Overview of Diffraction Model of Broadband Impedance for RF Cavities

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JLEIC Impedance Meeting

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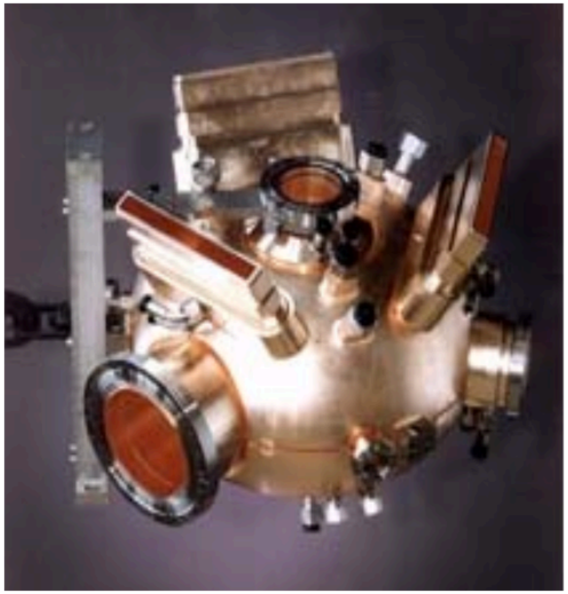
Outline

- Simplification of 3D RF cavity modeling
- Diffraction model of impedance for a pillbox cavity
- Longitudinal broadband impedance for a pillbox cavity
- Comparison of theory with simulations

Key Points

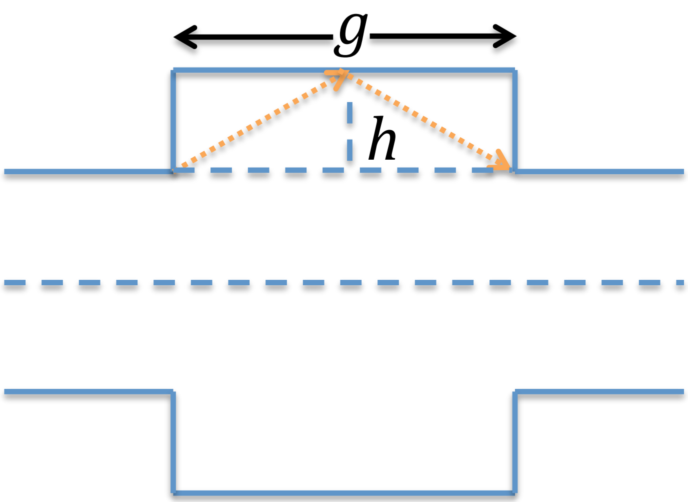
- For the broadband impedance, 3D RF geometry can be modeled by cylindrically symmetric pillbox model
- Diffraction theory for impedance can be applied to pillbox model
- Diffraction results depicts the average behavior of the actual broadband impedance
- CST or TBCI simulation shows good agreement with the diffraction results

Simplifications of 3D RF Cavity Modeling



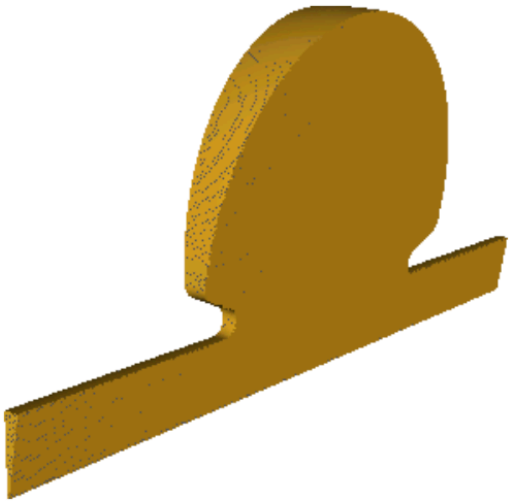
PEP II cavity
476 MHz, single cell,
1 MV gap with 150 kW,
strong HOM damping,

The couplers has no contribution to the broadband impedance because the reflected fields cannot catch up the bunch

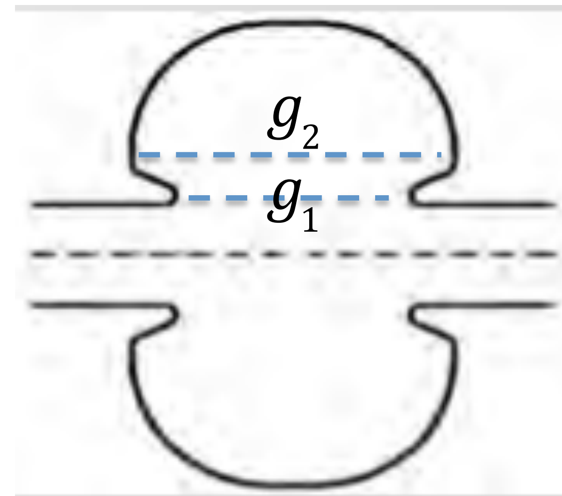


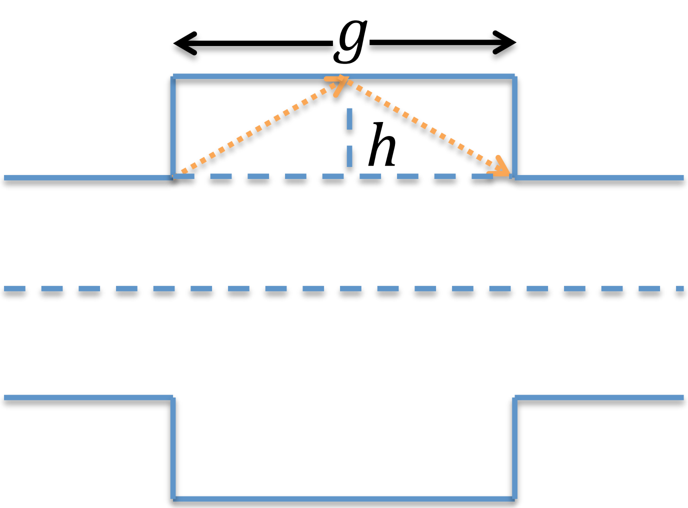
$$2\sqrt{\left(g/2\right)^2+h^2}-2g\geq\sigma_z \quad \text{or} \quad g\leq\frac{2h^2}{\sigma_z}$$

Simplifications of 3D RF Cavity Modeling



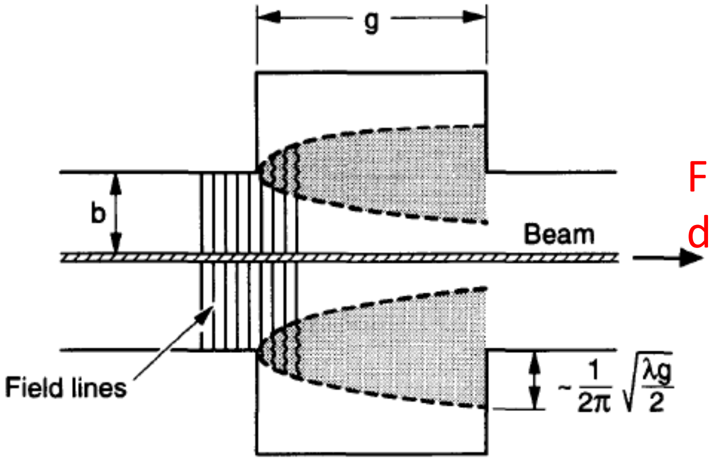
The PEP-II rf cavity, showing 10-deg slice in the azimuthal direction





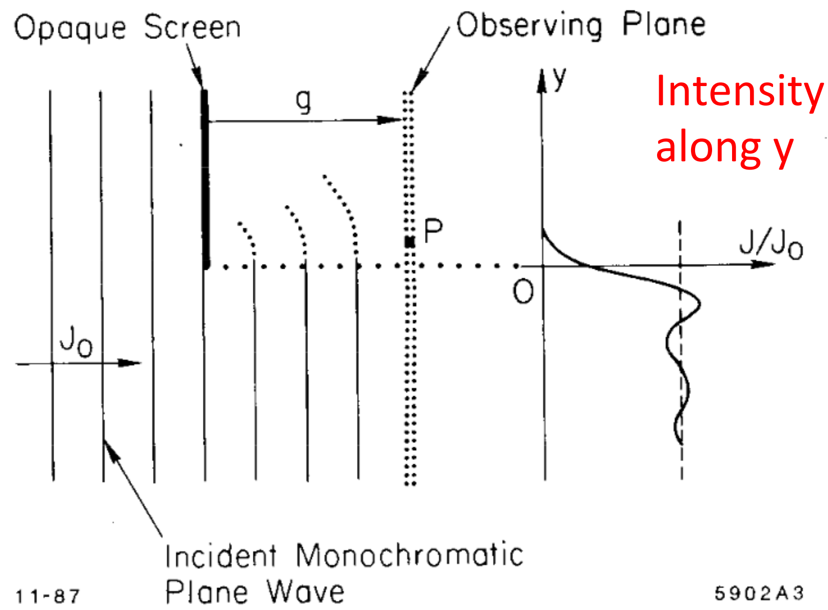
(Bane etc, SLAC–PUB–13999)

Diffraction Model of Impedance for a Pillbox Cavity



Fresnel diffraction

(Chao's book)



Intensity along y

(Bane and Sands, SLAC-PUB-4441)

Power loss: $\Delta P_D = 2\Delta l \int_0^\infty J(y)dy = R_\parallel(\omega)\cdot\langle I^2(\omega)\rangle$

Impedance: $R_\parallel(\omega) = \frac{Z_0 c}{2\pi^{3/2} a}\sqrt{\frac{g}{\omega}}$

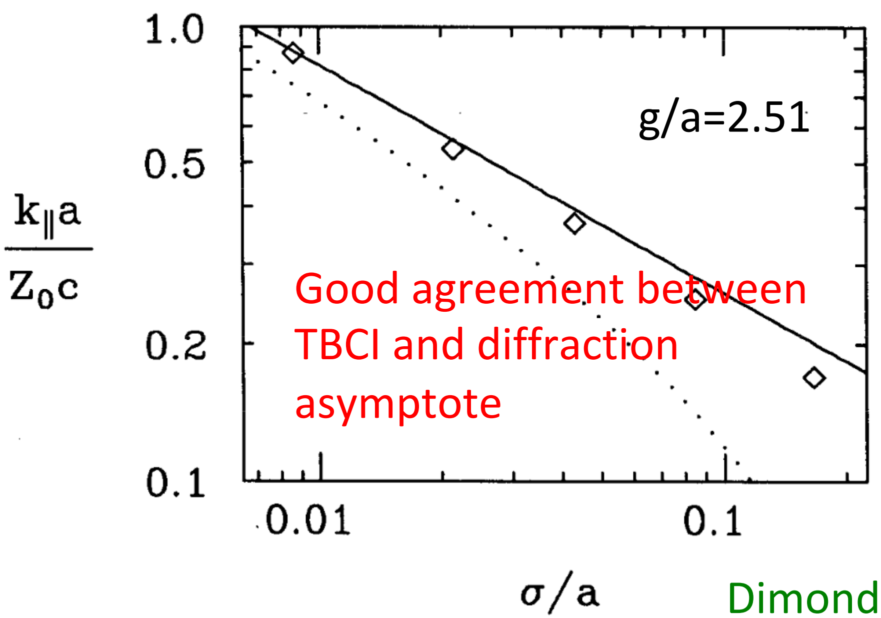
Numerical Modeling vs. Diffraction Theory

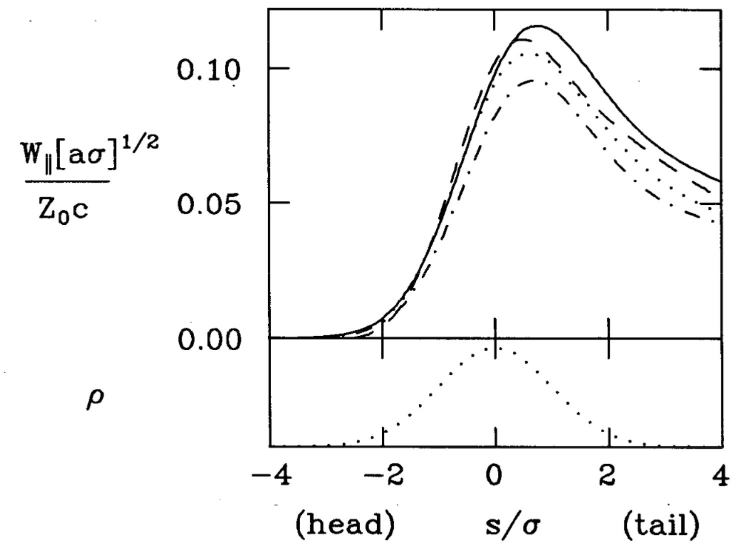
Loss factor:
(diffracted part)

$$\kappa_{\parallel}(\sigma)=\frac{1}{\pi}\int_{\omega_c}^{\infty} R_{\parallel}(\omega)\, e^{-(\omega\sigma/c)^2}\, d\omega = \frac{Z_0 c}{4\pi^{5/2} a}\sqrt{\frac{g}{\sigma}}\left[\Gamma\left(\frac{1}{4}\right)-4\sqrt{\frac{\omega_c \sigma}{c}}\right]$$

Asymptotic form:
(asymptote)

$$\kappa_{\parallel}(\sigma)=\frac{\Gamma(1/4)Z_0 c}{4\pi^{5/2} a}\sqrt{\frac{g}{\sigma}} \quad (\sigma \to 0)$$



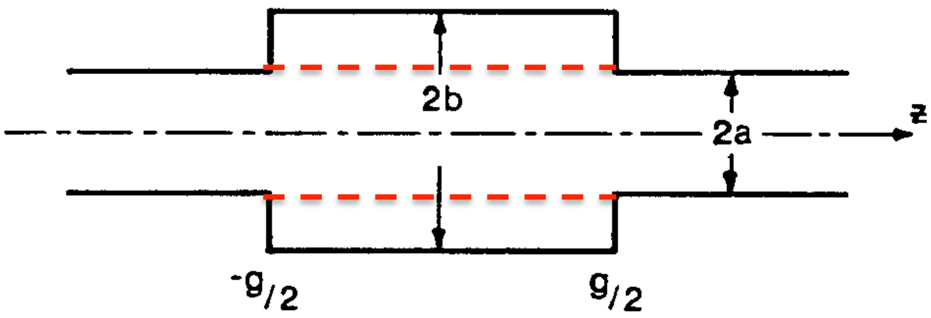


Dimond: TCBI results
Solid: Asymptotic form
Dotted: diffracted part

$\sigma/a = 0.0086,\ g/a = 2.51$
0.043,
0.167

(Bane and Sands, SLAC-PUB-4441)

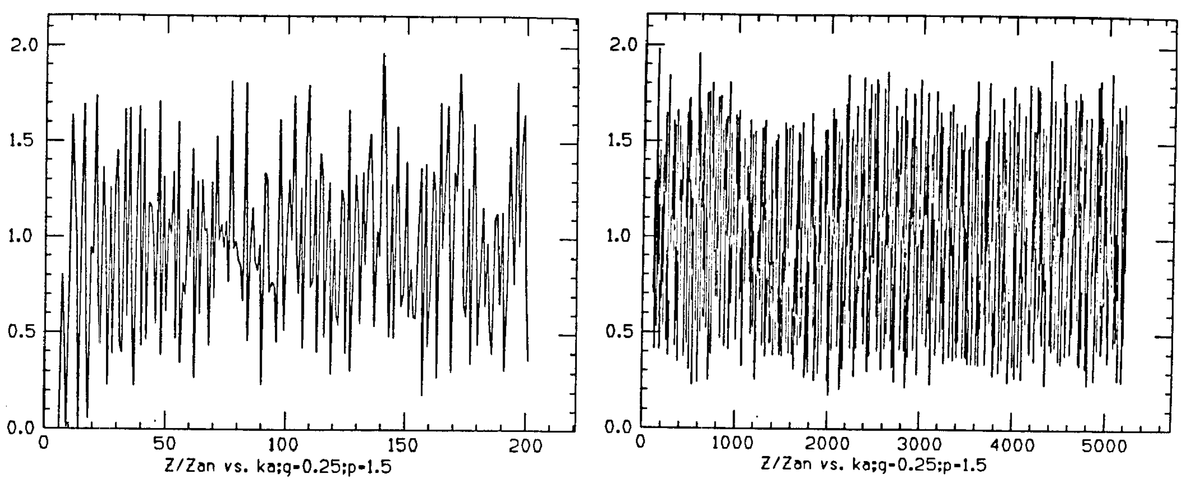
More Exact Solution from Mode Expansion



(Heifets and Kheifets, CEBAF-PR-87-030)

- Express field in r>a region in terms of discrete modes
- Express fields in r<a region in terms of source field and homogeneous field
- Match boundary condition at r=a and solve the fields

Re(Z) vs. ka

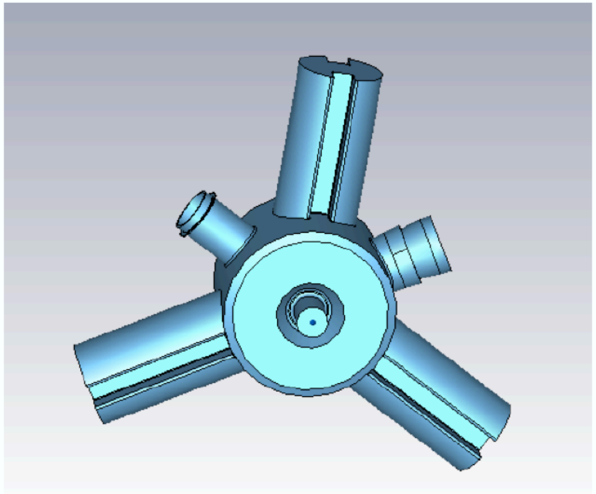


ALS-U Cavity Impedance Modeling

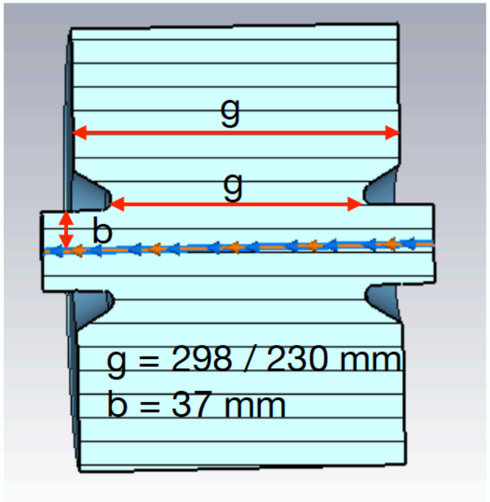
Calculation of cavity_1

(Wang, LBL)

Full model of RI cavity



model for short range simulation



1. Fundamental (high Q) mode 500 MHz;
2. Results will show that short range wakefield mainly come from the high frequency component (diffraction model);

$$\kappa_{||} \approx \frac{\Gamma(1/4) Z_0 c}{4\pi^{5/2} b}\sqrt{\frac{g}{\sigma_z}}$$

Approximation with **Pill-box** theory:

For a resonant cavity in a storage ring, $\kappa_{\parallel}$ is given by a sum over modes up to the cut-off frequency, plus a high frequency diffraction contribution (diffraction model [8], also Sec.3.2.4)

$$\kappa_{\parallel} \approx \frac{\Gamma(\frac{1}{4}) Z_0}{4\pi^{5/2} b}\sqrt{\frac{cg}{\sigma}} \quad (15)$$

← We find print mistake in the handbook :) →

ALS-U Cavity Impedance Modeling

Calculation of cavity_2

(Wang, LBL)

Loss factor table:

	1mm bunch length		5mm bunch length	
	CST	Pill-box theory	CST	Pill-box theory
loss factor V/pC	1.97	2.73 / 2.41	0.98	1.22 / 1.08

Loss factor / short range wakefield contributes from all modes: $\kappa_t = \sum_n \kappa_n$

Mode loss factor: $\kappa_n = \frac{\omega_n}{2} \frac{R}{Q} \cdot F(\omega_n, \sigma_z)$

Fundamental mode:

$$\omega_0 = 2\pi * 0.5[GHz]$$
$$R/Q = 237$$
$$\kappa_0 = 0.37\ [V/pC]$$

TABLE II: One RI Cavity longitudinal HOM parameters.

Plane	f_R (GHz)	Q	R_s (kΩ)
L	0.621682	19296.6	61.3946
L	2.65161	7151.77	29.7471
L	2.42536	21439.8	18.6311
L	1.57662	735.87	9.32046
L	1.91731	6417.2	8.85753
L	2.88224	5279.52	7.21098
L	0.597296	17478.3	6.6219
L	2.42832	12616.8	6.15527
L	0.621669	19715.2	5.34448
L	2.43037	6292.48	5.27058
L	2.6621	23788.4	5.26743
L	2.26339	10402.2	4.40352
L	1.98881	3305.57	4.36381
L	2.27578	4142.29	4.20015
L	2.68873	14827.	3.10371

eg. Pick one HOM:

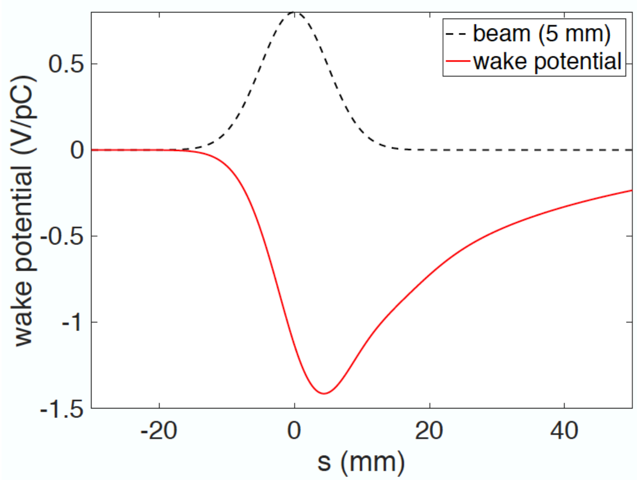
$$\omega = 2\pi * 1.58[GHz]$$
$$R/Q = 12.7$$
$$\kappa = 0.07\ [V/pC]$$

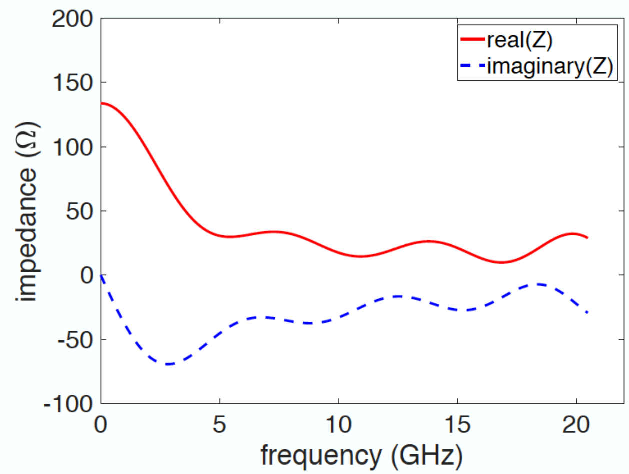
ALS-U Cavity Impedance Modeling

Calculation of cavity_3

(Wang, LBL)

Wake potential: resistive & capacitive property





Resistive component: $V_{ind} = -RI$

Capacitive component: $V_{ind} = -\frac{1}{C}\int I$, tend to shorten the bunch length.

- Karl Bane, Bunch lengthening in the SLC damping rings, 1990

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CST computation time: 10 hrs for 1mm bunch length

Summary

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