Multibunch Spectrum Analysis for Uneven Bunch Fill

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JLEIC Impedance Meeting

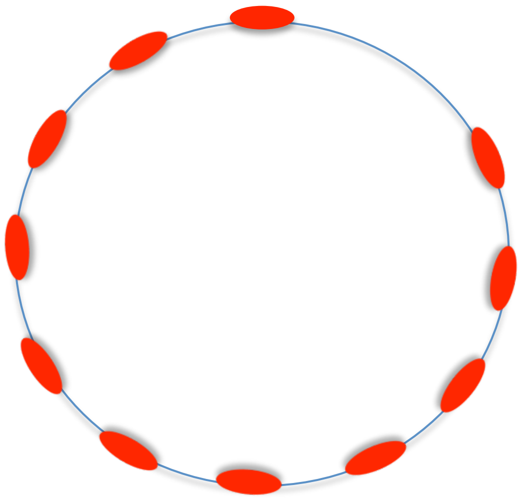
6-17-19

Outline

1. Motivation
2. Results from last meeting
   - By F. Marhauser and by G. Park
3. Bunch distribution for uneven fill
4. Current spectra for uneven fill
5. Example
6. Conclusion

1. Motivation

- Bunches in the storage ring often have one or more gaps for
  - Injection/ejection
  - ion clearing
  - e-clouds clearing



- This would impact the loss factor, power deposited to HOMs, and the growth rate of coupled-bunch instabilities.

- In particular, the growth rate for coupled bunch instability is usually estimated for an even bunch fill, since it is often worse than that for an uneven bunch fill

Loss Factor and Power Loss

- Loss factor is evaluated as

$$k = \frac{2}{q^2}\int_0^{\infty} d\omega \, Re[Z(\omega)] I^2(\omega)$$

$$U = 2\int_0^{\infty} d\omega \, Re[Z(\omega)] I^2(\omega)$$

(G. Park, 5/14/19 impedance meeting)

HOM power analysis

- The general analytical formula for induced voltage by *n*th mode in the cavity is

$$V_n(t) = 2k_n e^{(i\omega_n - 1/\tau_n)t}\int_{-\infty}^{t} dt' \, I(t') e^{-(i\omega_n - 1/\tau_n)t'}$$

Available from eigenmode solver simulation in CST-MWS

V_n is induced voltage across the cavity, k_n is loss factor by a single charge, I is beam current, ω_n is angular frequency of the mode, and τ_n is decay time of the cavity.

- The total power formula by n-th mode in the cavity and total average power are

$$P_n(t) = \frac{V_n^2(t)}{Q_L \cdot R_{\parallel,n}/Q_0} \qquad P_{ave} = \sum_n P_{n,ave} = \sum_n \frac{1}{T}\int_0^{1/f_b} dt \, P_n(t)$$

LCBI Growth Rate

LCBI growth rate: $$g_{\mu,a} = \left(\frac{a}{a+1}\right)\frac{I_b \omega_0^2 \eta}{3(L/2\pi R)^3 2\pi\beta^2 (E_T/e)\omega_s}\left[\frac{Z_{\parallel}}{n}\right]_{\text{eff}}^{\mu,a}$$

Effective impedance: $$\left[\frac{Z_{\parallel}}{n}\right]_{\text{eff}}^{\mu,a} = \int d\omega \left(\frac{Z_{\parallel}(\omega)}{\omega/\omega_0}\right) I(\omega)$$

Bunch spectra: $$I(\omega) = \frac{h_a(\omega_p'')}{\sum_p h_a} \sum_{p=-\infty}^{\infty} \delta\left(\omega - \omega_p''\right)$$

Weighted single bunch mode power spectra

Multibunch spectra for even bunch fill

Examples

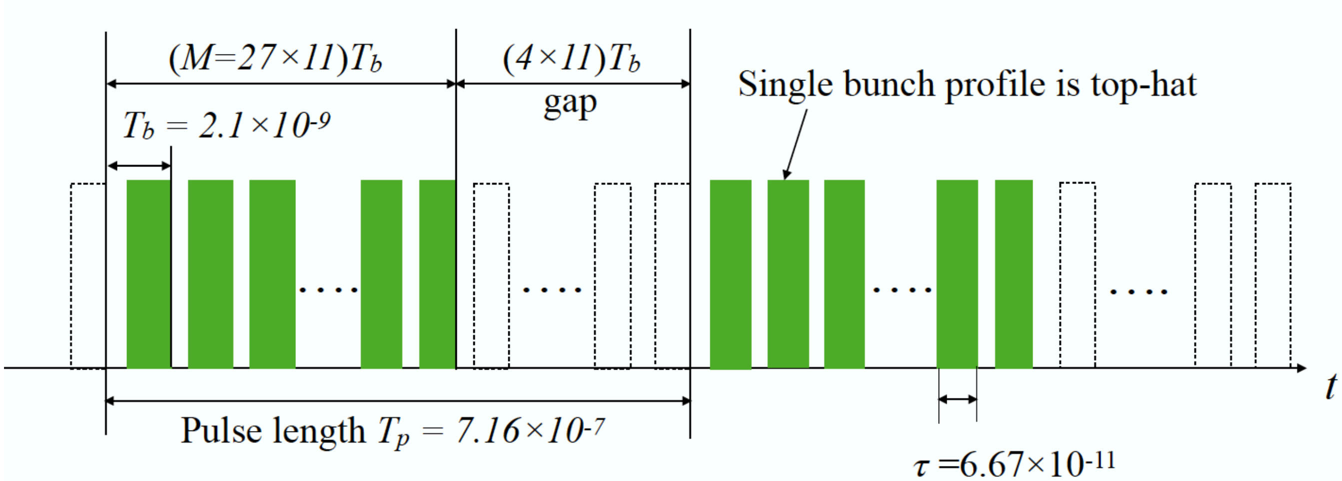
- Time structure in the JLEIC e-ring (Frank Marhauser)

$C=2366$ m , $f_0=c/C=0.127$ MHz, $f_{rep}=476.3$ MHz

$n_B=3759$, $n_{gap}=267$ (7.1% gap), $n_{fill}=n_B-n_{gap}=3492$,

$$\alpha_f = \frac{n_{fill}}{n_B} \approx 92.9\%$$

- Time structure in the JLEIC CCR (Gunn Tae Park)



14.8% gap

$$\alpha_f = \frac{n_{fill}}{n_B} \approx 85.2\%$$

Questions

- How does the bunch spectra of uneven fill differ from that of even fill? Analytic expression?

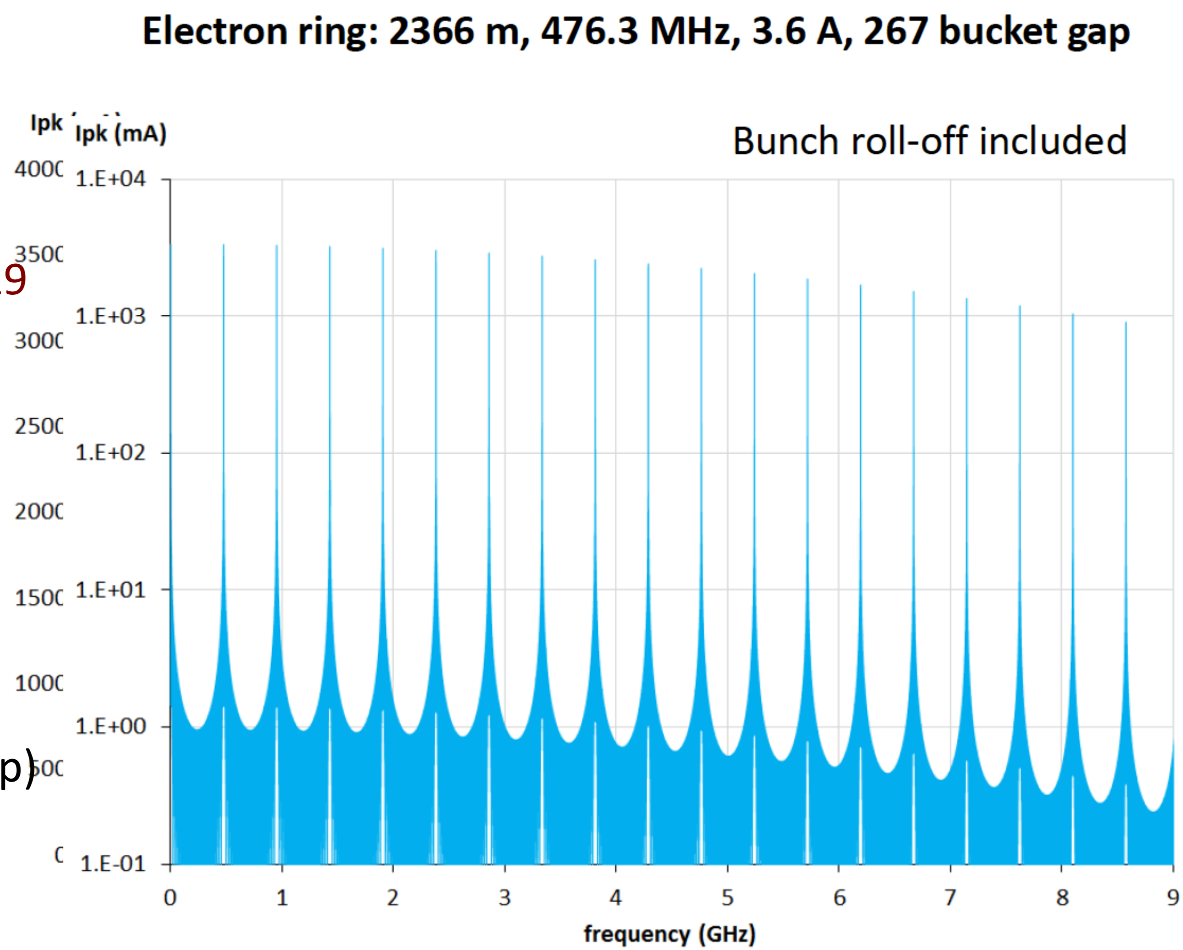
- What's the impact of uneven fill on the loss factor, deposited power, and the growth rate of coupled bunch instability?

- It is a common understanding that the even fill gives the most pessimistic CBI growth rate. Proof?

Examples from Last Meeting

- Status of IR chamber impedances analysis
  by Frank Marhauser
- Beam loading study of the harmonic kicker
  by Gunn Tae Park

(Example 1) Electron Ring Beam Spectrum



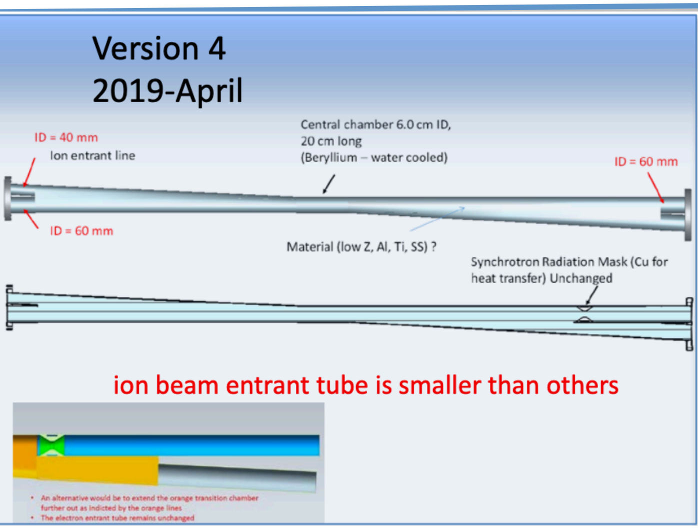
(F. Marhauser, 5/14/19 impedance meeting)

C=2366 m
f_0=c/C=0.127 MHz
f_{frep}=476.3 MHz

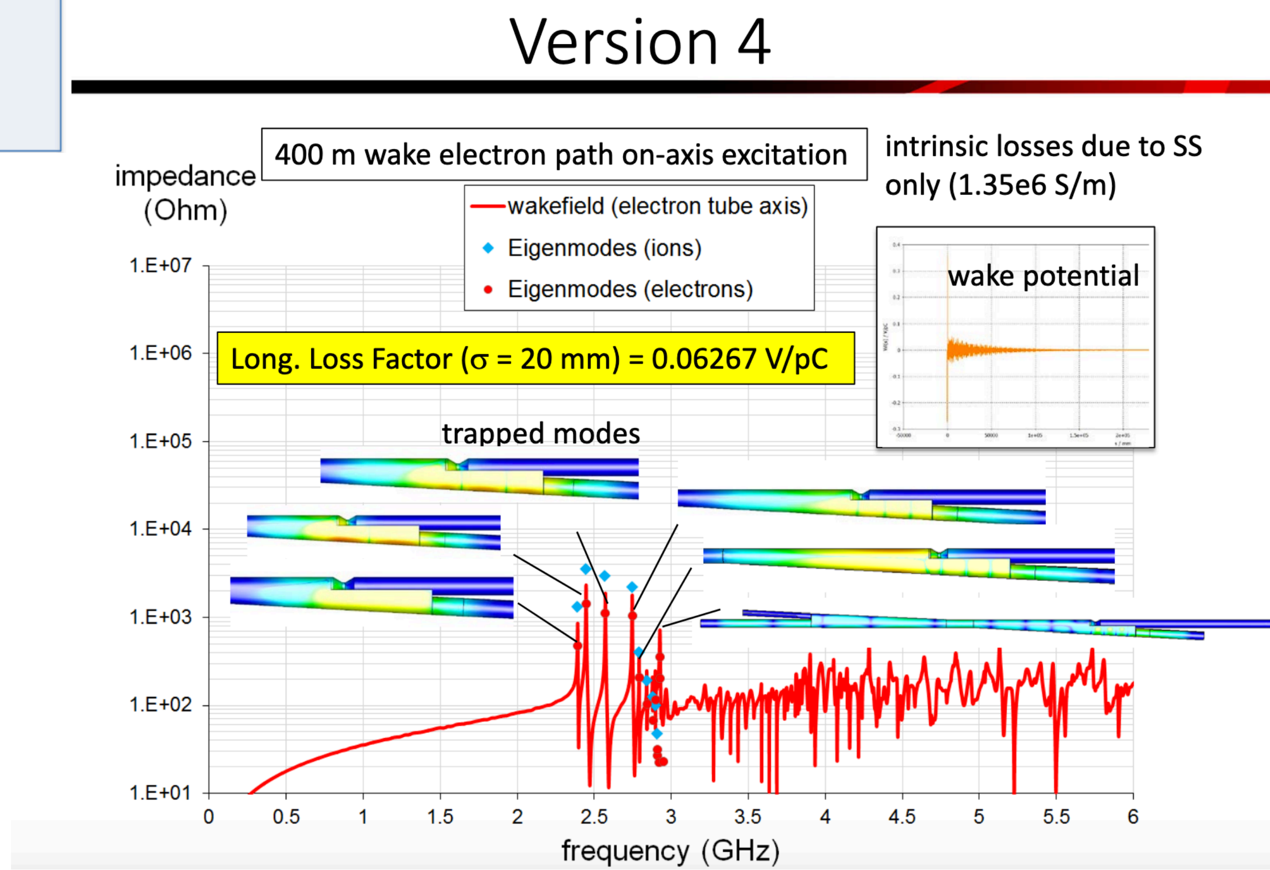
n_b=3759
n_{gap}=267 (7.1% of gap)
n_{fill}=3492

$$\alpha_f = \frac{n_{fill}}{n_B} \approx 92.9\%$$

IR Chamber Impedances



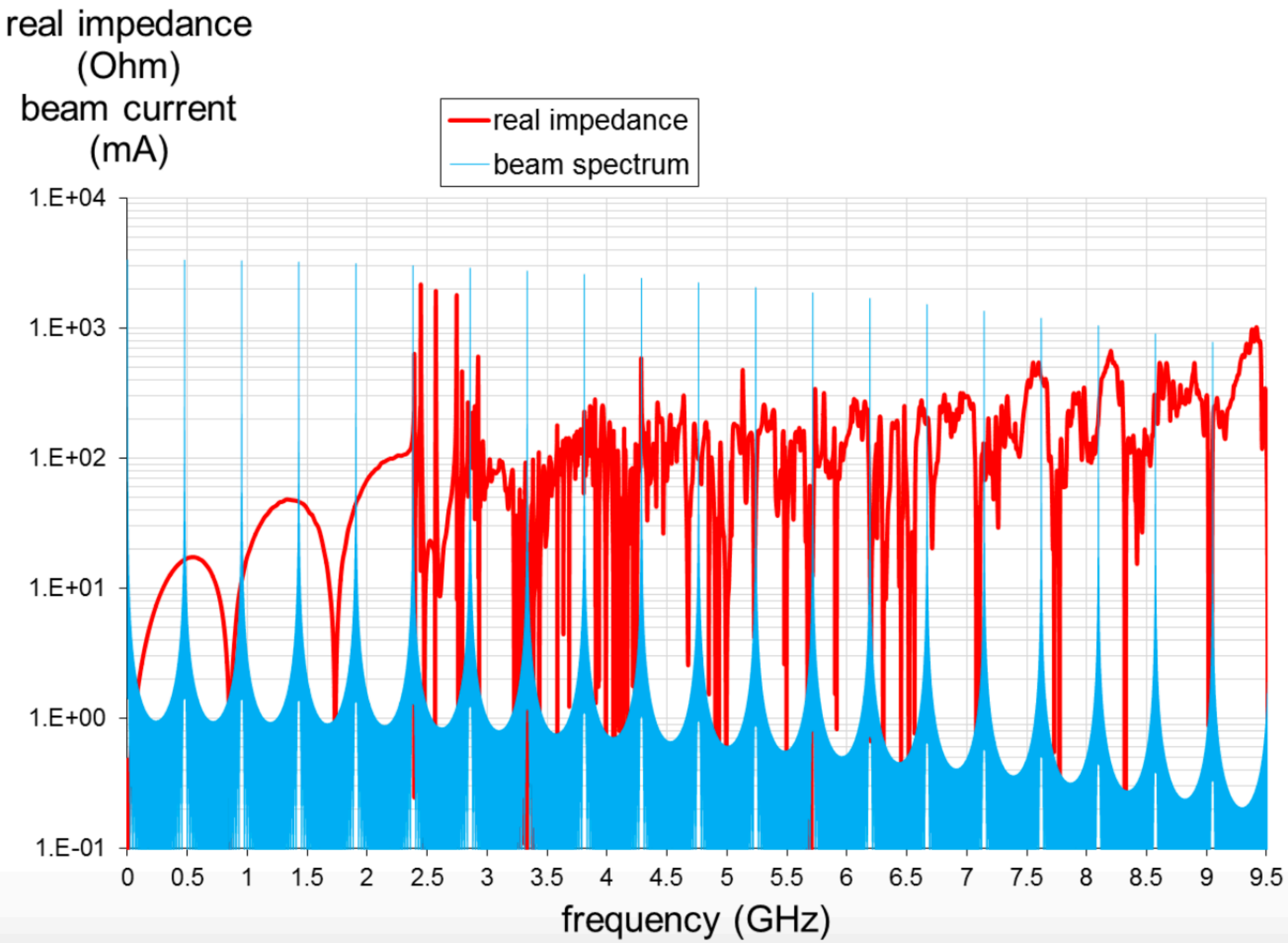
(F. Marhauser, 5/14/19
impedance meeting)



Electron Ring Real Impedance

Electron ring: 2366 m, 476.3 MHz, 3.6 A, 267 bucket gap

(F. Marhauser)

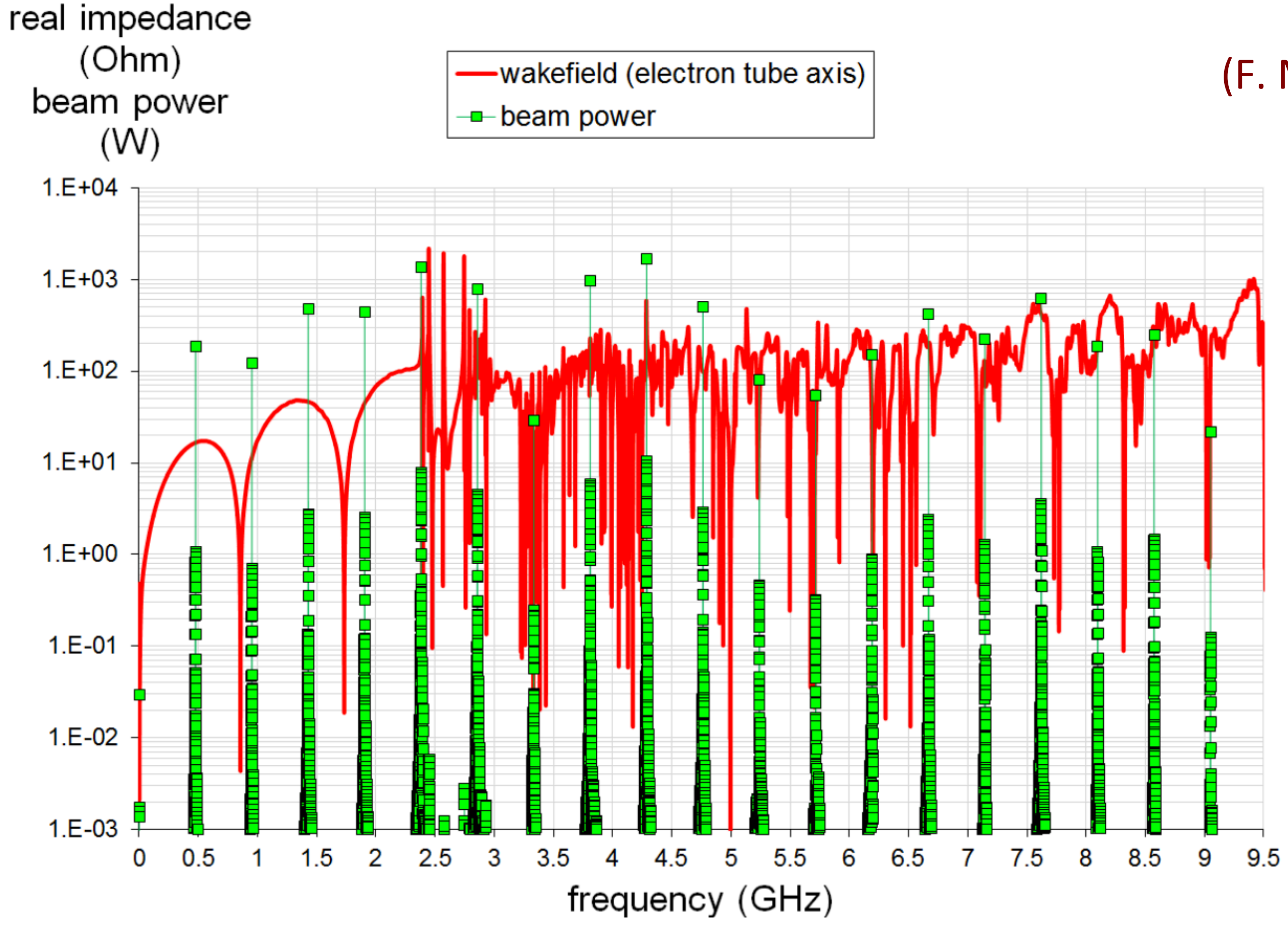


Power Deposited

Electron ring: 2366 m, 476.3 MHz, 3.6 A, 267 bucket gap

Up to 9.5 GHz sum of 9.2 kW power deposited by electrons

(F. Marhauser)

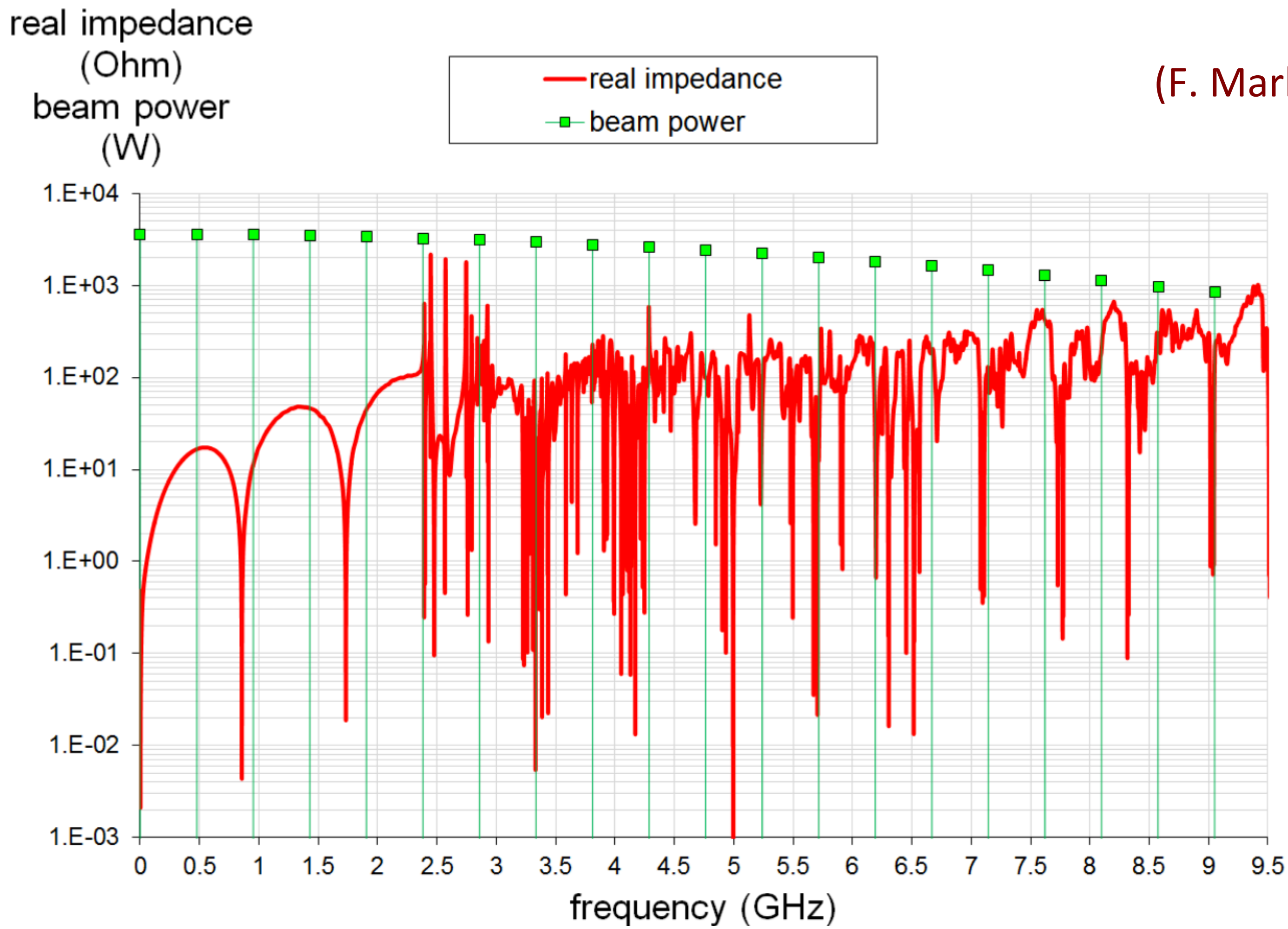


Power Deposited

Electron ring: 2366 m, 476.3 MHz, 3.6 A, NO GAP

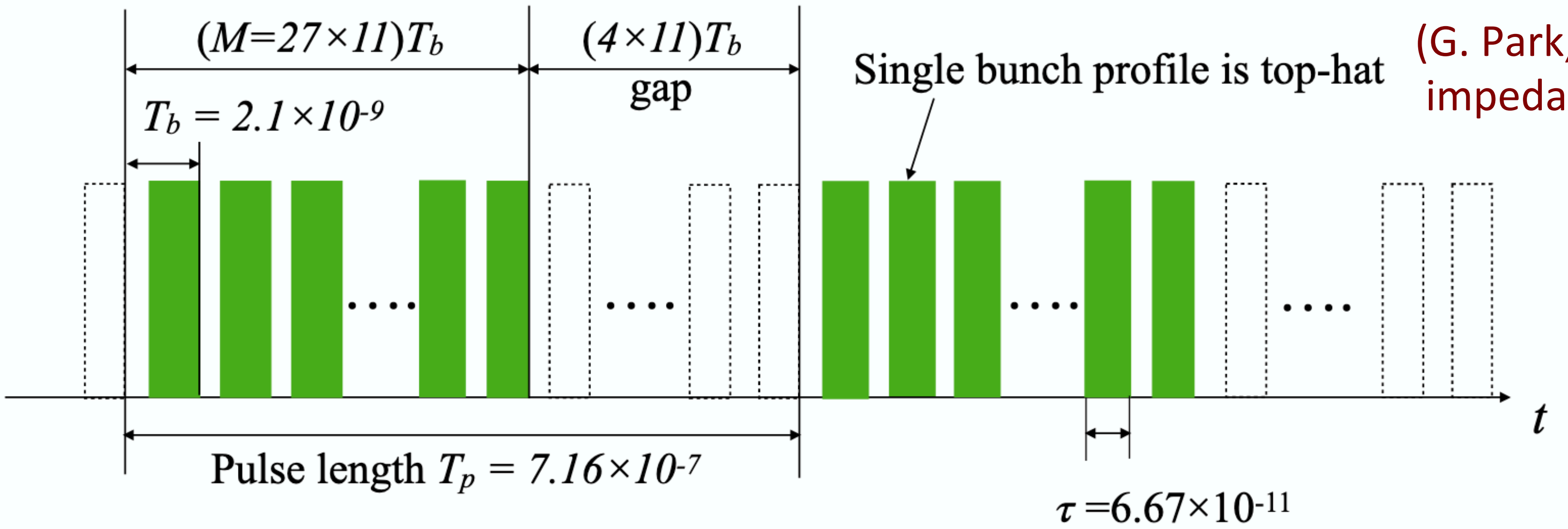
Up to 9.5 GHz sum of 9.9 kW power deposited by electrons

(F. Marhauser)



Time structure of beam current in CCR (Example 2)

(G. Park, 5/14/19 impedance meeting)



$$I(t) = \frac{2Q_b}{T_p} M + \Sigma A_n \cos(n\omega_p t) + B_n \sin(n\omega_p t)$$

n is not multiple of 31×11

$$A_n = \frac{4Q_b}{n\omega_p T_p \tau} \sin\left(\frac{n\omega_p\tau}{2}\right)\left[\cos\left(\frac{n\omega_p\tau}{2}\right)\left\{\frac{1}{2} + \frac{\sin(M-1/2)\delta_n}{2\sin\delta_n/2}\right\} - \sin\left(\frac{n\omega_p\tau}{2}\right)\left\{\frac{1}{2}\cot\frac{\delta_n}{2} - \frac{\cos(M-1/2)\delta_n}{2\sin\delta_n/2}\right\}\right]$$

$$= \frac{4Q_b}{n\omega_p T_p \tau} M \sin\left(\frac{n\omega_p\tau}{2}\right)\cos\left(\frac{n\omega_p\tau}{2}\right)$$ n is multiple of 31×11

n is not multiple of 31×11

$$B_n = \frac{4Q_b}{n\omega_p T_p \tau} \sin\left(\frac{n\omega_p\tau}{2}\right)\left[\sin\left(\frac{n\omega_p\tau}{2}\right)\left\{\frac{1}{2} + \frac{\sin(M+1/2)\delta_n}{2\sin\delta_n/2}\right\} + \cos\left(\frac{n\omega_p\tau}{2}\right)\left\{\frac{1}{2}\cot\frac{\delta_n}{2} - \frac{\cos(M+1/2)\delta_n}{2\sin\delta_n/2}\right\}\right]$$

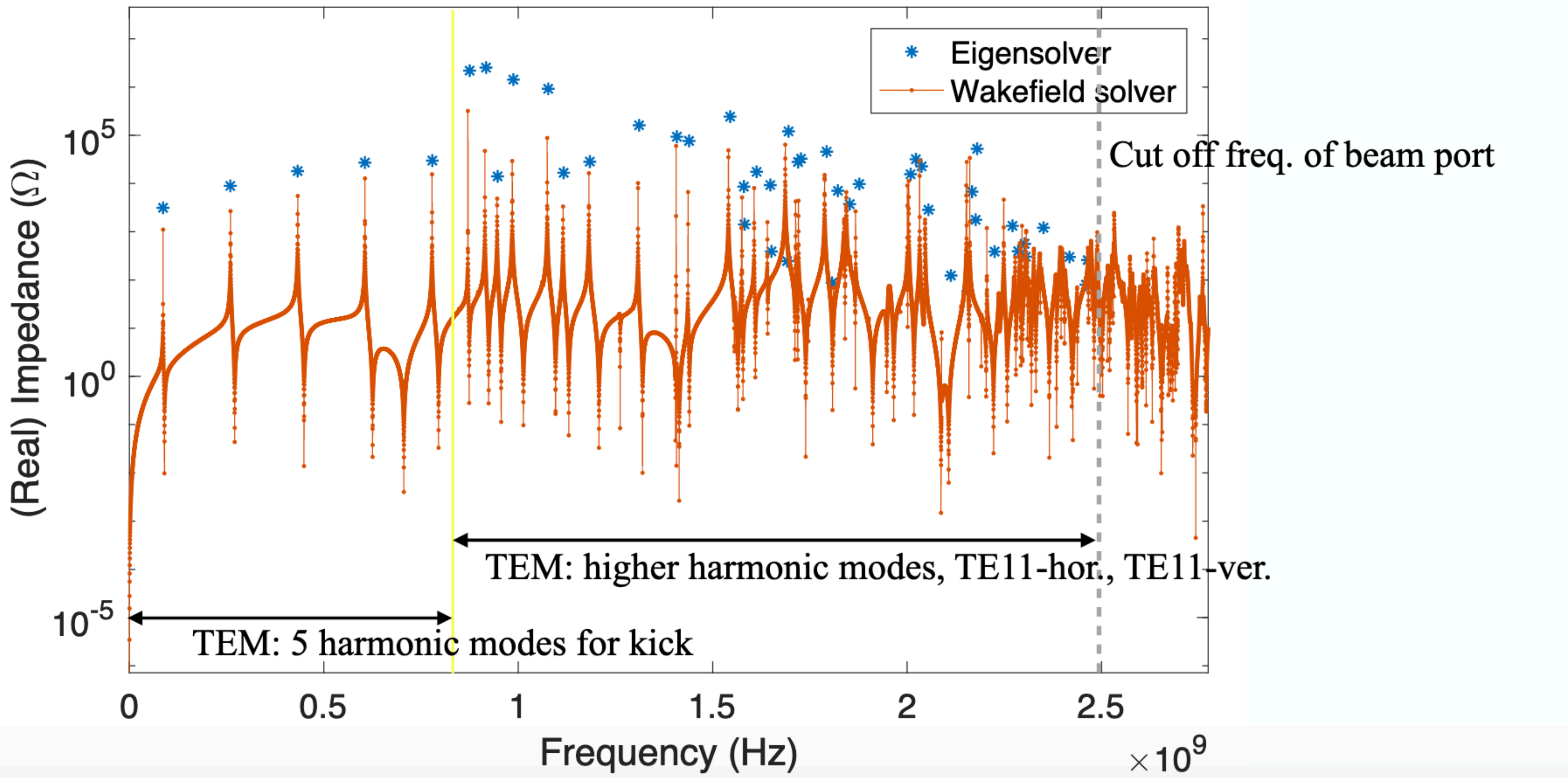
$$= \frac{4Q_b}{n\omega_p T_p \tau} M \sin^2\left(\frac{n\omega_p\tau}{2}\right)$$ n is multiple of 31×11

Impedance spectrum (real value) by charge on beam axis

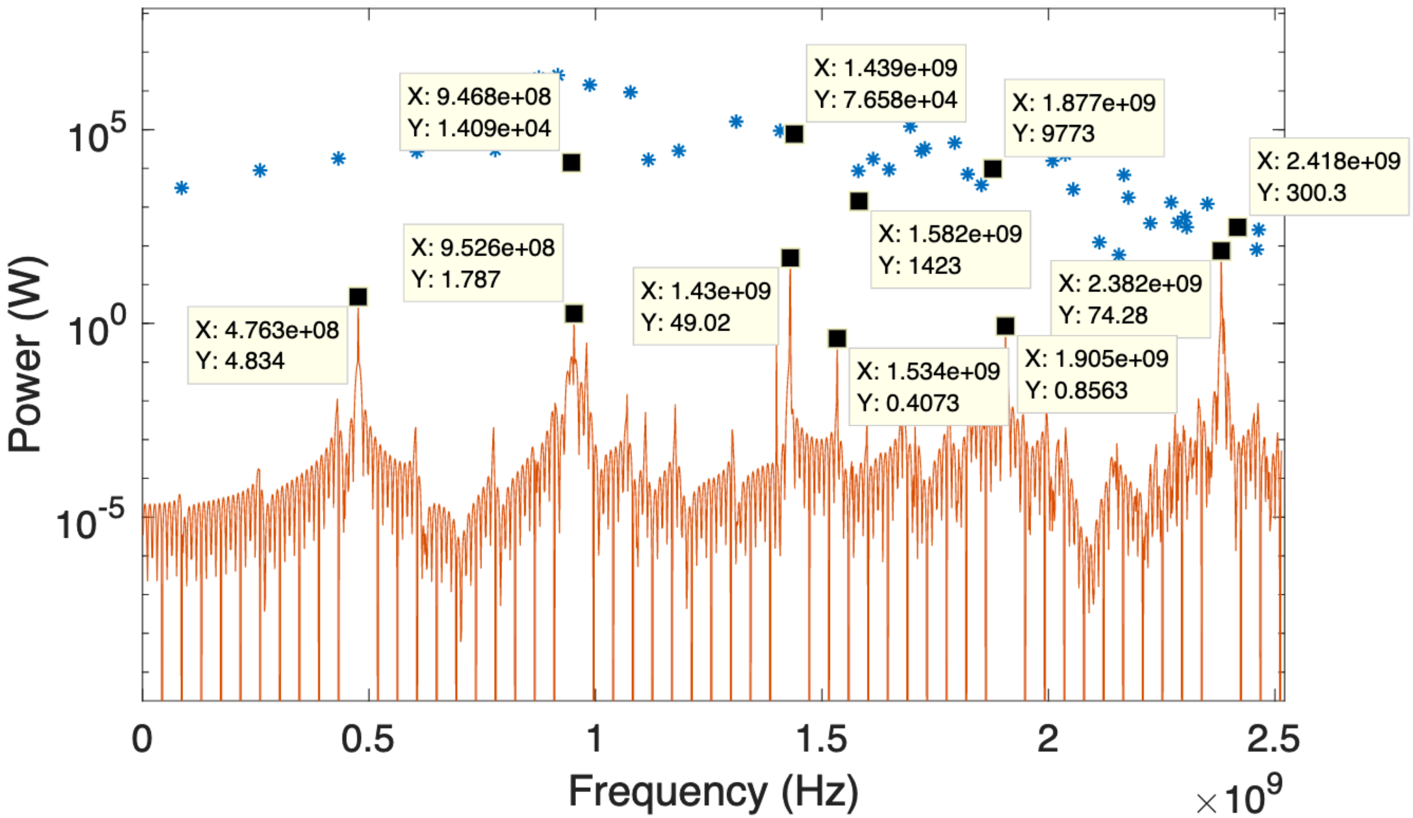
- the peak locations in (real) impedance spectrum are identified with resonant frequencies and the peak values are the effective shunt impedance, which is more accurately evaluated by CST-MWS (Eigensolver).

(G. Park)

$$\mathrm{Re}\,[\,Z\,] = Q_e \times R_{\mathrm{long}}/Q_0$$



$$k = \frac{2}{q^2} \int_0^\infty d\omega \, Re[Z(\omega)] I^2(\omega) \qquad U = 2 \int_0^\infty d\omega \, Re[Z(\omega)] I^2(\omega)$$



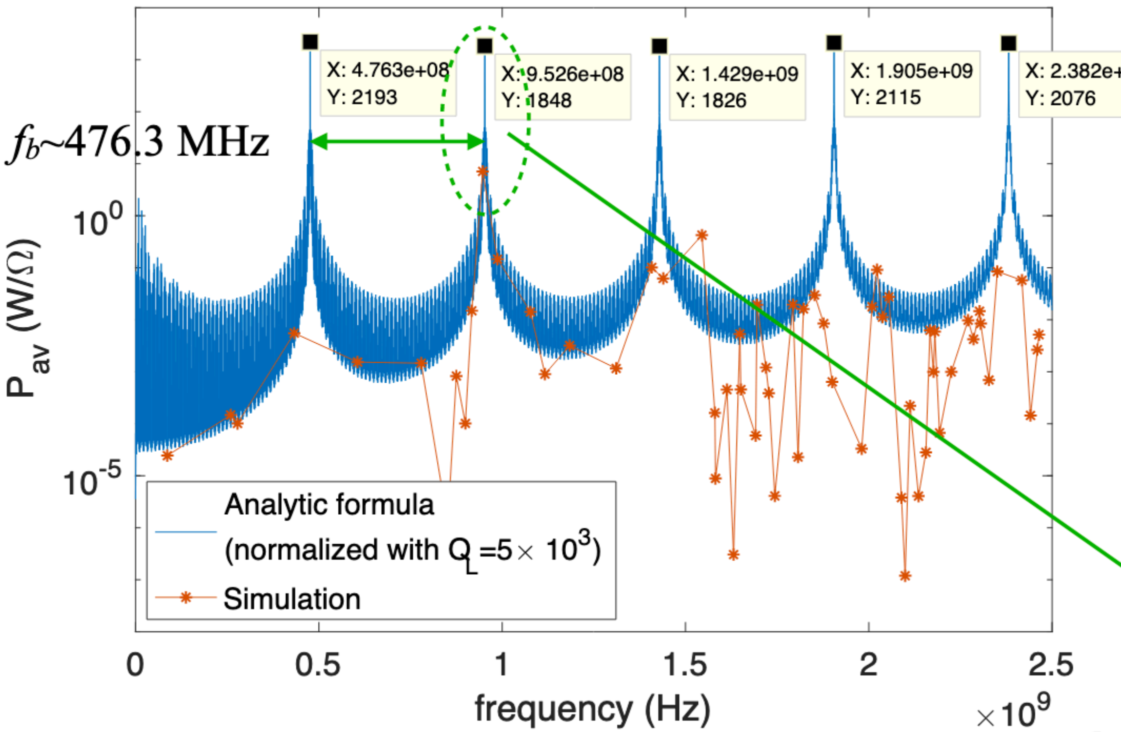
(G. Park)

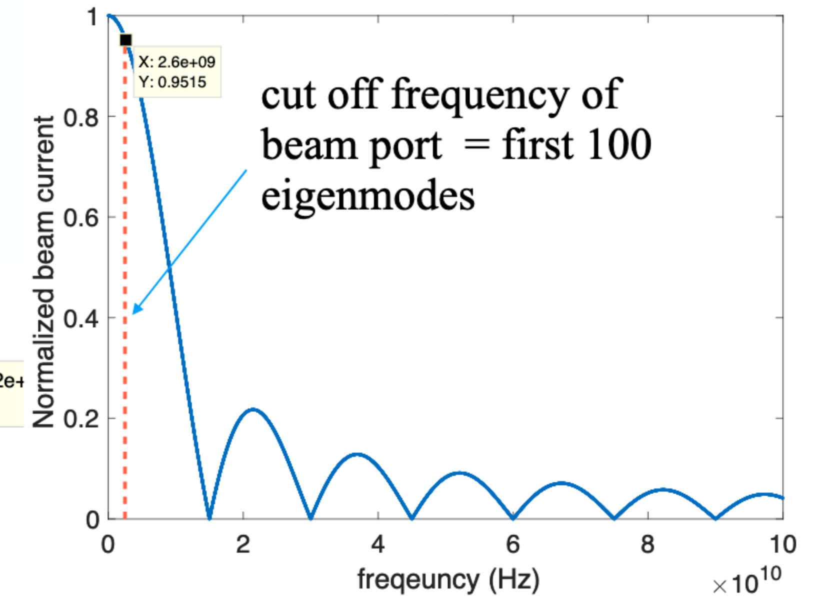
$P_{loss} = 311$ W.

$k_{loss} = 0.02$ V/pC (for single bunch)

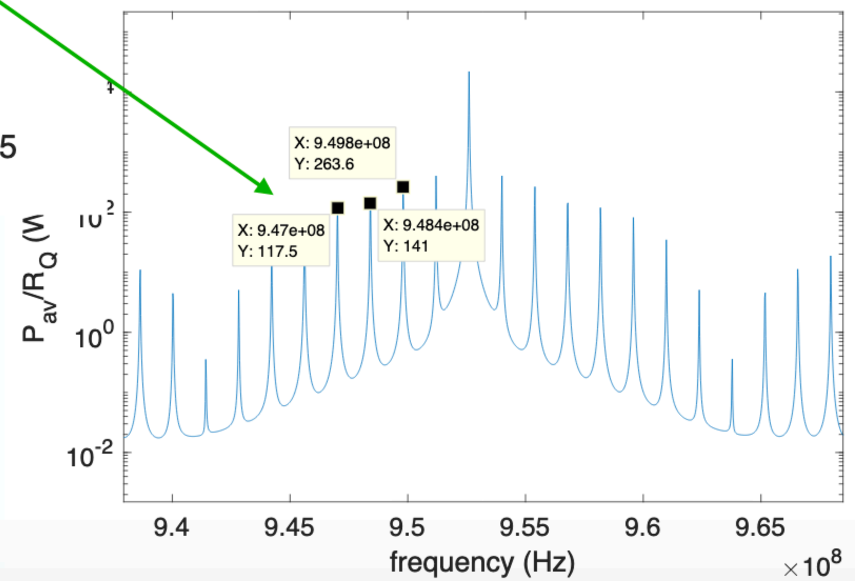
- (average) Power of HOM

$P_{\text{ave,total}} \sim 10\text{W}$ ($P_{\text{ave,loss}} \sim 4\text{W}$)



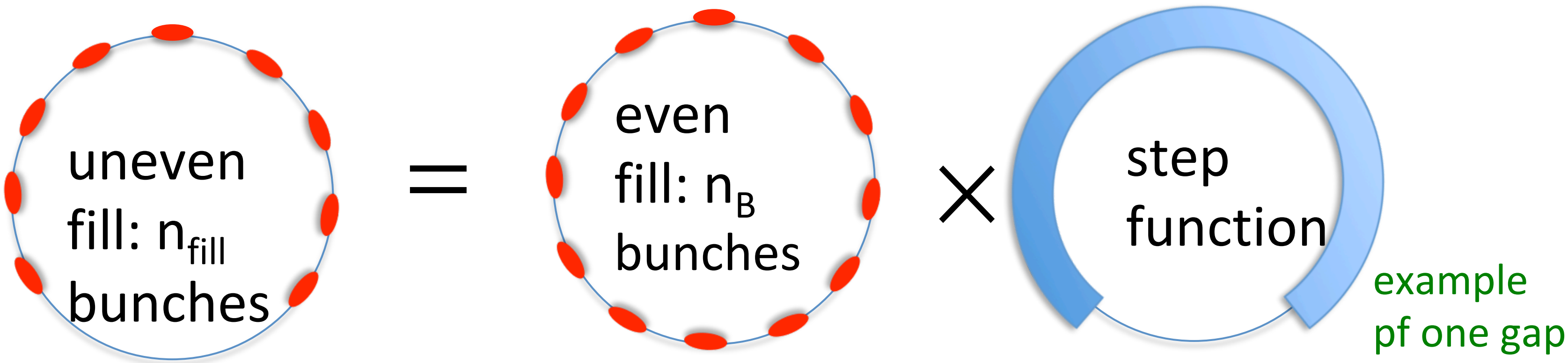


Frequency spectrum of top-hat beam current



(G. Park)

2. Bunch Distribution for Uneven Fill



$$I(t) = F(t) \times H(t)$$

Periodic Functions:

$$F(t) = \sum_{n=0}^{n_B} f(t - nT_B)$$

$$F(t) = F(t + T_B)$$

$$H(t) = \begin{cases} 1 & (|t| < T_{fill}/2) \\ 0 & (T_{fill}/2 \le |t| < T_0/2) \end{cases}$$

$$H(t) = H(t + T_0)$$

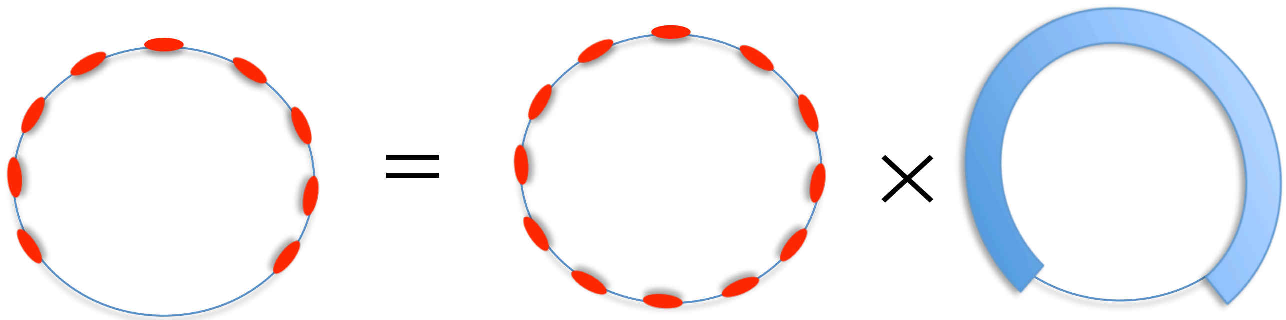
Bunch spacing: T_B

Even fill: n_B bunches

Revolution period: T_0

Portion filled: T_{fill}

Bunch Distribution for Uneven Fill



$$I(t) \quad = \quad F(t) \quad \times \quad H(t)$$

$$F(t) = \sum_{n=0}^{n_B} f(t - nT_B), \quad f(t)\text{: single bunch distribution,} \quad \alpha_f = \frac{n_{fill}}{n_B} = \frac{T_{fill}}{T_0}$$

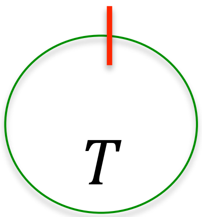
Normalization: $\displaystyle\int_{-T_0/2}^{-T_0/2} f(t)\,dt = N_b, \qquad \int_{-T_0/2}^{-T_0/2} I(t)\,dt = \alpha_f N_b n_B$

3. Current Spectra for Uneven Fill

- Total current spectra

$$I(t)=F(t)\cdot H(t), \qquad \bar{I}(\omega)=\int_{-\infty}^{\infty} d\omega' \, \bar{F}(\omega-\omega')\bar{H}(\omega')$$

- Fourier spectra for a periodic delta function



  - Periodic delta function

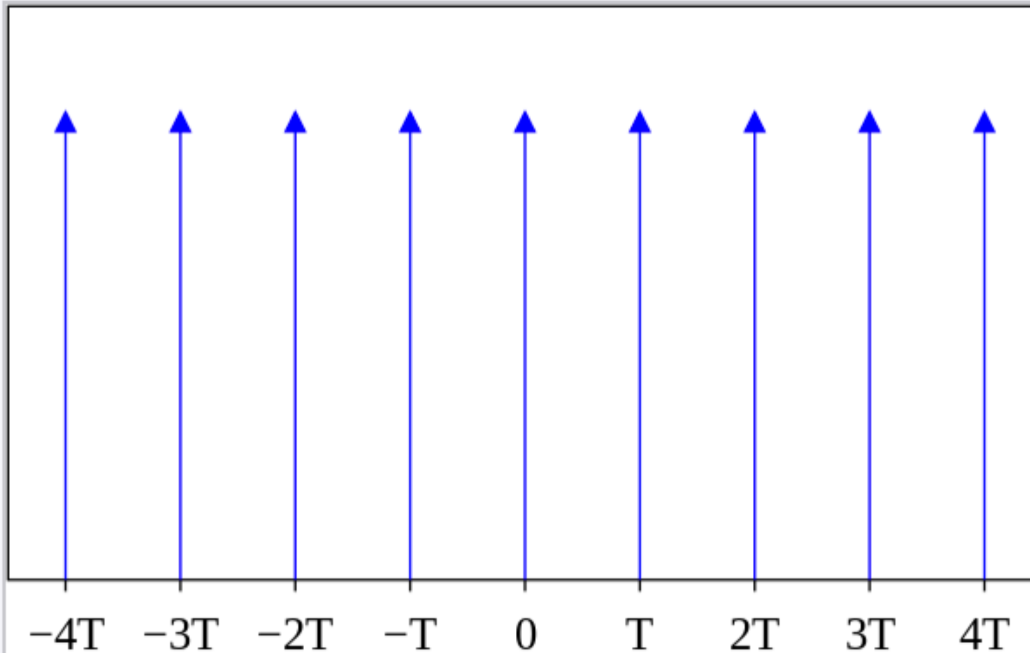
$$\delta^{(T)}(t)=\delta(t) \;\; (\text{for } |t| \leq T/2); \quad \delta^{(T)}(t)=\delta^{(T)}(t+T)$$

  - Its Fourier transform

$$\bar{\delta}^{(T)}(\omega)=\frac{1}{T}\sum_{l=-\infty}^{\infty} \delta(\omega-l\omega_0) \quad \text{for} \quad \omega_0=\frac{2\pi}{T}$$

Equivalent Dirac Comb

$$\text{III}_T(t) \triangleq \sum_{k=-\infty}^{\infty} \delta(t - kT)$$



A Dirac comb is an infinite series of Dirac delta functions spaced at intervals of T

Fourier Expansion:

$$\text{III}_T(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{i2\pi n \frac{t}{T}} .$$

Fourier Transform:

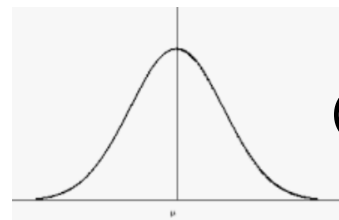
$$\sum_{k=-\infty}^{\infty} \delta\left(t - kT\right) \rightarrow \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\omega_0\right) \quad \text{for} \quad \omega_0 = \frac{2\pi}{T}$$

Fourier Spectra for an Even Fill

- Convolution with single bunch distribution and Dirac comb

$$F(t)=\sum_{n=0}^{n_b} f(t-nT_B)=\int_{t'=-T_0/2}^{t'=T_0/2} f(t-t')\sum_{n=0}^{n_B}\delta^{(T_b)}(t'-nT_B)\,dt',\quad F(t)=F(t+T_B)$$

$$\bar{F}(\omega)=\frac{n_B}{T_0}\sum_{l=-\infty}^{\infty}\bar{f}(\omega)\,\delta(\omega-l\omega_B)$$



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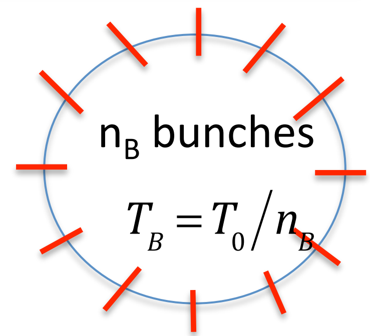


- Single bunch distribution

Gaussian bunch:

$$f(t)=\frac{N_b}{\sqrt{2\pi}\sigma_\tau}e^{-\frac{t^2}{2\sigma_\tau^2}},\qquad \bar{f}(\omega)=\frac{N_b}{2\pi}e^{-\omega^2\sigma_\tau^2/2}$$

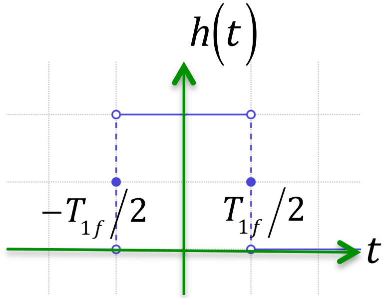
Step bunch:

$$f(t)=\begin{cases}1 & (|t|<\sigma_\tau/2)\\ 0 & (\text{otherwise})\end{cases},\qquad \bar{h}(\omega)=\frac{1}{\pi\omega}\sin\left(\frac{\omega\sigma_\tau}{2}\right)$$

Spectra for Bunch Train with Gaps

- Fourier spectra for the periodic step function

  (T_1 as bunch train period, T_1=T_0/n_{tr}, n_{tr}=3 for 3 bunch trains)



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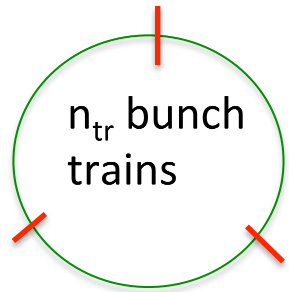


$$H(t) = \int_{t'=-T_1/2}^{t'=T_1/2} h(t-t')\, \delta^{(T_1)}(t';T_1)\, dt', \quad \mathrm{H}(t) = H(t+T_1)$$

T_{1f} : filled portion in T_1

Filling factor: $\alpha_f = T_{1f}/T_1$

$$\bar{H}(\omega) = \frac{1}{T_0} \sum_{m=-\infty}^{\infty} \bar{h}(\omega)\, \delta(\omega - m\omega_1)$$

for $\omega_1 = n_{tr}\omega_0$

Step bunch: $h(t) = \begin{cases} 1 & (|t| < T_{1f}/2) \\ 0 & (\text{otherwise}) \end{cases}$, $\bar{h}(\omega) = \frac{1}{\pi\omega}\sin\left(\frac{\omega T_{1f}}{2}\right)$

Beam Spectra for Uneven Fill

for $\omega_B = n_B \cdot \omega_0$

$\omega_{tr} = n_{tr} \omega_0$

- Total current spectra for an uneven fill

$$\bar{I}(\omega) = \int_{-\infty}^{\infty} d\omega' \, \bar{F}(\omega - \omega') \bar{H}(\omega')$$

$$= \frac{\alpha_f n_B N_b}{(2\pi)^2 T_0} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-l^2 \omega_B^2 \sigma_b^2 / 2} \left[\frac{\sin(m\pi\alpha_f)}{m\pi\alpha_f} \right] \delta(\omega - l\omega_B - m\omega_{tr})$$

- Total beam power spectra for an uneven fill

$$\bar{I}^2(\omega) == \left(\frac{\alpha_f n_B N_b}{(2\pi)^2 T_0} \right)^2 \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-(l\omega_B \sigma_\tau)^2} \left[\frac{\sin(m\pi\alpha_f)}{m\pi\alpha_f} \right]^2 \delta(\omega - l\omega_B - m\omega_{tr})$$

Beam Spectra for Even Fill

- Total beam power spectra for an uneven fill

$$\bar{I}^2(\omega) == \left(\frac{\alpha_f n_B N_b}{(2\pi)^2 T_0} \right)^2 \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-(l\omega_B \sigma_\tau)^2} \left[\frac{\sin(m\pi\alpha_f)}{m\pi\alpha_f} \right]^2 \delta(\omega - l\omega_B - m\omega_1)$$

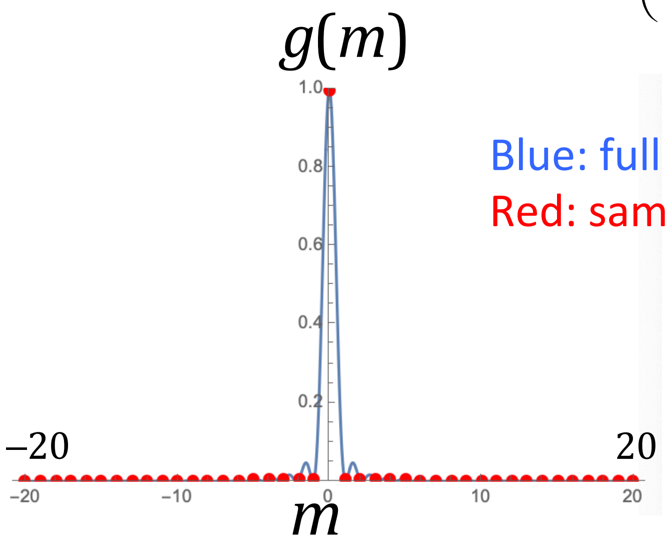
- Total beam power spectra for an even fill

$\alpha_f = 1$, and only $m = 0$ term remains

$$\bar{I}^2(\omega) = \left(\frac{n_B N_b}{4\pi^2 T_0} \right)^2 \sum_{l=-\infty}^{\infty} e^{-l^2 \omega_b^2 \sigma_\tau^2} \delta(\omega - l\omega_B)$$

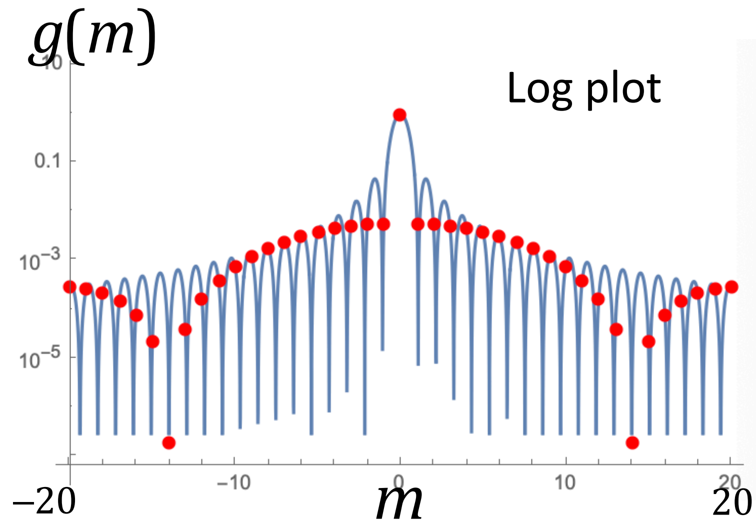
Behavior of g(m) in Beam Spectrum

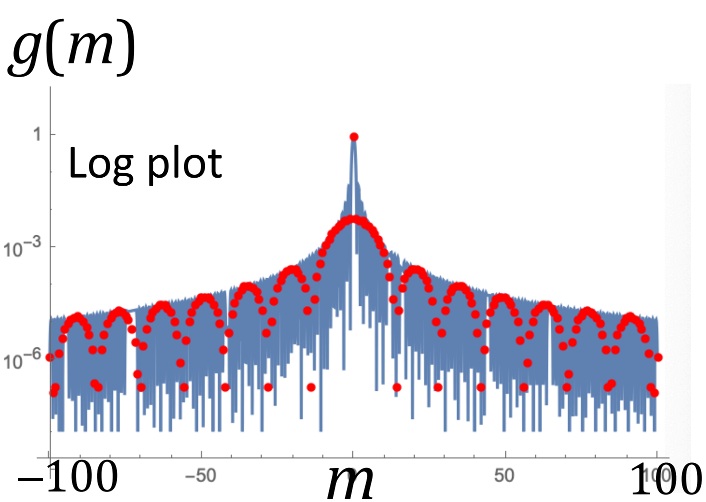
Spectrum for step function: $g(m) = \left(\frac{\sin(m\pi\alpha_f)}{m\pi\alpha_f} \right)^2$ vs. m $(\text{for } \alpha_f = 0.929)$

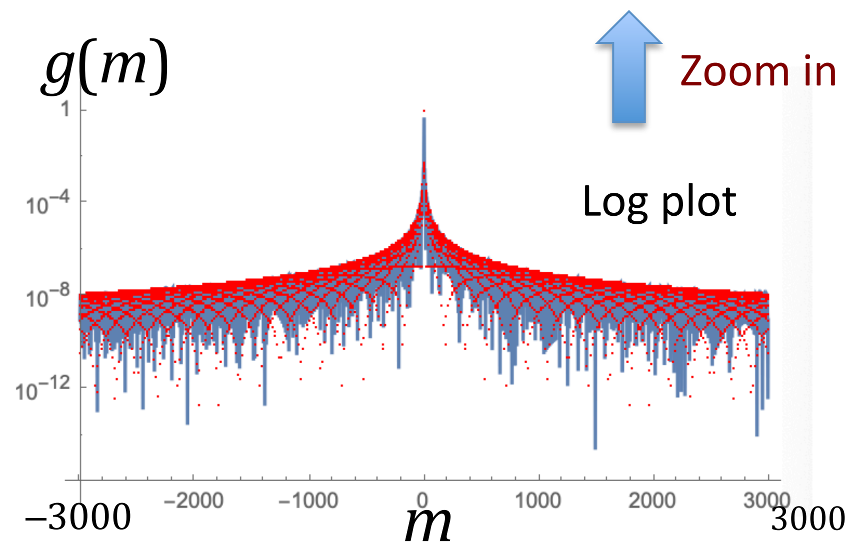


Blue: full function

Red: sampled sideband

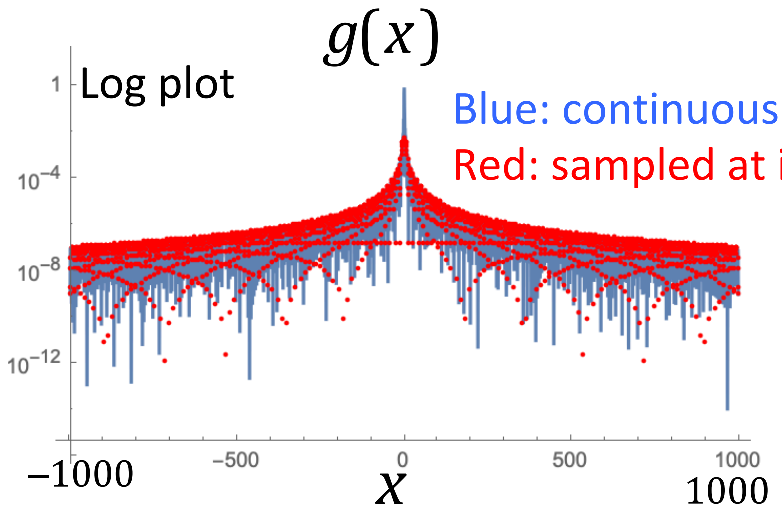






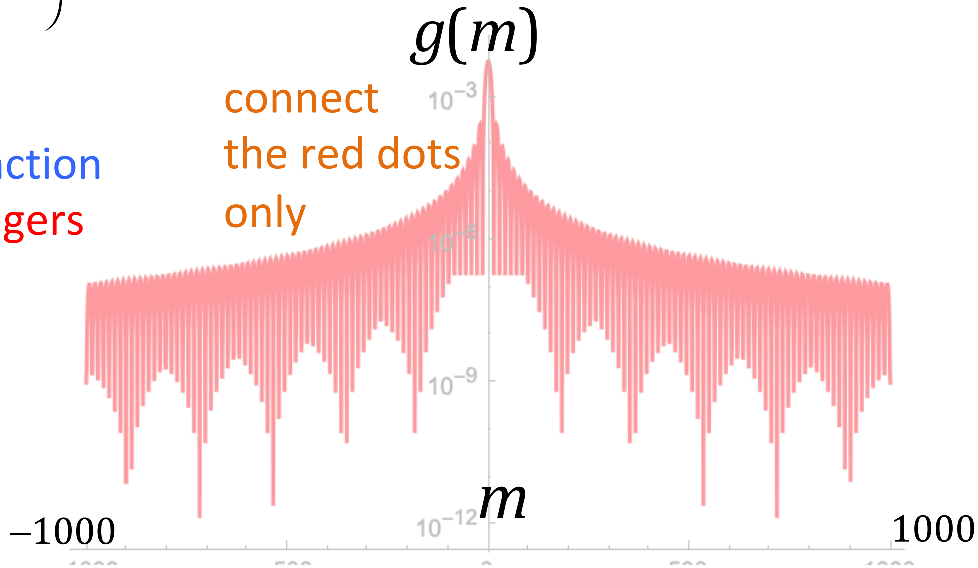
Behavior of g(m) in Beam Spectrum

Spectrum for step function: $g(m)=\left(\frac{\sin\left(m\pi\alpha_f\right)}{m\pi\alpha_f}\right)^2$ vs. m $\left(\text{for } \alpha_f = 0.929\right)$



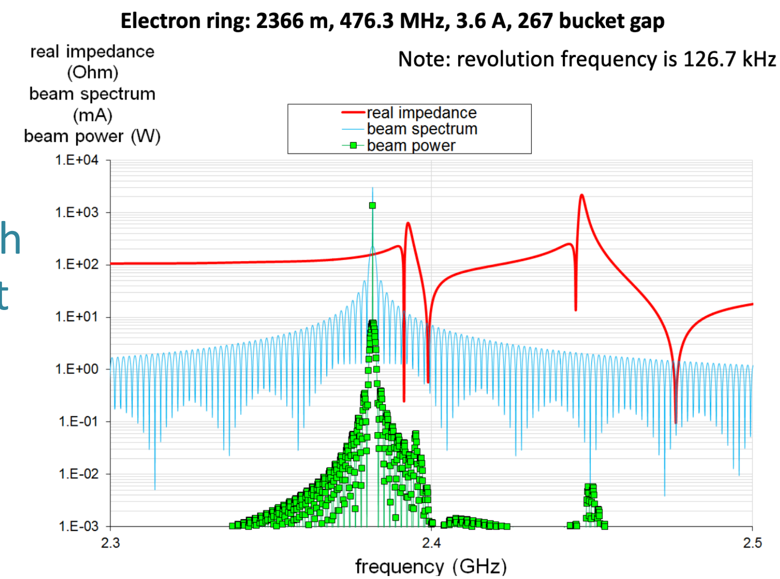
Blue: continuous function

Red: sampled at integers



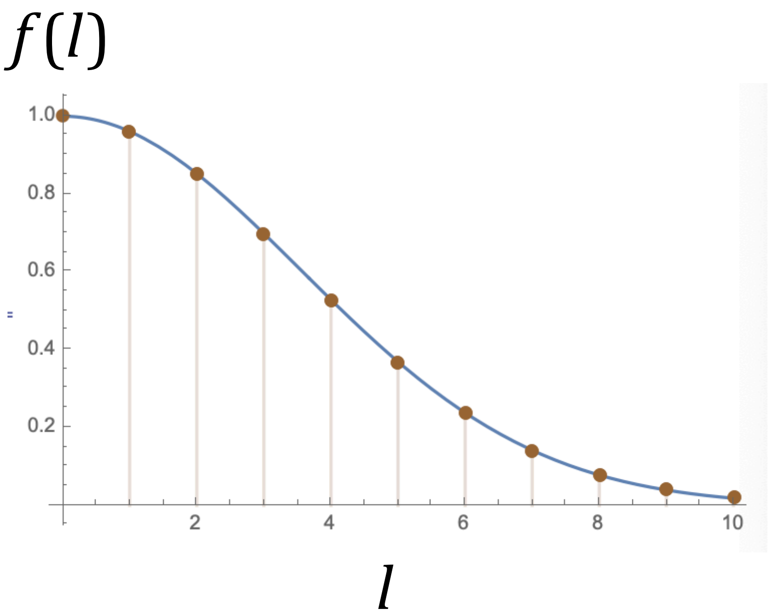
connect the red dots only

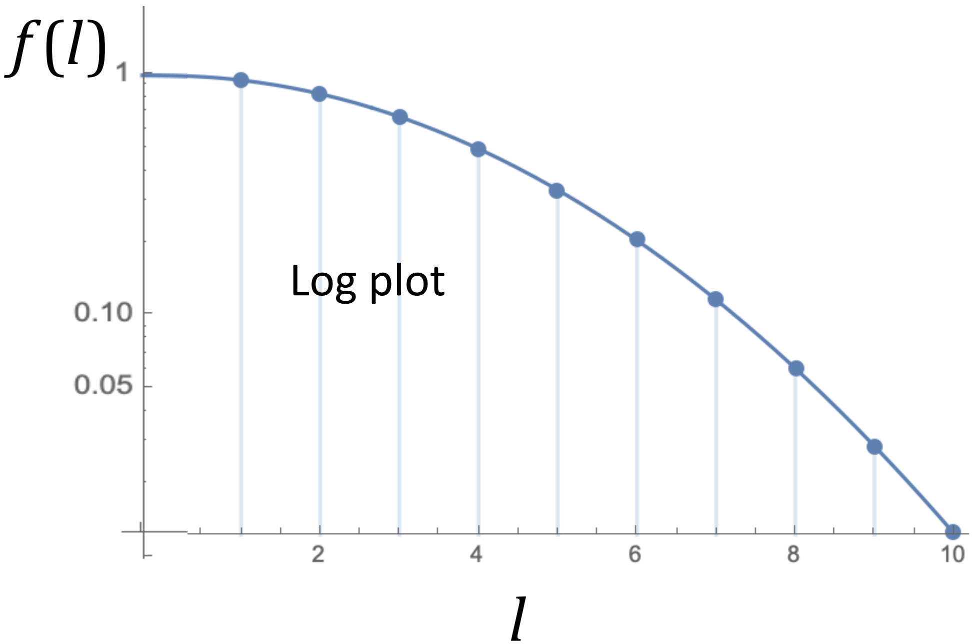
Compare with Frank's result in light blue



Behavior of f(l) in Single Beam Spectrum

Spectrum for Guassian function: $\text{f(l)}=e^{-(l\omega_B\sigma_\tau)^2/2}$ vs. l $\left(\text{for } \sigma_\tau = 10 \text{ mm/c}\right)$





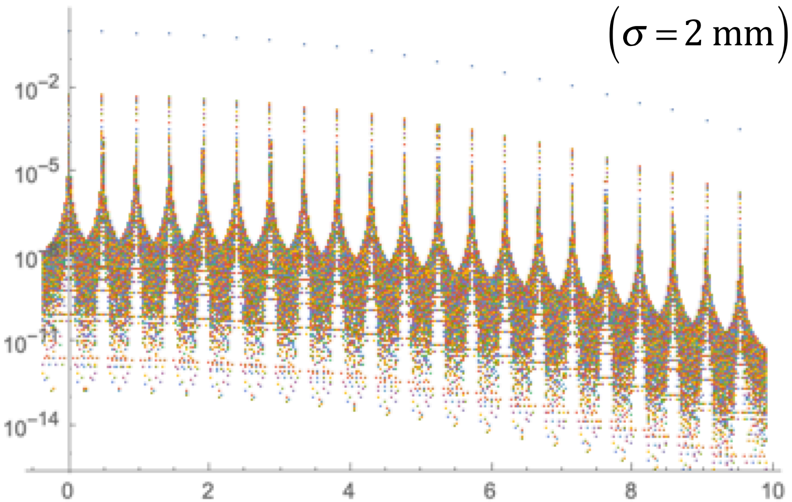
Final Results

- Bunch Power Spectra: weighted sidebands around higher harmonics of bunch rep rate

$$\bar{I}^2(\omega)=\left(\frac{\alpha_f n_B N_b}{(2\pi)^2 T_0}\right)^2 \sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} e^{-l^2\omega_B^2\sigma_b^2}\left[\frac{\sin(m\pi\alpha_f)}{m\pi\alpha_f}\right]^2 \delta(\omega-l\omega_B-m\omega_{tr})$$

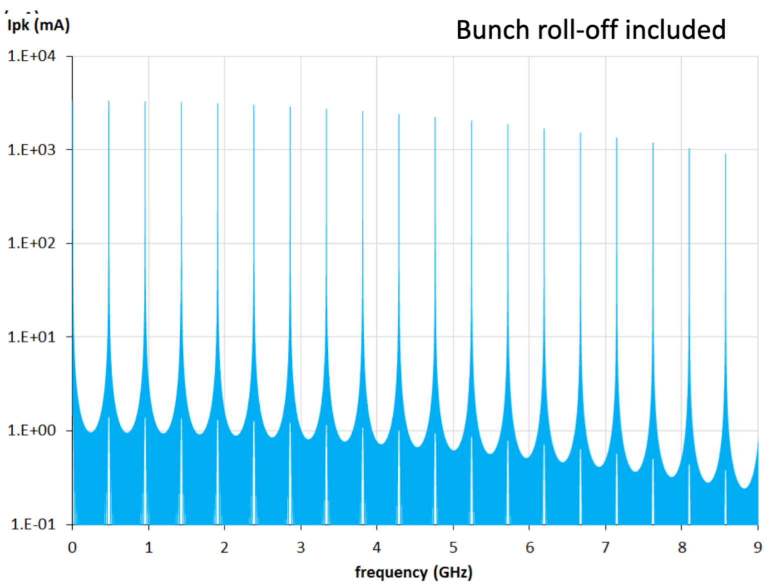
Beam power spectra from analysis

$(\sigma = 2\text{ mm})$



Frequency (GHz)

F. Marhauser's result



7. Conclusion

- Preliminary analysis of the spectrum for uneven bunch distribution are performed, it needs further check, and more plots are needed
- It should be compared with numerical results from Frank and Gunn Tae for complete understanding
- The study for the impact of uneven bunch distribution on the power loss and CBI growth rate will be continued