Study of wiggler CSR effect on electron ring of MEIC

Cheng-Ying Tsai

March 23, 2015

Motivation

- In high-luminosity collider machines, to lower down the lepton beam emittance, people usually use damping wigglers to achieve the goal by extracting a large fraction of the synchrotron radiation and thus increasing the radiation damping rate (or, shortening the damping time).
- Fanglei's previous presentation (Feb. 5, MEIC R&D Meeting) about potential damping wiggler design for MEIC reminds me about J. Wu *et al*. work:
  - J. Wu, T. O. Raubenheimer, and G. V. Stupakov, Calculation of the coherent synchrotron radiation impedance from a wiggler, PRST-AB 6, 040701 (2003)
  - J. Wu, G. V. Stupakov, T. O. Raubenheimer, and Z. Haung, Impact of wiggler coherent synchrotron radiation impedance on the beam instability and damping ring optimization, PRST-AB 6, 104404 (2003)
- The following slides demonstrate some results reproduced from the above two papers, and apply the developed code to MEIC e-Ring case.

Outline

- Basic theory and assumption
- CSR impedance from a wiggler
- Underlying physics
- Application: MEIC e-Ring with damping wiggler
- Summary

Theoretical background

- step 1: Vlasov equation, with longitudinal equations of motion,

$$\frac{\partial \rho}{\partial s} - \eta\delta\frac{\partial \rho}{\partial z} - \frac{r_0}{\gamma}\frac{\partial \rho}{\partial \delta}\int_{-\infty}^{\infty} dz'\, d\delta'\, W(z - z')\rho(\delta', z', s) = 0$$

- step 2: adding a (harmonic) perturbation

$$\rho = \rho_0(\delta) + \rho_1(\delta, z, s) \qquad \rho_1 = \hat{\rho}_1 e^{-i\omega s/c + ikz}$$

- step 3: linearization of Vlasov equation

$$(\omega + ck\eta\delta)\hat{\rho}_1 = i\frac{r_0 c}{\gamma}\frac{\partial \rho_0}{\partial \delta} Z(k)\int d\delta\, \hat{\rho}_1(\delta)$$

- step 4: dispersion equation

$$1 = \frac{i r_0 c Z(k)}{\gamma}\int \frac{d\delta\,(d\rho_0/d\delta)}{\omega + ck\eta\delta}$$

$$\boxed{1 = -\frac{iZ(k)\Lambda}{\sqrt{2\pi}k}\int_{-\infty}^{\infty} dp\, \frac{p e^{-p^2/2}}{\Omega \pm p}}$$

$$\boxed{Z(k) = Z_D(k)\frac{\Theta R}{C} + Z_W(k)\frac{L_W}{C}}$$

$$Z_D(k) = -iA\frac{k^{1/3}}{R^{2/3}},$$

(low-frequency approximation)

$$Z(k) = -i2k_w\frac{k}{k_0}\left[\gamma_{\mathrm{E}} + \log\left(\frac{4k}{k_0}\right) + i\frac{\pi}{2}\right]$$

(high-frequency approximation)

$$Z(k) = -i\frac{6\Gamma[\frac{11}{6}]}{5\sqrt{\pi}\Gamma[\frac{4}{3}]}A\left(\frac{Kk_w}{\gamma}\right)^{2/3}k^{1/3}$$

CSR impedance from a dipole

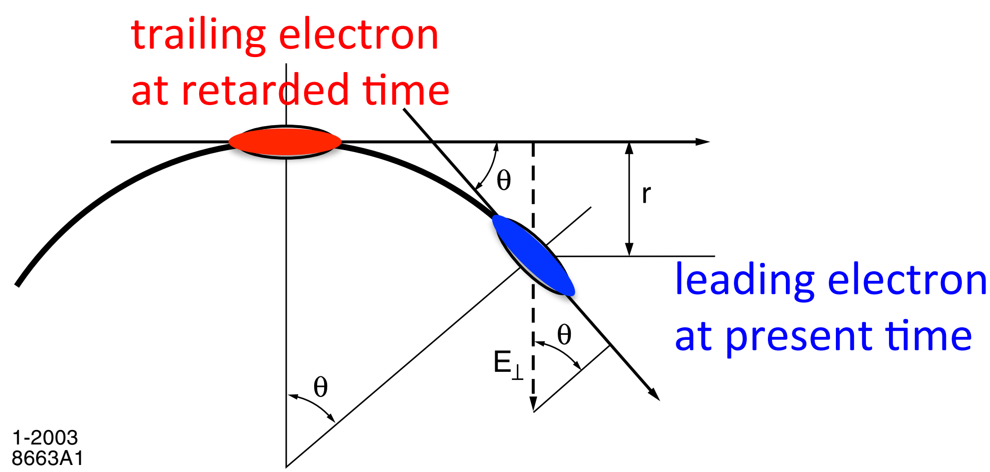
- Assume steady-state and ultrarelativistic beam, the CSR impedance in a dipole can be expressed as

$$Z_D(k) = -iA\frac{k^{1/3}}{R^{2/3}} \quad \text{with} \quad A = 3^{-1/3}\Gamma(\tfrac{2}{3})(\sqrt{3}i - 1).$$

- Physical picture:

$$\frac{dE}{cdt} = W_{\parallel}(s,z) = eE_{\perp}(s,z)\sin\theta$$

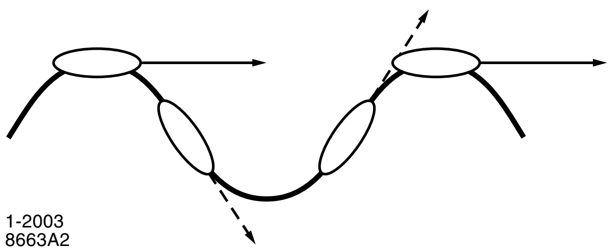
$$E_{\perp}(s,z) \approx \frac{2Ne\lambda(s,z)}{r(s,z)}$$



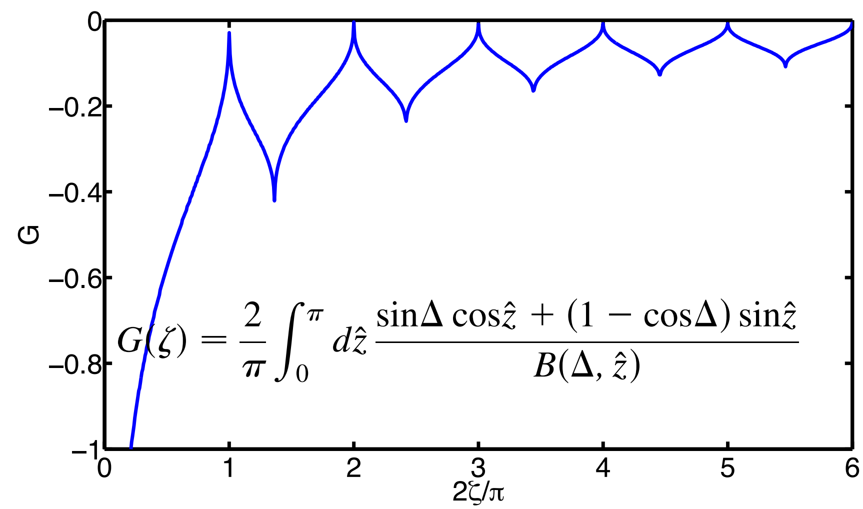
CSR impedance from a wiggler

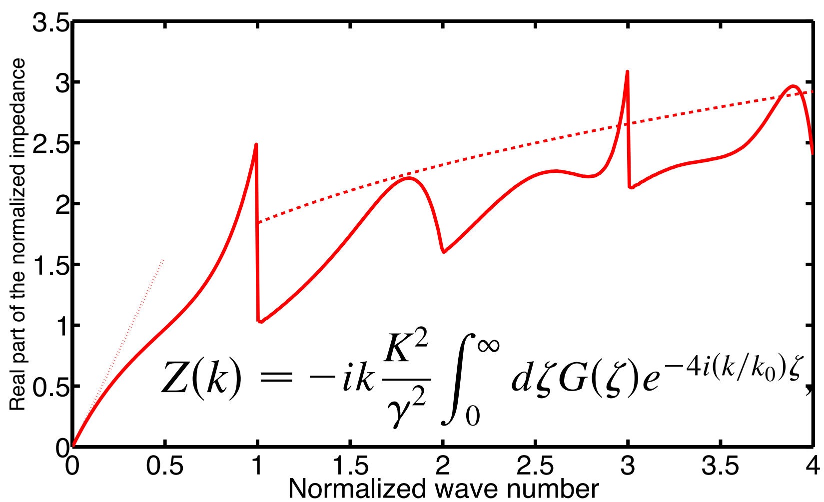
- <u>Physical picture</u>:

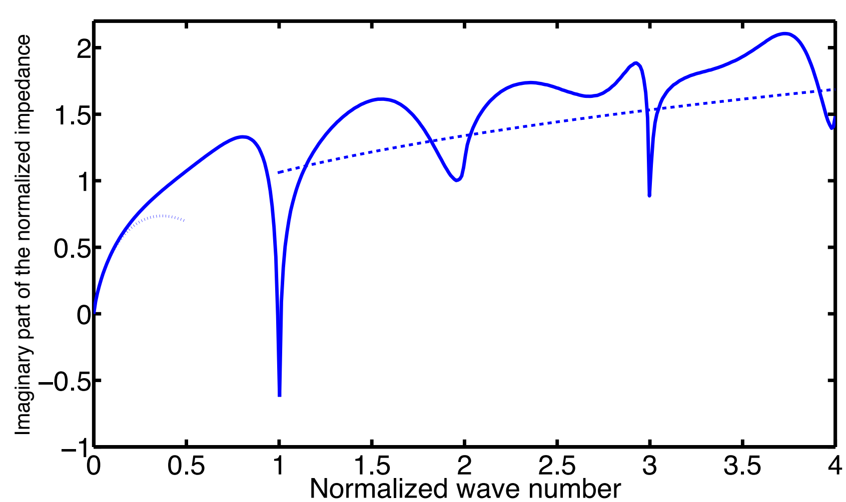
  separated by m λ_w, $W_{||}$ always 0



  separated by [(m+1)/2] λ_w, $W_{||}$ max.







J. Wu, T. O. Raubenheimer, and G. V. Stupakov, Calculation of the coherent synchrotron radiation impedance from a wiggler, PRST-AB 6, 040701 (2003)

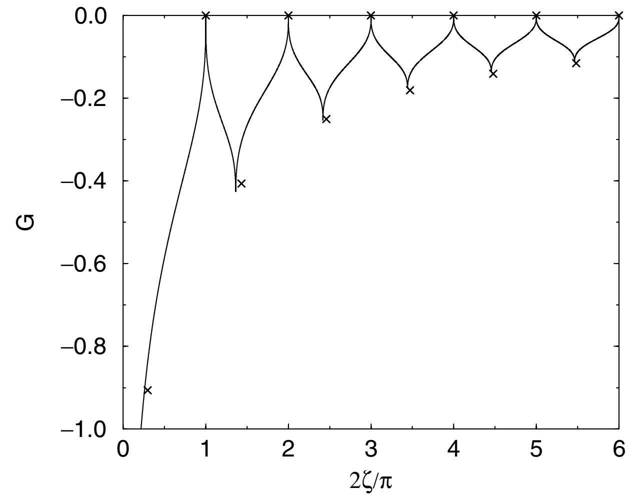


FIG. 1. The solid curve represents the $G(\zeta)$ defined in Eq. (11) as a function of the normalized coordinate $2\zeta/\pi$. The ($\times$) signs are the approximation given in Eq. (12).

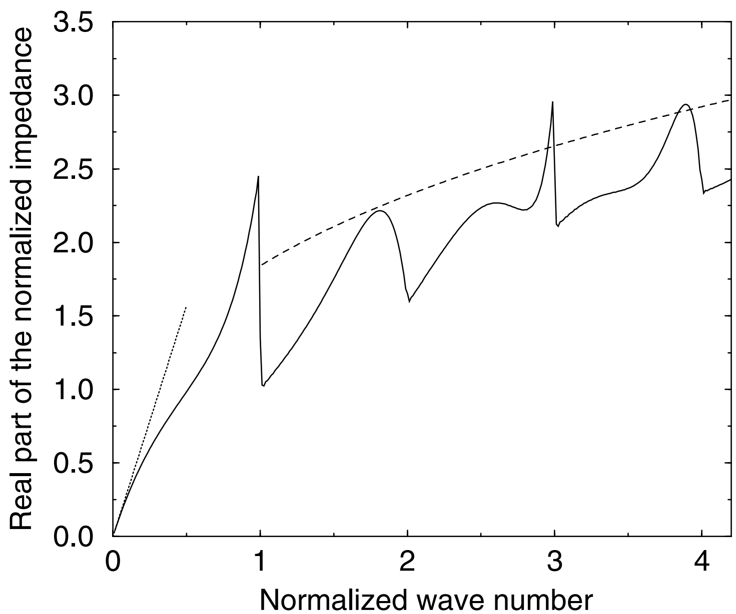


FIG. 5. The real part of the normalized impedance $Z(k)/k_w$ as a function of the normalized wave number k/k_0. Solid line: numerical solution from Eq. (23); dotted line: analytical low-frequency asymptotic behavior from Eq. (26); and dashed line: analytical high-frequency asymptotic behavior from Eq. (27).

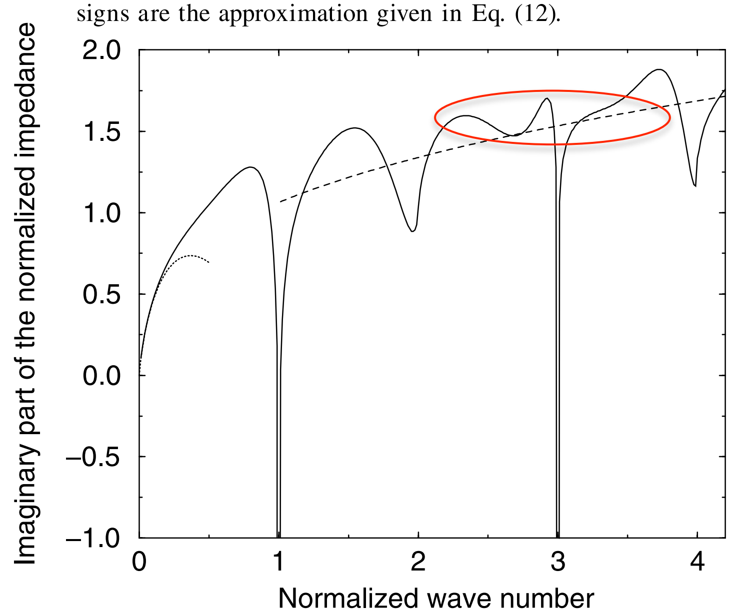


FIG. 4. The imaginary part of the normalized impedance $Z(k)/k_w$ as a function of the normalized wave number k/k_0. Solid line: numerical solution from Eq. (23); dotted line: analytical low-frequency asymptotic behavior from Eq. (26); and dashed line: analytical high-frequency asymptotic behavior from Eq. (27).

Simplest case: cold beam

- When Landau damping is negligible, i.e. cold beam case, the dispersion relation

$$1 = \frac{ir_0 cZ(k)}{\gamma} \int \frac{d\delta\,(d\rho_0/d\delta)}{\omega\ +\ ck\eta\delta} \quad \text{with} \quad \rho_0(\delta = \Delta p/p) = \delta(\delta_0)$$

  can be greatly simplified and the growth rate can be estimated to be

$$\omega = \sqrt{ic^2 n_0 r_0 \eta \boxed{kZ(k)}/\gamma},$$

  and the instability growth rate $\tau^{-1} \propto \mathrm{Im}(\omega) \propto \sqrt{kZ(k)}$.
- Usually, $Z(k) \propto k^{\varepsilon}$, with $\varepsilon > 0$.. For example, for dipole CSR, $\varepsilon = 1/3$
- Thus, $k \nearrow$ $(\lambda \swarrow)$, $\tau^{-1} \nearrow$
- However, there are damping mechanisms, e.g. Landau damping due to finite energy spread and beam emittance, and synchrotron radiation damping (of course, quantum excitation would be involved.).

NLC main damping ring

Table 1: MDR Damping Wiggler Parameters

Beam energy	1.98 GeV
Wiggler peak field	2.15 T
Wiggler period	0.27 m
Total wiggler length	46.25 m
Energy loss/turn from dipoles	247 keV
Energy loss/pass from wiggler	530 keV
Damping times $\tau_{x,y,\varepsilon}$	4.85, 5.09, 2.61 ms

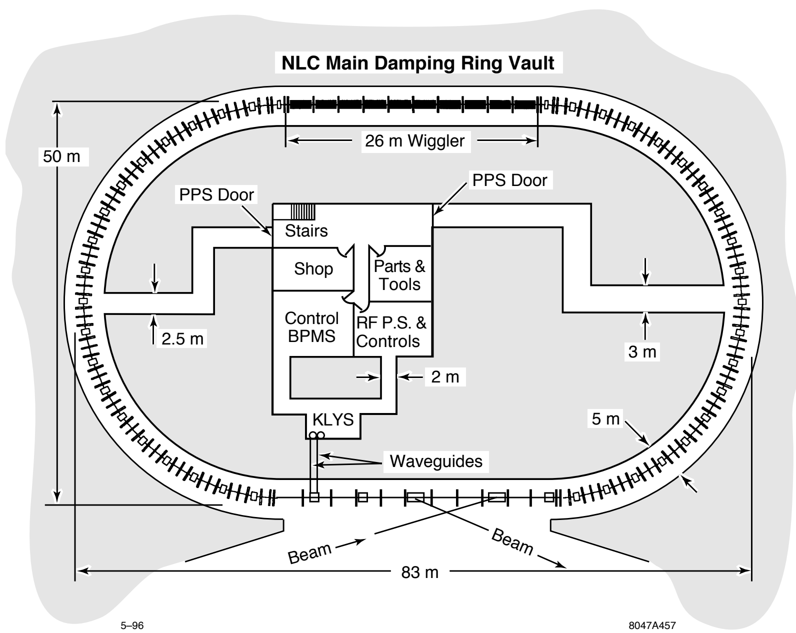
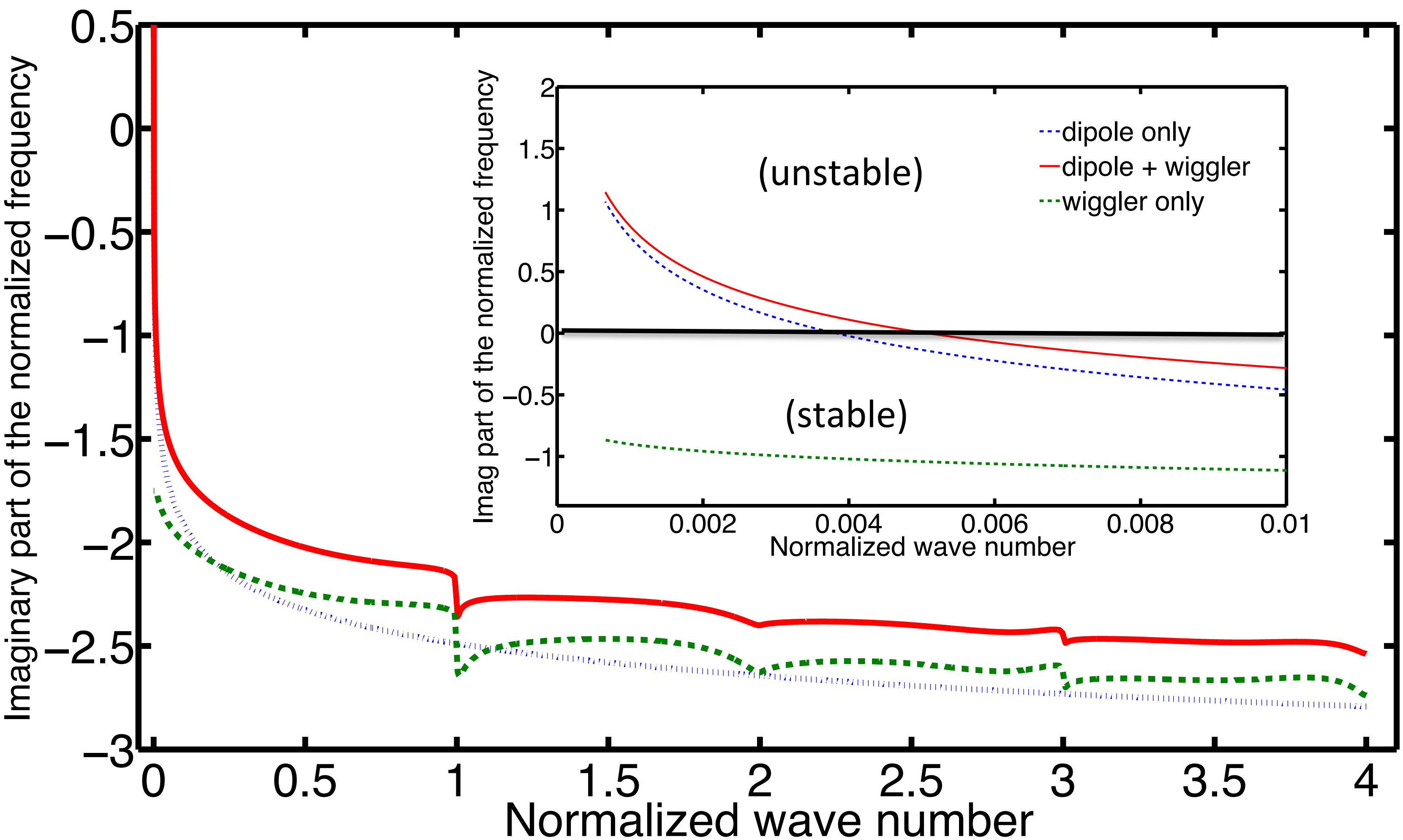


Figure 4-7. *Layout of main damping ring.*

Table 2: Principal Parameters of the Main Damping Rings

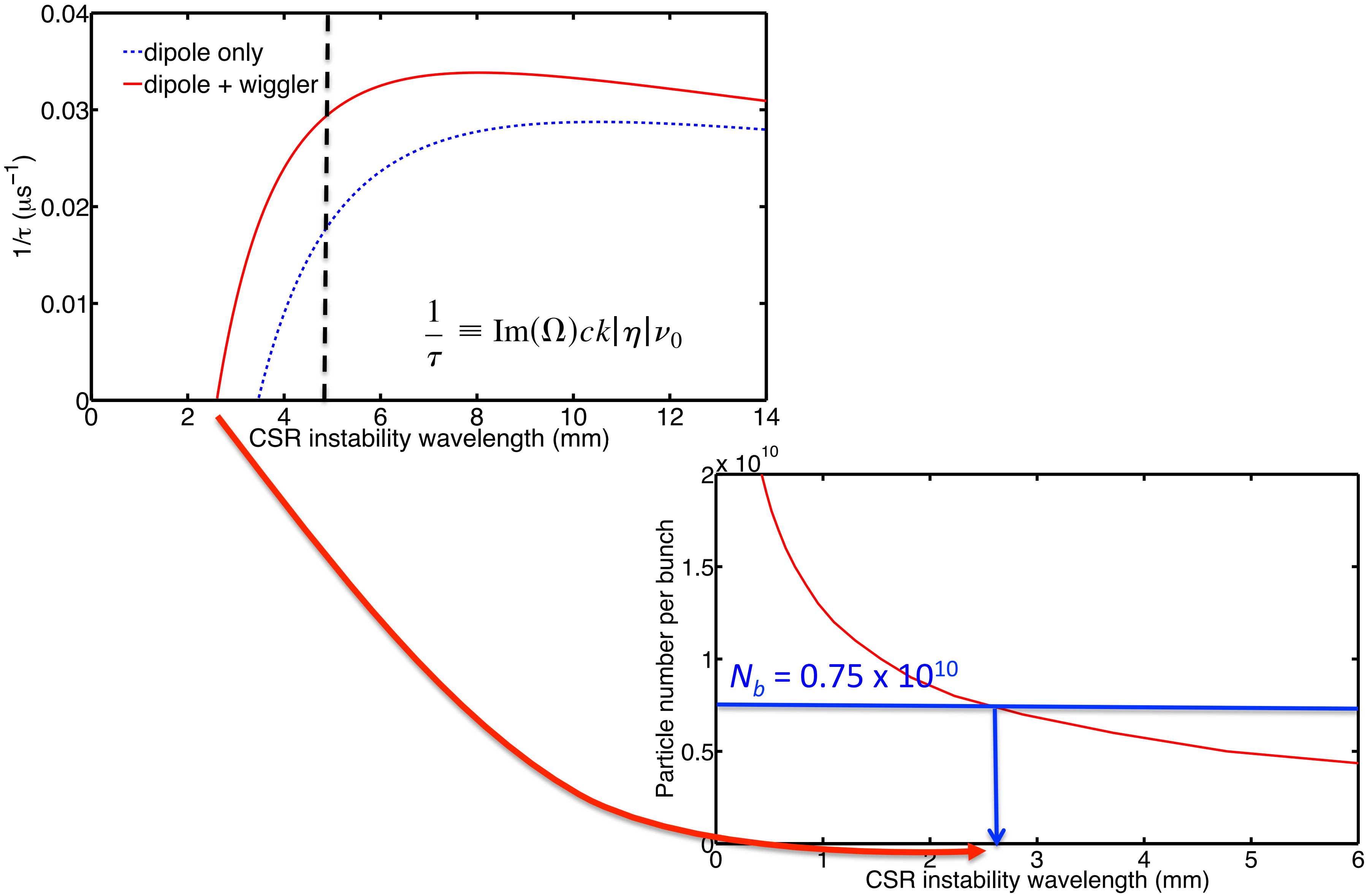
Energy	E	1.98 GeV
Circumference	C	299.792 m
Number of stored trains		3
Natural emittance	$\gamma\varepsilon_0$	2.17 mm-mrad
Tunes	ν_x, ν_y	27.2616, 11.1357
Natural chromaticity	ξ_x, ξ_y	-37.12, -28.24
Momentum compaction	α	2.95×10^{-4}
RF voltage	V_{RF}	1.07 MV
RF acceptance	ε_{RF}	1.5 %
Energy spread (rms)	σ_δ	0.091 %
Bunch length (rms)	σ_z	3.60 mm
Integrated wiggler field	$\int B_w^2 ds$	106.9 T^2m
Energy loss/turn	U_0+U_w	247+530 keV
Damping times	$\tau_{x,y,\varepsilon}$	4.85, 5.09, 2.61 ms

NLC main damping ring



To obtain the same results as in the paper, somehow we need to slightly adjust the rms energy spread from $\underline{9.09 \times 10^{-4}}$ to $\underline{8.6 \times 10^{-4}}$.

NLC main damping ring

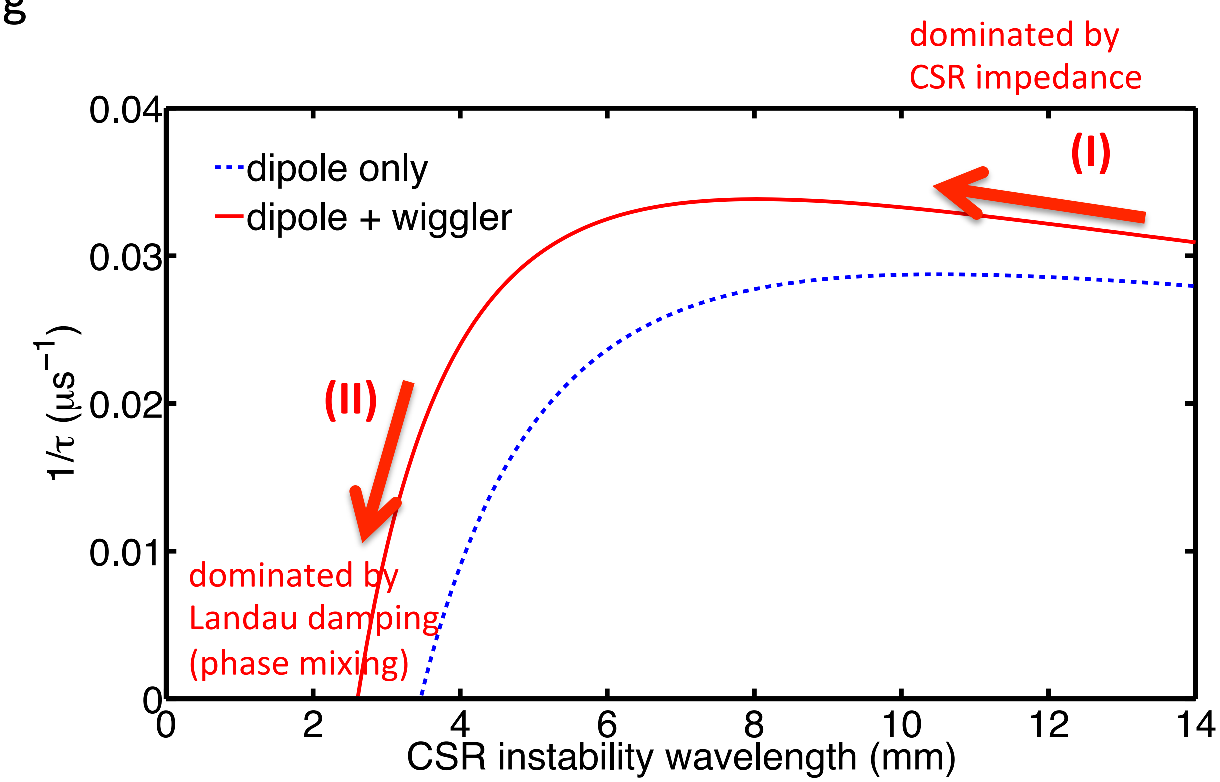


Underlying physics

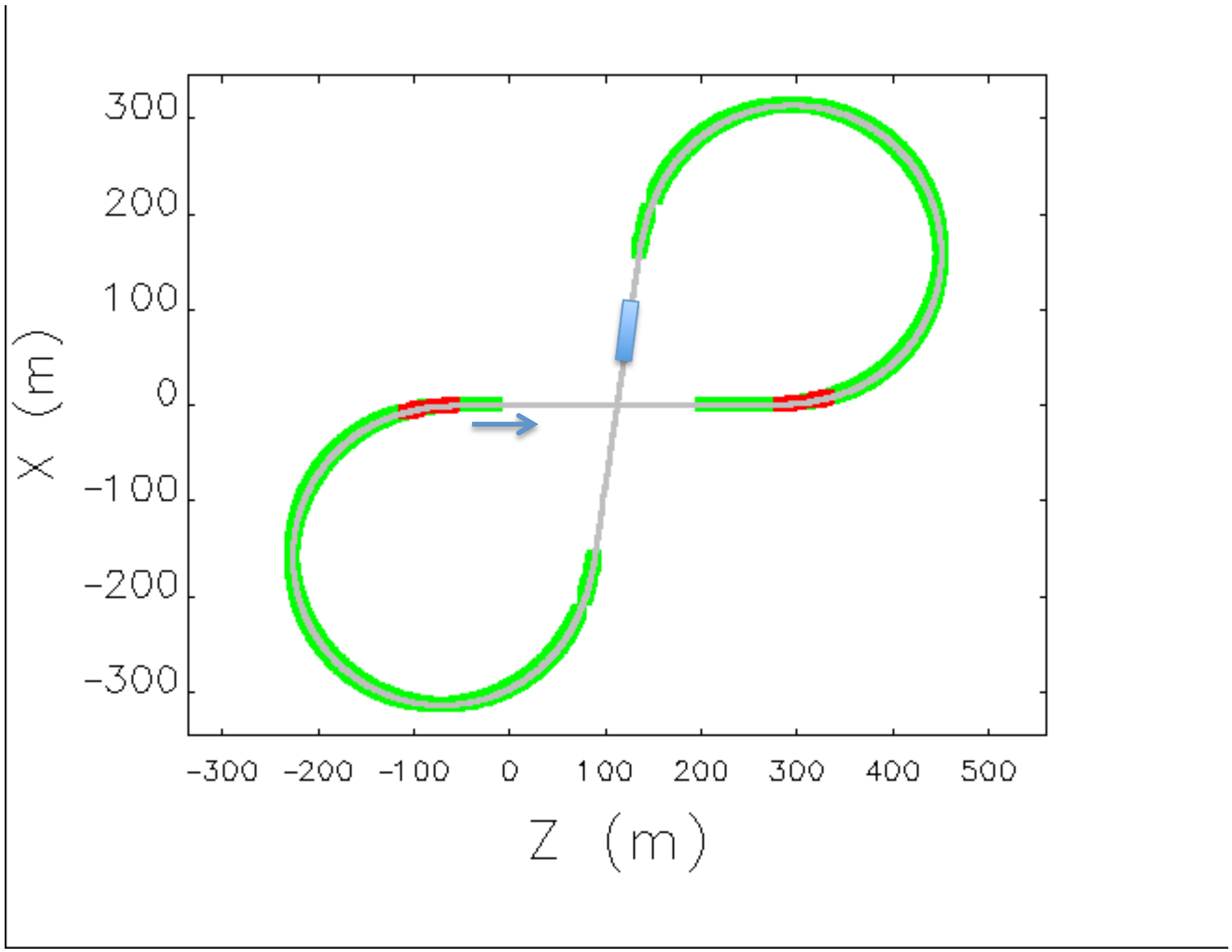
- Region (I): long wavelength, negligible Landau damping

$$\tau^{-1} \propto \mathrm{Im}(\omega) \propto \sqrt{kZ(k)}.$$

- Region (II): shorter wavelength, more Landau damping, more phase mixing



Application: MEIC e-Ring



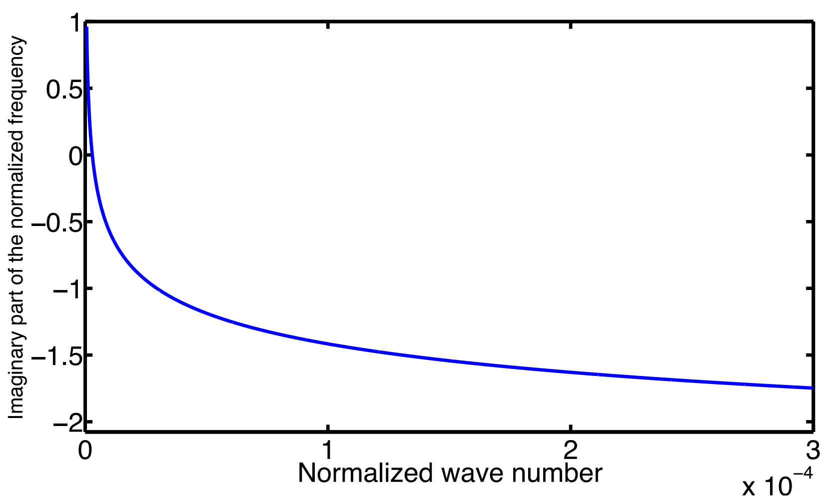
Note: beam and lattice parameters are provided by Fanglei.

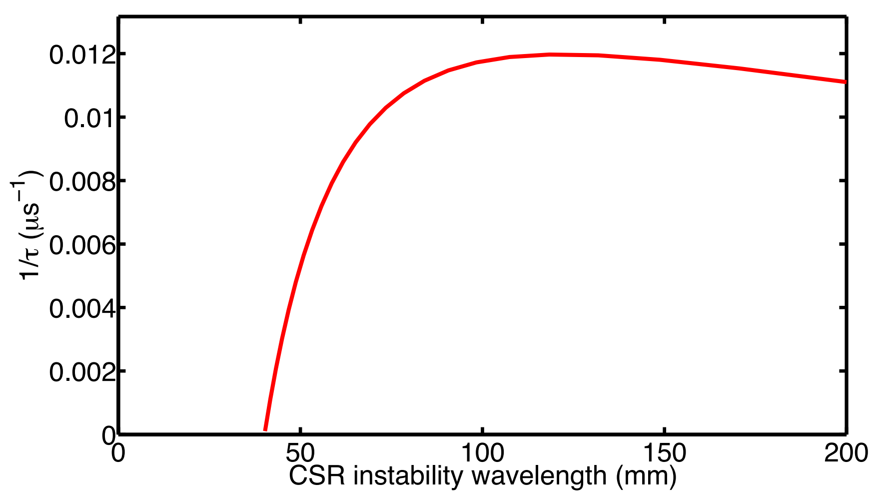
Application: MEIC e-Ring

Name	Value	Unit
Circumference	2.154	km
Dipole radius	64 (average)	m
Total bending angle	540	deg
Momentum compaction factor, α_c	0.00215	
Wiggler peak field	1.6	Tesla
Wiggler period	0.2	m
Wiggler total length	24	m
Beam energy	10	GeV
Particles in a bunch	5×10^{10} (vary)	
RMS fractional energy spread	1.14×10^{-3}	

Application: MEIC e-Ring

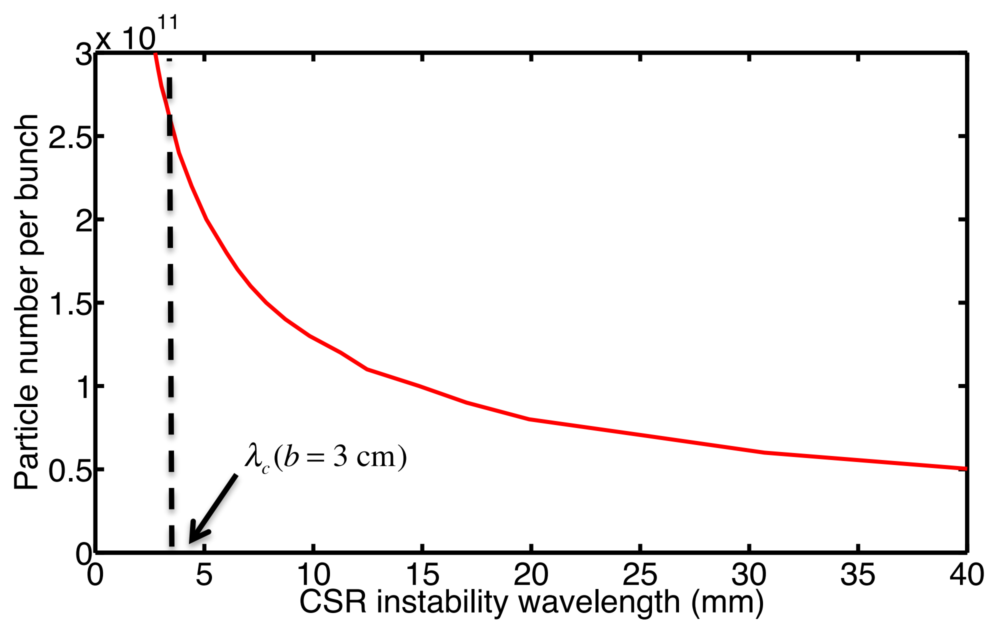
- Numerical simulation shows that, based on the given parameters, there is no such instability found in the e-Ring system.
- Comparing MEIC e-Ring with NLC damping ring, we found:
  - dipole radius is much larger, causing smaller CSR effect
  - rms fractional energy spread is a bit larger, resulting in more Landau damping
  - smaller fraction of wiggler total length to the whole ring (or, much larger ring circumference)
- What if the energy spread becomes half of the given number? (N_b = 5 × 10^{10}, σ_δ = 0.57 × 10^{-3})





Application: MEIC e-Ring

- Would the instability be suppressed by shielding of vacuum chamber?
  $\lambda < \lambda_c \sim 4\sqrt{2}b\sqrt{\frac{b}{R}}$ [R. Warnock and P. Morton, Part. Accel. 25, 113 (1990)]
- Given the dipole radius R = 64 m and $N_b = 5 \times 10^{10}$, to effectively suppress the instability requires the pipe radius $b \leq 14$ cm, which can be easily achieved.
- Assume pipe radius b = 3 cm (which is a usual case?), what about the intensity threshold, given $\sigma_\delta = 0.57 \times 10^{-3}$?



Application: MEIC e-Ring

Name	**Value**	**Unit**
Cutoff wavelength (assume b = 3 cm)	3.67	mm
Intensity threshold at cutoff (wiggler on)	25×10^{10}	
Growth time at cutoff (wiggler on) assume $N_b = 27 \times 10^{10}$	22.7	µsec
Synchrotron oscillation frequency	1~10 (assume)	kHz
Longitudinal radiation damping time	4.12	msec

Summary

- By solving the dispersion relation derived from (linearized) Vlasov equation, we can estimate the instability growth rate, given a set of beam parameters.
- Given further the information of vacuum chamber, i.e. pipe radius b, we can estimate the threshold intensity $N_{\mathrm{b,th}}$.
- Since the calculation is fast, it can be used to optimize the e-Ring design with insertion of damping wiggler.

Possible improvement of the presented work

- e-Ring optimization with damping wiggler [see Section IV of J. Wu et al., PRST-AB 6, 104404 (2003)]
- 1-D linearized Vlasov formulation ➔ 2-D
  - to account for Landau damping effect from finite transverse emittance
- coasting beam approximation ➔ bunched-beam effect should be taken into account
- only {steady-state CSR + wiggler} impedances are included ➔ {entrance transient + exit propagation effects} of CSR can be considered, as well as {wall shielding effect} should be incorporated
- Vlasov equation ➔ Vlasov-Fokker-Planck equation (VFP)
  - VFP takes into account the effect of synchrotron radiation induced quantum excitation
- lumped model ➔ distributed model

TABLE I. Parameters and results for the NLC main damping ring [16], the TESLA damping ring [17], and the KEK ATF prototype damping ring [18]. The parameter F_w, defined in Eq. (19), is the ratio of the ISR power emitted in the wiggler to that emitted in the arc bending magnets.

	NLC	TESLA	ATF
Circumference C/km	0.3	17	0.14
Dipole radius R/m	5.5	80	5.7
Total bending angle $\Theta/2\pi$	1	5/3	1
Momentum compaction $\alpha/10^{-4}$	2.95	1.2	19
Synchrotron frequency Q_s/kHz	3.5	0.8	17.4
Extracted X emittance $\gamma\epsilon_x/10^{-6}$ m	3	8	5
Extracted Y emittance $\gamma\epsilon_y/10^{-8}$ m	2	2	5
Energy E/Gev	1.98	5	1.3
Energy rms spread $\nu_0/10^{-4}$	9.09	9	6
Bunch rms length σ_z/mm	3.6	6	5
Particles in a bunch $N_e/10^{10}$	0.75	2	1
Wiggler peak field B_w/T	2.15	1.5	1.88
Wiggler period λ_w/m	0.27	0.4	0.4
Wiggler total length L_w/m	46.24	432	21.2
Wiggler β function $\beta_{x,w}/\text{m}$	1.87	6.67	6
Pipe radius b/cm	1.6	2	1.2
F_w	2.2	13.4	1.8
Cutoff wavelength λ_c/mm	4.9	1.8	3.1
Threshold at cutoff (wiggler off) $N_t/10^{10}$	0.60	27.44	0.95
Threshold at cutoff (wiggler on) $N_t/10^{10}$	0.52	24.56	0.76
Growth time at cutoff (wiggler off) $\tau/\mu\text{s}$	54.9	N/A	34.3
Growth time at cutoff (wiggler on) $\tau/\mu\text{s}$	32.9	N/A	6.5