EQUILIBRIA AND SYNCHROTRON STABILITY IN TWO ENERGY STORAGE RINGS*

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Abstract

In a dual energy storage ring, the electron beam passes through two loops at markedly different energies E_L, and E_H, i.e., energies for low energy loop and high energy loop respectively. These loops use a common beamline where a superconducting linac at first accelerates the beam from E_L to E_H and then decelerates the beam from E_H to E_L in the next pass. There are two basic solutions to the equilibrium problems possible, i.e., "Storage Ring" (SR) equilibrium and "Energy Recovery Linac" (ERL) equilibrium. SR equilibrium mode more resembles the usual single loop storage ring with strong synchrotron motion and ERL equilibrium mode is the case where RF in two beam passes nearly cancels. Calculations based on linear transfer matrix formalism show that longitudinal stability exists for both SR mode and ERL mode in two energy storage rings.

INTRODUCTION

To enhance the luminosity over a broad center of mass energy range in a collider, one may need to cool the ion beams with a proper method [1]. Cooling allows small transverse beam sizes at the interaction point and enhanced luminosity. For a heavy charged particle, there is no natural radiation damping. However, electron beams do have natural synchrotron radiation damping and this property of electrons can be used to cool the ion beams [2]. Hence, an efficient way of ion damping based on electron cooling is possible. One of the future possible electron coolers may be dual energy storage ring cooler.

DUAL-ENERGY STORAGE RING COOLER

Concept

The proposed storage-ring electron cooler consists of cooling and damping rings connected by an energy recovering superconducting Radio-Frequency (RF) structure. The schematic diagram for such a cooler system is given in Figure1.

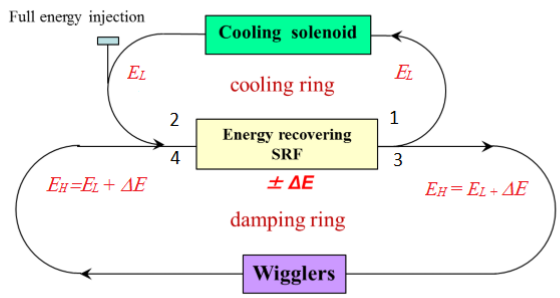

Figure 1: Schematic drawing of a dual-energy storage ring cooler [3].

This is new concept and expands the range of applicability of storage-ring based electron cooling of ion beams. The electron energy, bunch length, and energy spread in the cooling section are determined for optimum cooling of a stored beam. Cooling and damping rings are at significantly different energies which is connected and conducted by superconducting RF structure. Hence in a damping ring, where the electron beam has been boosted to a relatively higher energy and passes through the wiggler to enhance the damping. Going to cooler ring, an electron beam decelerates to the lower energy and finally passes through the cooling solenoid.

In cooling solenoid, an electron beam is brought into thermal contact with an ion beam. Heat in the transverse degree of freedom is transferred from the ion beam to the electron beam leading to beam cooling. After each pass, the electron beam comes to the damping ring. At the damping ring, electron beam passes through the wigglers where the electron beam damps due to synchrotron radiation. This results the electron beam temperature to decrease after passing through the wigglers. This cooled electron beam then can be reusing to cool the ion beam in cooling solenoid. The process finally leads to the cooled ion beam with smaller emittance. This greatly enhances the luminosity in the collider experiments.

The first step in analyzing such a system is the stability. We study the longitudinal stability in two energy storage rings and derive the conditions necessary for the stability in terms of accelerating rf phase angle $\phi_{s,a}$.

Stability Criteria in Two- Energy Storage Rings

We apply the linear matrix transfer approach to study the stability in two energy storage rings.

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The schematic diagram to understand the stability in two-energy storage ring is presented in Fig. 1. Electron beam passing through energy recovering SRF from point 4 to 1 decelerates to the energy 55 MeV while going from point 2 to 3 accelerates to the energy 155 MeV. The whole picture can be represented by matrices:

$$M_{1\to 2} = \begin{pmatrix} 1 & M_{56}^{ler} \\ 0 & 1 \end{pmatrix}$$

$$M_{2\to 3} = \begin{pmatrix} 1 & 0 \\ M_{65}^{acc} & M_{66}^{acc} \end{pmatrix}$$

$$M_{3\to 4} = \begin{pmatrix} 1 & M_{56}^{her} \\ 0 & 1 \end{pmatrix}$$

$$M_{4\to 1} = \begin{pmatrix} 1 & 0 \\ M_{65}^{dec} & M_{66}^{dec} \end{pmatrix}$$

Then the total transfer matrix takes the form:

$$M = M_{4\to 1} M_{3\to 4} M_{2\to 3} M_{1\to 2} (1)$$

Here we have assumed that $|M_{65}^{dec}| = M_{65}^{acc} . \frac{P_2}{P_1}$, and $M_{66}^{acc} = \frac{P_1}{P_2}$, $M_{66}^{dec} = \frac{P_2}{P_1}$.
$M_{65}^{acc} = -\frac{\Delta P}{P_2} \frac{\cos\phi_{s,a} 2\pi}{\sin\phi_{s,a} \lambda}$ and $P_2 = 155$ MeV/c and $P_1 = 55$ MeV/c, c is the velocity of light. In our case, we assume that $M_{56}^{ler} = M_{56}^{her}$ = -1.0 m.
The stability criteria is given by:

$$|2\cos\mu| = |M_{11} + M_{22}| < 2 \quad (2)$$

SR Mode Equilibrium

In SR mode equilibrium, we assume that $M_{65}^{dec} = M_{65}^{acc} . \frac{P_2}{P_1}$. From (1), we calculate the total transfer matrix for the system for one turn and check the stability criteria defined by (2). This results the inequality $-\frac{4}{B} < X(A + X) < 0$, where X=$M_{56} M_{65}^{acc}$, B = $\frac{P_2}{P_1}$, A = $\frac{2(1+B)}{B}$.

(i) $X(A + X) < 0, -A < X < 0.$
since $M_{56} < 0, 0 < M_{65}^{acc} < \frac{A}{|M_{56}|}$.
This gives the limitation on the accelerating phase angle:

$$\tan\phi_{s,a} < -\Delta P \frac{|M_{56}| 2\pi}{P_2\, \lambda_{rf}\, A} \qquad (3)$$

(ii) $X^2 + AX > -\frac{4}{B}$,

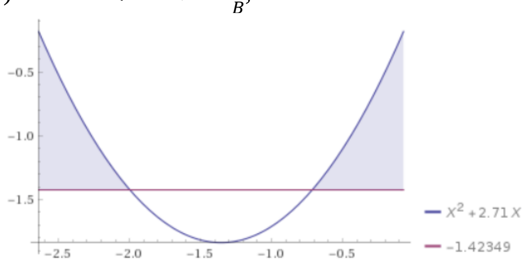

Figure 2: Plot of $X^2 + AX > -\frac{4}{B}, A = 2.71, B = 2.81$

The RHS simply represents the equation of parabola. Solving the above inequality, we get two roots of: $X^2 + AX + \frac{4}{B} = 0$

$$X_{1,2} = \frac{-A \pm \sqrt{A^2 - \frac{16}{B}}}{2}$$

Fig. 2 shows two solutions for the given inequality. The above two conditions allow us to choose the right phase to accelerate or decelerate the beam in storage ring mode. Hence the final constraints on phase angle $\phi_{s,a}$ are:

$$\pi - \tan^{-1}\left(\Delta P \frac{2|M_{56}| 2\pi}{P_2 \lambda_{rf} \left(A + \sqrt{A^2 - \frac{16}{B}} \right)} \right) < \phi_{s,a}$$

$$< \pi \tan^{-1}\left(\frac{\Delta P |M_{56}| 2\pi}{P_2 A \lambda_{rf}} \right) (4)$$

And

$$\frac{\pi}{2} < \phi_{s,a} < \pi - \tan^{-1}(\Delta P \frac{2|M_{56}| 2\pi}{P_2 \lambda_{rf} \left(A - \sqrt{A^2 - \frac{16}{B}} \right)}) \ (5)$$

The corresponding decelerating phase angle (going from 4 to 1 as in Fig. 1) is given by $\phi_{s,d} = (\pi - \phi_{s,a}) + \pi$.
Now, using equations (4) and (5), we make a plot of phase angle $\phi_{s,a}$ versus wavelength λ which is shown below.

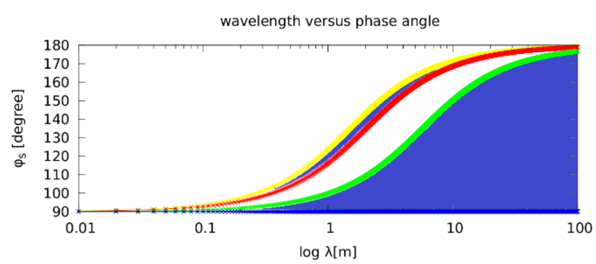

Figure 3: $\phi_{s,a}$ versus λ plot showing the stable region (blue) and unstable region (white) in two – energy storage rings for SR mode.

Plot shows that there exists unstable region (white shaded) which is “Phase – gap” for SR mode.

ERL Mode Equilibrium

We follow the same procedure to study the stability criteria for ERL mode. In this mode, $M_{65}^{dec} = -M_{65}^{acc} . \frac{P_2}{P_1}$.
Then solving the stability criteria given by (2), we get the following condition: $BX^2 < 4$.

$$X^2 < \frac{4}{B}, i.e. X = \pm \frac{2}{\sqrt{B}}$$

Hence from the above relation, $-\frac{2}{\sqrt{B}} < X < \frac{2}{\sqrt{B}}$, we get the stability criteria:

$$|M_{65}^{acc}| < \frac{2}{\sqrt{B}\, |M_{56}|}$$

This implies that M_{65}^{acc} may be both positive or negative.
$M_{65}^{acc} < 0$, defocusing and $M_{65}^{acc} > 0$, focusing.

(i) $M_{65}^{acc} > 0$, focusing

This condition results:

$$\tan^{-1}\phi_{s,a} < -\Delta P\ \pi\ \sqrt{B}\,\frac{|M_{56}|}{P_2\lambda_{rf}}$$

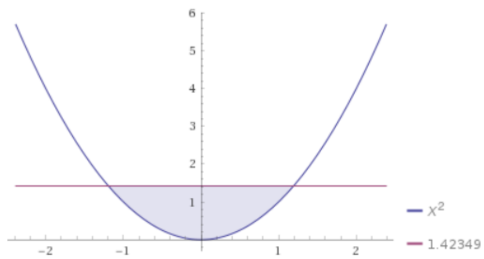

Figure 4: Plot of $X^2 < \frac{4}{B}, B = 2.81$

Similarly, for defocusing, i.e., $M_{65}^{acc} < 0$,

$$\tan^{-1}\phi_s > \Delta P\ \pi\ s\sqrt{B}\,\frac{|M_{56}|}{P_2\lambda_{rf}}$$

combining above defined two criteria, we get the stability condition:

$$\tan^{-1}\left(\Delta P\ \pi\,\frac{\sqrt{B}\,|M_{56}|}{P_2\lambda_{rf}}\right) < \ \phi_{s,a} < \tan^{-1}\left(-\Delta P\ \pi\sqrt{B}\,\frac{|M_{56}|}{P_2\lambda_{rf}}\right) \quad (6)$$

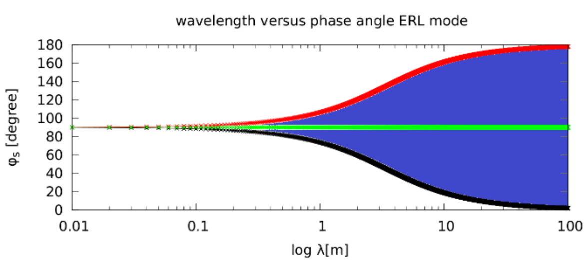

Figure 5: $\phi_{s,a}$ versus λ plot showing the stable (blue) and unstable (green line) regions in two energy storage rings for ERL mode.

Plot shows that there exist two stable regions (blue shaded). At $\phi_{s,a} = \frac{\pi}{2}$, ERL mode is unstable (green line).

Periodic Solution

Periodic solution exists for both SR mode and ERL mode. We consider an accelerating phase angles defined by above stability criteria and choose a specific wavelength $\lambda_{rf} = 0.62$ m (476 MHz) in our elegant simulations. First, we create a longitudinal beam distribution using corresponding twiss parameters and emittance from our calculations. Then we performed elegant simulations for different number of passes. All passes result the same phase space distribution (Figure 6) which ensures that the periodic solution exists.

A plot of $\frac{\Delta P}{P}$ versus Δz is shown for the SR mode. We repeat the elegant simulation and for each pass we get the same beam phase space distribution with $\frac{\Delta P}{P} = 1.5 \times 10^{-3}$ and bunch length $\Delta z \approx 2.0$ mm.

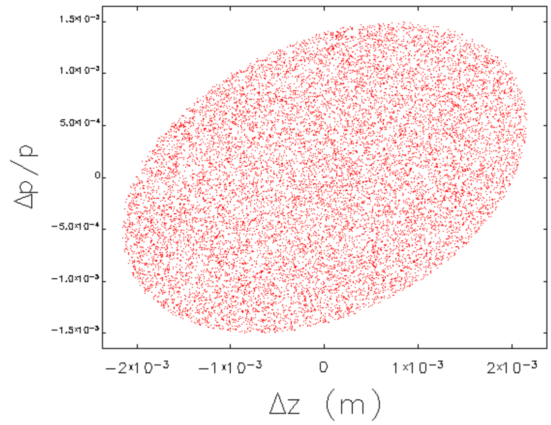

Figure 6: $\Delta P\ /P$ versus Δz plot showing the periodic solution exists. This is for SR mode $\Phi_{s,a} = 91^0, \lambda_{rf} = 0.62$ m (Fig. 1, point 1).

CONCLUSION

This paper describes the concept of longitudinal stability in a dual energy storage ring. Analytical calculations and the corresponding elegant simulations show that stable solutions exist for both SR mode and ERL mode. The limitations on accelerating phase angles in both cases are calculated and verified using elegant simulations. All our calculations are based on $\lambda_{rf} = 0.62$ m ≈ 476 MHz. Elegant simulations show that periodic solution exists. It means under certain accelerating phase angles limitation both SR mode and ERL mode are stable. Calculations show that ERL mode has larger stable regions.

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