BEAM-BEAM EFFECT: CRAB DYNAMICS CALCULATION IN JLEIC

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Abstract

The electron and ion beams of a future Electron Ion Collider (EIC) must collide at an angle for detection, machine and engineering design reasons. To avoid associated luminosity reduction, a local crabbing scheme is used where each beam is crabbed before collision and de-crabbed after collision. The crab crossing scheme then provides a head-on collision for beams with a non-zero crossing angle. We develop a framework for accurate simulation of crabbing dynamics with beam-beam effects by combining symplectic particle tracking codes with a beam-beam model based on the Bassetti-Erskine analytic solution [1]. We present simulation results using our implementation of such a framework where the beam dynamics around the ring is tracked using Elegant and the beam-beam kick is modeled in Python.

MOTIVATION

The current Jefferson Lab Electron-Ion Collider (JLEIC) design relies upon short bunches and high repetition rates to achieve the desired luminosity unlike most ion colliders which rely on longer bunches with higher space charge. Crab crossing is an integral part of JLEIC design. Collider luminosity formulas assume head-on collisions, thus giving the maximum luminosity for a given beam intensity. The JLEIC design features a crossing angle of 50 mrad leading to a Piwinski angle of 16.5 rad. Without compensation of the crossing angle at the interaction point (IP), the beams no longer collide head-on and JLEIC design would result in an unacceptable loss of luminosity due to the beam-beam kicks generating synchro-betatron resonances. Considering this effect of crabbing, for the JLEIC design, a local crabbing scheme is used and thus each beam is crabbed before collision and de-crabbed after collision. JLEIC crab-crossing scheme is similar to what has been used at KEKB [2]. In detail, the compensation JLEIC is achieved by "crabbing," or tilting, each beam by half of the crossing angle such that the two beams collide head-on in the center of momentum frame (see Figure 1). Because JLEIC crab crossing scheme provides a head-on beam-beam collision for beams with a nonzero crossing angle, it can achieve high luminosity while meeting the detection and physics program requirements.

Beam-beam effects are one of the most dominant effects limiting the luminosity in electron-ion colliders. As discussed in [3], the simulations of crabbing dynamics for the current symplectic tracking codes such as Elegant do not include beam-beam effects.

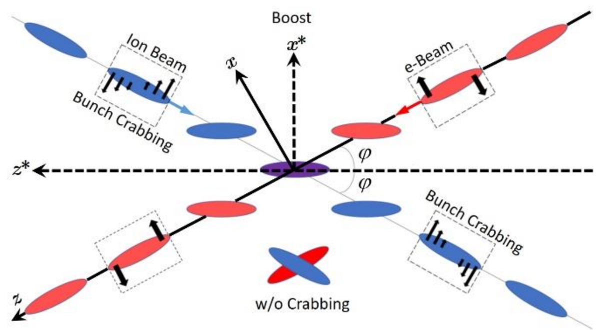

Figure 1: Local crabbing for JLEIC: a schematic of the crab crossing required to restore head-on collisions

CASA BeamBeam is a beam-beam interaction package. Based on Python GUI (Graphical User Interface), CASA BeamBeam is developed by Jefferson Lab CASA. We made use of CASA BeamBeam's implementation which models the crossing angle, bunch tilt and bunch offset at the interaction point. The simulation framework is as follows: The particle distribution is initiated at the start point of the tracking simulation. The beam is tracked through the first crab cavity and is transported to the IP. At the IP, the particle distribution information is written out and fed into a Python script for applying the beam-beam interaction. The kicked distribution is fed back into Elegant for continued tracking through the second crab cavity and the rest of the collider ring optics. The simulation flow is illustrated in Figure 2.

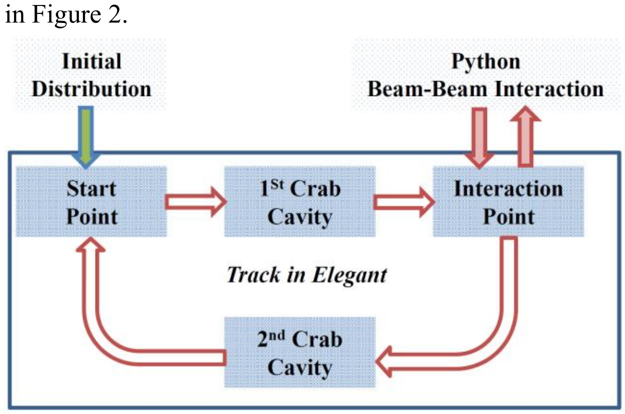

Figure 2: Simulation flow for beam-beam interaction.

The beam-beam interaction model is described in the following sections.

CRAB KICKING MODEL

Our calculation model, "CASA BeamBeam", is based on the Bassetti-Erskine analytic solution of the beam-beam interaction. It is extended to finite-length bunches using a symplectic algorithm proposed by Hirata [5, 6]. In the CASA BeamBeam model, we assume one IP in a ring located at $s = 0$, where s is the azimuthal coordinate. At the IP, based on Lorentz transformation, the coordinates of a particle are boosted so that the collision becomes head on, then the particle interacts with the other beam in this boosted frame. For numerical calculation, each of the colliding finite-length bunches is split into multiple longitudinal slices. Then the beam-beam interaction reduces to consecutive pair-wise collisions of these thin slices. The algorithm calculates the longitudinal position of each collision and properly propagates the slice parameters to that point from the IP. The beam-beam kick is then applied to each particle in the slice using the Bassetti-Erskine formula. In detail, our model includes the following parts:

Laboratory frame to Boost frame. – In Cartesian coordinate, the system $(x, p_x, y, p_y, z, p_z; h, s,)$ and the system $(x^*, p_x^*, y^*, p_y^*, z^*, p_{z;}^* h^*, s^*,)$ corresponds to the laboratory frame and the boost frame, respectively.

Crab Kick in Boost frame. – A particle will be kicked when "Crabbing". According to Bassetti-Erskine, the unitless force felt by a particle is:

$$f_{x^\pm, y^\pm}(x^\pm, y^\pm) = \frac{N^\mp r^\pm}{\gamma_0} F_{x^\pm, y^\pm}\left(x^\pm - \overline{x^\mp}, y^\pm - \overline{y^\mp}, \sigma_{x^\mp}, \sigma_{y^\mp}\right) \tag{1}$$

here "$\pm$" represents the particle $(+)$ and the particle $(-)$, respectively; r is the classical radius of particle, N is the number of particles; and

$$F_y\left(x, y, \sigma_x, \sigma_y\right) + iF_x\left(x, y, \sigma_x, \sigma_y\right) = \sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}}\left[w\left(\frac{x + iy}{\sqrt{2\left(\sigma_x^2 - \sigma_y^2\right)}}\right) - \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) w\left(\frac{\frac{\sigma_y}{\sigma_x} x + i\frac{\sigma_x}{\sigma_y} y}{\sqrt{2\left(\sigma_x^2 - \sigma_y^2\right)}}\right)\right] \tag{2}$$

where, w is named as Faddeeva function or Kramp function.

In Boost frame, the interaction between two thin slices of bunches with the center position of the slice z^+ and z^- takes place at $S^* = (z^{*+} - z^{*-})/2$. It makes the change for all of RMS slice sizes, $\sigma^*_{x^\pm}, \sigma^*_{y^\pm}$ at $s = S^*$. For "weak-weak interaction" or "strong-strong interaction"

$$\sigma^*_{x^\pm}(S^*) = \sqrt{\left(\sigma^{\pm\,2}_{x^*}\right)_{s=0} + 2\left(\sigma^\pm_{x^* p_{x^*}}\right)_{s=0} S^* + \left(\sigma^{\pm\,2}_{p_{x^*}}\right)_{s=0} {S^*}^2} \tag{3}$$

$$\sigma^*_{y^\pm}(S^*) = \sqrt{\left(\sigma^{\pm\,2}_{y^*}\right)_{s=0} + 2\left(\sigma^\pm_{y^* p_{y^*}}\right)_{s=0} S^* + \left(\sigma^{\pm\,2}_{p_{y^*}}\right)_{s=0} {S^*}^2} \tag{4}$$

and for "weak-strong interaction",

$$\sigma^*_{x^\pm}(S^*)\Big|_{s=0} = \sqrt{\left\langle x^{*\pm 2} - \bar{x}^{*2}\right\rangle + 2\left\langle x^{*\pm} p_{x^{*\pm}} - \overline{x^{*\pm} p_{x^{*\pm}}}\right\rangle S^* + \left\langle p_{x^{*\pm}}^{\;2} - \overline{p_{x^{*\pm}}}^{2}\right\rangle {S^*}^2} \tag{5}$$

$$\sigma^*_{y^\pm}(S^*)\Big|_{s=0} = \sqrt{\left\langle y^{*\pm 2} - \bar{y}^{*2}\right\rangle + 2\left\langle y^{*\pm} p_{y^{*\pm}} - \overline{y^{*\pm} p_{y^{*\pm}}}\right\rangle S^* + \left\langle p_{y^{*\pm}}^{\;2} - \overline{p_{y^{*\pm}}}^{2}\right\rangle {S^*}^2} \tag{6}$$

Here, $x^{*\pm} p_{x^{*\pm}}$ and $y^{*\pm} p_{y^{*\pm}}$ are the crab tilt terms.

Boost frame to Laboratory frame. – After kicking in the boost frame, the transformation from the phase space $(x^{*bst}, p_x^{*bst}, y^{*bst}, p_y^{*bst}, z^{*bst}, p_z^{*bst})$ to lab phase space $\left(x^{lab}, p_x^{lab}, y^{lab}, p_y^{lab}, z^{lab}, p_z^{lab}\right)$ will be carried out.

NUMERICAL CALCULATION AND RESULTS

Including the hourglass effect [4], the beam-tilt effects and the beam offset effects, the luminosity [7] and the rms size are calculated by the summation of CASA BeamBeam. In all of CASA BeamBeam numerical calculation processes, the colliding bunched beams are cut into many slices whose normal direction is parallel to the longitudinal direction in the boosted frame.

Table 1: JLEIC Parameters

CM Energy (GeV)	**21.9**	
Collision Freq. (MHz)	476	
Crossing angle (mrad)	50	
Beam	**proton**	**electron**
Beam Energy (GeV)	40	3
Particles per bunch (10^{10})	0.59	3.9
σ_z (cm)	2.5	1.0
ε_{Nx} (mm mrad)	0.5	1.8
ε_{Ny} (mm mrad)	0.2	3.6
β_x^* (cm)	8	30
β_y^* (cm)	1.3	9.8
Cutting slices per bunch	23	21

First, based on the JLEIC parameters (see Table 1), the benchmark has been carried out by comparing BeamBeam3D [8] with CASA BeamBeam. As we can see in Figure 3, CASA BeamBeam results matched BeamBeam3d very well. Comparing with BeamBeam3D, CASA BeamBeam reduced the numerical noise significantly.

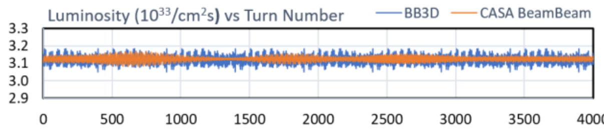

Figure 3: Benchmark between BeamBeam3d and CASA BeamBeam: Luminosity vs. Turn-number including hourglass effect for head-on collision without crossing angle and offset.

We initiated studies of the crab cavity amplitude and phase noise effects. Before each beam-beam interaction, both bunches receive random radial offsets consistent with the phase noise of the crab cavities or random horizontal tilts consistent with the crab cavity amplitude noise. The offsets and tilts are removed after the beam-beam interaction to avoid random walk of the two beams away from each other, which is usually prevented by a feedback system. The rms value of $\sigma_{\Delta x}$ and $\sigma_{\Delta x'}$, in the simulations are based on the Table 1, we have

$$\sigma_{\Delta x} = \left(\frac{dx}{dz}\right)_{IP} \lambda_{RF} \frac{\sigma_{\Delta\varphi}}{2\pi} = 1.6\times10^{-6} \text{ (m)} \quad (7)$$

$$\sigma_{\Delta x'} = \left(\frac{dx}{dz}\right)_{IP} \frac{\sigma_{\Delta v}}{v} = 2.5\times10^{-6} \quad (8)$$

where, λ_{RF} is the collision wavelength which is inversely proportional to the collision frequency.

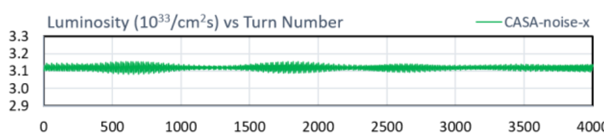

Figure 4: CASA BeamBeam (σ_x noise introduced): Luminosity vs. Turn-number including hourglass effect for head-on collision without crossing angle and offset.

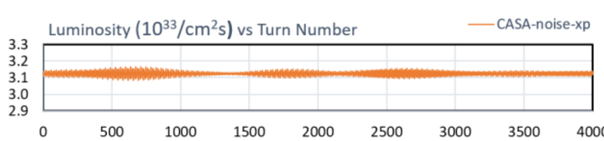

Figure 5: CASA BeamBeam (σ_{xp} noise introduced): Luminosity vs. Turn-number including hourglass effect for head-on collision without crossing angle and offset.

In Figure 4 and Figure 5, for CASA BeamBeam, we can see that both the $\sigma_{\Delta x}$ and $\sigma_{\Delta x'}$ will not affect the luminosity calculation significantly due to the statistical Gaussian distribution by using the Bassetti-Erskine formula.

For JLEIC case described in Table 1, Figure 6 demonstrates that, for proton-electron beam collision, the normalized emittances' variation with the electron bunch will dominate the luminosity fluctuations in the collision process.

For collision frequency = 476 MHz, based on the table 1, CASA BeamBeam numerical result shows that the luminosity for JLEIC is about $3.1 \times 10^{33}/cm^2s$ and the hourglass reduction is 84.78%, which is consistent with the analytic solutions.

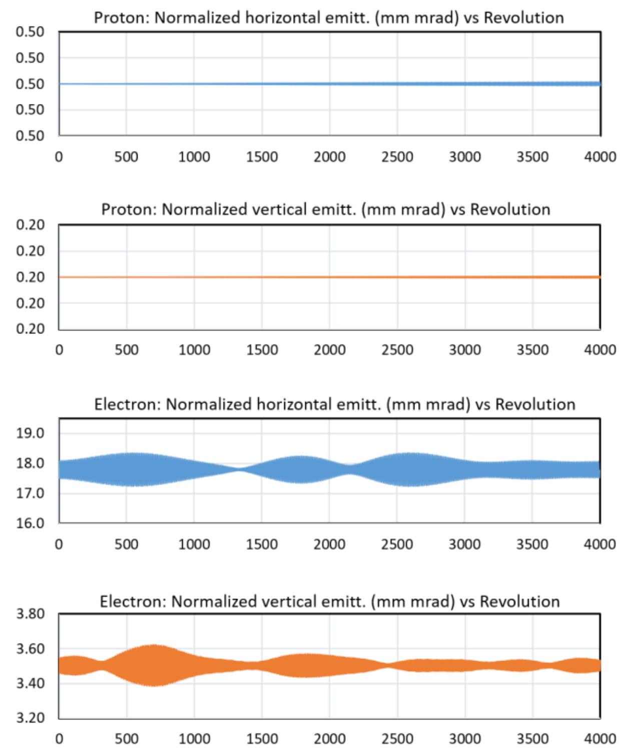

Figure 6: Normalized emittance vs Turn-number.

CONCLUSIONS

We benchmarked the numerical results by using our "CASA BeamBeam" package and the particle in cell methods (PIC) based package "Beambeam3D". CASA BeamBeam results are highly consistent with BeamBeam3D. CASA BeamBeam has been implemented for the design of the JLEIC collider rings. This code has combined Elegant's accurate simulation of the beam dynamics in the collider lattice with a somewhat simplified but sufficiently accurate beam-beam interaction model that captures the main physical features of the process. The beam parameters used in CASA BeamBeam code can be extracted from the tracking data, such as SDDS format data, Twiss format data and etc. The Python-based scripts of CASA BeamBeam can deal with kicks via Bassetti-Erskine for an individual particle and hence the results of the kicked beam distribution will be used in Elegant simulation.

ACKNOWLEDGMENTS

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