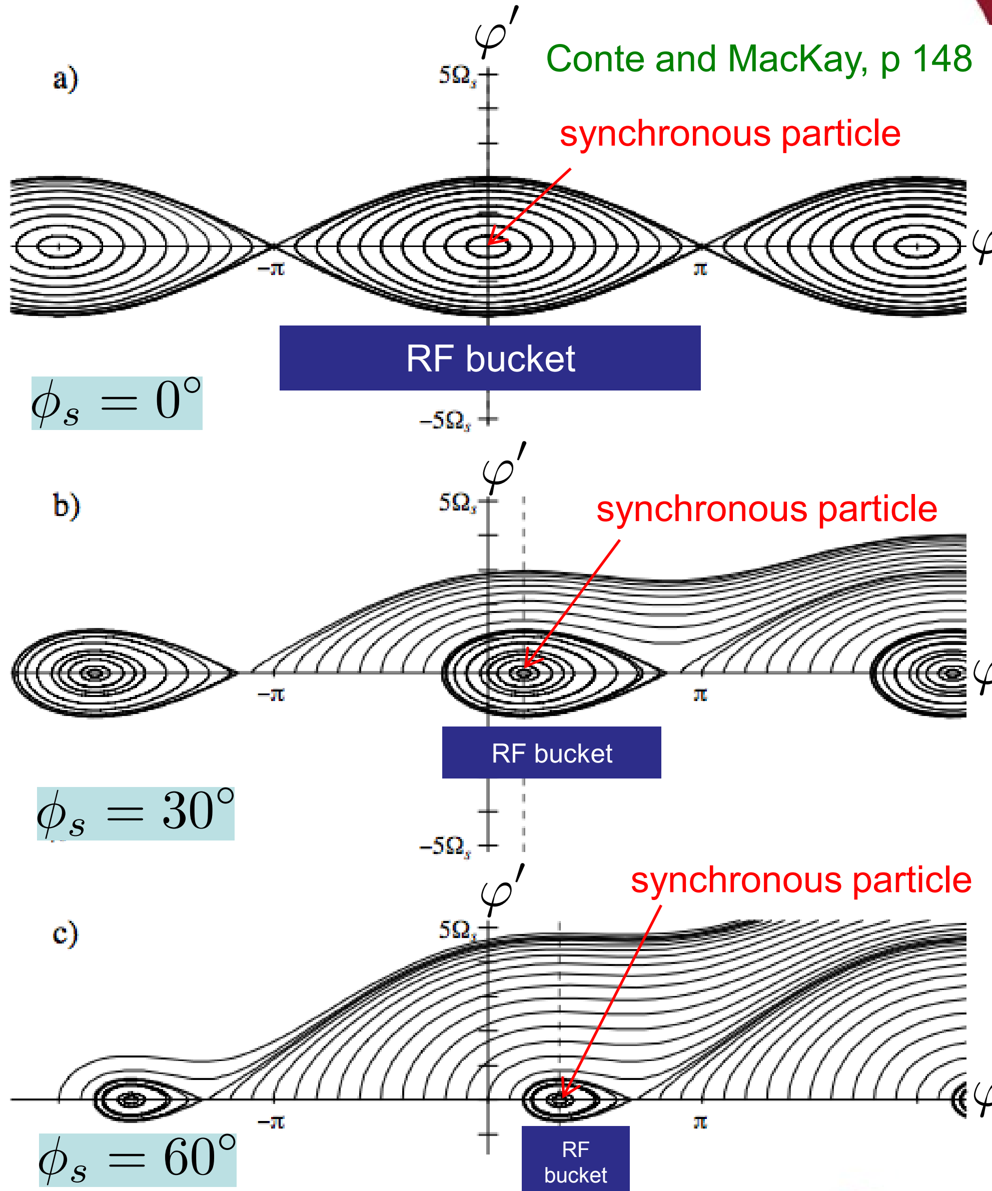
  - Integrate with a constant $\phi_0 \equiv \phi(t=0)$

$$\frac{1}{\Omega_s}\dot{\phi} = \pm\sqrt{\frac{2(\cos\phi - \cos\phi_0)}{\cos\phi_s} + 2(\phi - \phi_0)\tan\phi_s + \frac{1}{\Omega_s^2}\dot{\phi}_0^2}$$

  - This is not closed-form integrable but you can write a computer program to iterate initial conditions to find $(\varphi(t), \varphi'(t))$

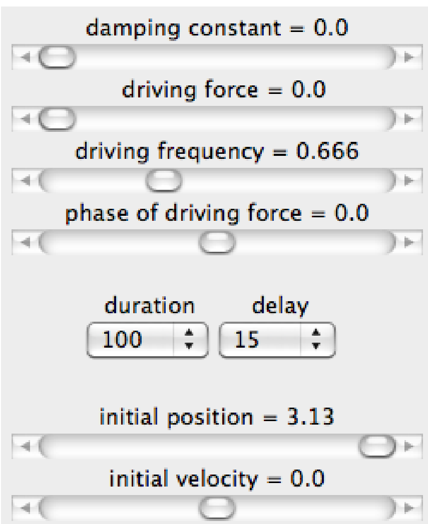
Synchrotron Oscillation Phase Space

- Start particles at $\varphi \neq 0$ and $\varphi' = 0$
- φ' is how phase moves
  - Related to momentum offset δ
- Area of locally stable motion is called **RF bucket**
  - Move like stable biased pendula
- Synchronous particle and nearby particles are stable
  - But some particles "spin" through phases like unstable biased pendula
  - $\Rightarrow \varphi', \delta$ grow, particle is lost at momentum aperture



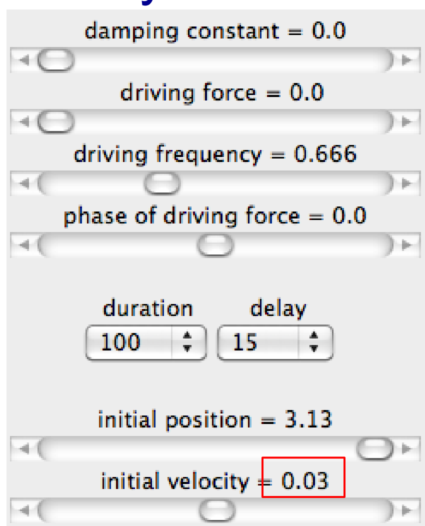
(Pendulum Motion and Nonlinear Dynamics)

- Time variations of the RF fields (particularly voltage or phase modulation) can cause very complicated dynamics
  - Driven pendula are classic examples in nonlinear dynamics
  - See http://www.physics.orst.edu/~rubin/nacphy/JAVA_pend
  - Some demos for your amusement



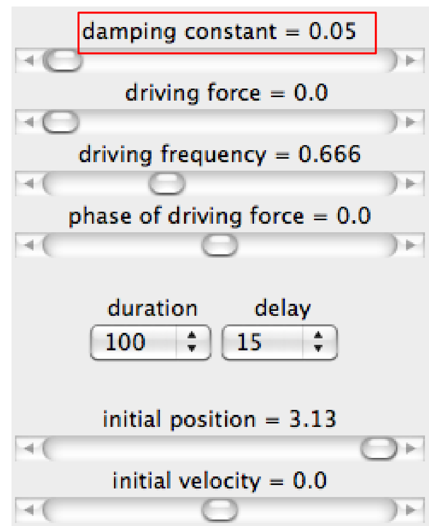
no damping, no drive

pendulum separatrix



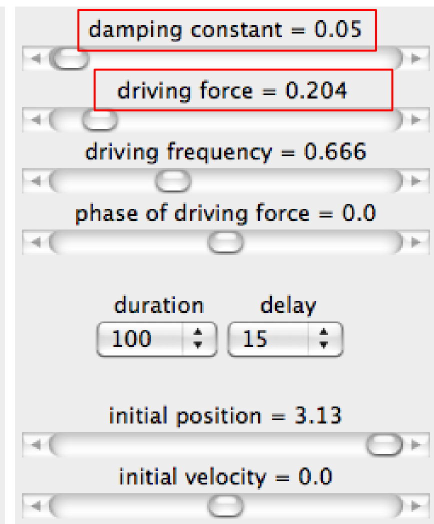
no damping, no drive

precessing pendulum



damping, no drive

damped pendulum



damping, driven

chaotic pendulum