**Control of proton and deuteron polarizations in
the JLEIC ion collider ring using 3D spin rotators at 100 GeV/c**

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**1 Proton and deuteron polarization in an ideal lattice of the JLEIC ion collider ring**

Figures 1a and 1b show graphs of the spin components in an ideal lattice of the collider ring for proton and deuteron beams. The beam momentum is 100 GeV/c. The particle was launched along the closed orbit with vertically (Fig.  1a) and longitudinally (Fig. 1b) oriented spin.

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| **Figure 1a:** Proton spin components in an ideal collider lattice. The initial conditions are: *Sy*=1, *x*0=*y*0=0 μm, *x*’0= *y*’0=0 mrad. | **Figure 1b:** Deuteron spin components in an ideal collider lattice. The initial conditions are: *Sz*=1, *x*0=*y*0=0 μm, *x*’0 = *y*’0=0 mrad. |
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| **Figure 2a:** Proton vertical spin component in an ideal collider lattice when including betatron oscillations (the initial condition is: *Sy* = 1).  | **Figure 2b:** Proton spin components in an ideal collider lattice when including betatron oscillations (the initial condition is: *Sz* = 1). |

As can be seen from Figs.  1a and 1b, when a particle is launched along the ideal closed orbit, the spin does not change at all, which means that the coherent component of the spin resonance strength equals zero in an ideal collider lattice.

Figs. 2a and 2b show calculations of the proton spin dynamics for the transverse beam size at the interaction point of 19.4×3.9 μm2. The spin was started in vertical (Fig.  2a) and longitudinal (Fig. 2b) directions. The graphs in Figs.  2a and 2b yield that the incoherent resonance strength component is directed vertically and its value for protons is about *w*incoh = 1.3 10-4.

Note that, for the particle offset from the closed orbit, the modulation of its vertical spin component occurs at a doubled frequency. This confirms that the average spin field due to betatron oscillations is determined by the second order of the averaging method (compare Figs. 2a and 2b).

Similar calculations for deuterons show that their incoherent resonance strength component is also directed vertically and its value is about *w*incoh = 4 10-10.

*Obtaining transverse polarization at the collider’s interaction point*

As an example let us consider obtaining radial proton polarization at the collider’s interaction point. Figures 6a-6d show graphs of changes in the spin components in an ideal collider lattice. The 3D rotator parameters are: *nx*=1, νsol=0.01. The beam momentum is 100 GeV/c. The particle is launched along the closed orbit with radial (Fig. 6a) and longitudinal (Fig. 6b) spin directions. Figures 6c and 6d show similar calculations for the transverse beam size at the interaction point of 19.4×3.9 μm2. As can be seen from Figs. 6b and 6d, the proton spin tune value induced by the 3D rotator is practically independent of betatron oscillations and remains equal to 10-2.

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| **Figure 6a:** Proton spin components in an ideal collider with a 3D rotator setting of *nx*=1, νsol=0.01. The initial conditions are: *Sx*=1, *x*0=*y*0=0 μm, *x*’0= *y*’0=0 mrad. | **Figure 6b:** Proton spin components in an ideal collider with a 3D spin rotator setting of *nx*=1, νsol=0.01. The initial conditions are: *Sz*=1, *x*0=*y*0=0 μm, *x*’0 = *y*’0= 0 mrad |
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| **Figure 6c:** Proton radial spin component in the presence of betatron motion with a 3D rotator setting of *nx*=1, νsol=0.01. The initial conditions are: *Sx*=1, *x*0=19.3 μm, *y*0=3.9 μm, *x*’0= *y*’0=0 mrad. | **Figure 6d:** Proton spin components in the presence of betatron motion with a 3D rotator setting of *nx*=1, νsol=0.01. The initial conditions are: *Sz*=1, *x*0=19.3 μm, *y*0=3.9 μm, *x*’0= *y*’0=0 mrad |

Our calculations show that, when setting vertical polarization at the collider’s interaction point, the spin dynamics is similar to the case of the radial polarization setting considered above.

*Obtaining longitudinal polarization at the collider’s interaction point*

As an example, let us consider obtaining longitudinal deuteron polarization at the collider’s interaction point. Graphs of change in the spin components in an ideal collider lattice are shown in Figs. 7a-7d. Parameters of the 3D rotator are: *nz*=1, νsol=10-4. The beam momentum is 100 GeV/c. The particle is launched along the closed orbit with longitudinal (Fig. 7a) and vertical (Fig. 7b) spin directions. Similar calculations for the transverse beam size at the interaction point of 19.4×3.9 μm2 are shown in Fig. 7c and 7d. As can be seen from Figs. 7b and 7d, the deuteron spin tune value induced by the 3D rotator is practically independent of betatron oscillations and remains equal to 10-4.

Note the exceptional stability of deuterons in regard to the incoherent spin resonance strength: the longitudinal component changed by only 5⋅10-6 in the presence of betatron oscillations.

The presented calculations confirm the stability of polarization in the ion collider ring when using a 3D spin rotator, which provides the spin tune values of 10-2 for protons and 10-4 for deuterons.

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| **Figure 7a:** Deuteron spin vector in an ideal collider with a 3D rotator setting of *nz*=1, νsol=10-4. The initial conditions are: *Sz*=1, *x*0=*y*0=0 μm, *x*’0 = *y*’0 = 0 mrad. | **Figure 7b:** Deuteron spin vector in an ideal 3D collider with a 3D rotator setting of *nz*=1, νsol=10-4. The initial conditions are: *Sy*=1, *x*0=*y*0=0 μm, *x*’0 = *y*’0 = 0 mrad |

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| **Figure 7c:** Deuteron longitudinal spin component in the presence of betatron motion with a 3D rotator setting of *nz*=1, νsol=10-4. The initial conditions are: *Sy*=1, *x*0=19.4 μm, *y*0=3.9 μm, *x*’0= *y*’0=0 mrad | **Figure 7d:** Deuteron spin components in the presence of betatron motion with a 3D rotator setting of *nz*=1, νsol=10-4. The initial conditions are: *Sy*=1, *x*0=19.4 μm, *y*0=3.9 μm, *x*’0= *y*’0=0 mrad |

**2 Impact of lattice imperfections on polarization in the JLEIC ion collider ring**

In real conditions, there are always errors in the manufacture of collider magnetic lattice elements as well as errors in alignment of these elements along the collider’s design orbit. These lattice imperfections lead to a change in the collider’s closed orbit. As a result, particle spins experience additional coherent rotations caused by perturbing magnetic field when the particles are moving along the distorted periodic closed orbit. The combined effect of these magnetic fields on the spin determines the coherent component of the resonance strength.

*Coherent component of the resonance strength in a non-ideal collider lattice*

One of the main reasons for appearance of the coherent resonance strength component are random quadrupole shifts resulting in a change in the collider’s closed orbit.

Figures 8 and 9 show diagrams of random quadrupole shifts, which are used in calculations of the proton spin motion in the collider. The sizes of the quadrupole shifts in the vertical and radial directions are given in units of their rms deviation equal to 5 µm. The diagrams also indicate the locations of the control 3D rotator (1st 3D-rotator) and of the compensating 3D rotator (2nd 3D-rotator). The indicated quadrupole alignment errors result in a closed orbit distortion in the arcs of a few hundred µm (see Fig. 10).



**Figure 8:** Diagram of vertical quadrupole alignment errors in the collider ring distributed normally
with $σ\_{rms}= Δy\_{quad}=5 μm$.



**Figure 9:** Diagram of radial quadrupole alignment errors in the collider ring distributed normally
with $σ\_{rsm}=Δx\_{quad}=5 μm$.

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**Figure 10:**Radial and vertical orbit excursions with random misalignments of all quadrupoles in the collider ring.

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| **Figure 11a:** Proton spin components at the interaction point in an non-ideal collider lattice with the 3D rotators off. | **Figure 11b:** Deuteron spin components at the interaction point in an non-ideal collider lattice with the 3D rotators off. |

To determine the coherent resonance strength component, figure 11 demonstrates graphs of the proton and deuteron spin components versus the number of particle turns in the collider with the random quadrupole shifts as per the diagrams in Figs. 8 and 9. The particle is launched from the beam interaction point along the closed orbit with longitudinal spin.

The proton spin completes 9 oscillations in 985 particle turns. The coherent strength component is then about $w\_{coh}^{prot}≈9.14⋅10^{-3}$. The deuteron spin completes 1 oscillation in $10^{4}$ particles turns, which gives the coherent strength component equal to about $w\_{coh}^{deut}≈10^{-4}$.

As noted above, the coherent resonance strength component itself does not cause beam depolarization. On the contrary, by finding the unknown direction of the coherent component, which is determined by random quadrupole misalignments, one can stabilize the particle’s spin.

To find the direction of the precession axis $\vec{n}$ induced by the coherent resonance strength component, one can also use the graphs in Fig. 11. Since the spin component along the $\vec{n}$ axis is an invariant of the spin motion, the average spin value $\left〈\vec{S}\right〉$ is directed along the $\vec{n}$ axis:

$$\left〈\vec{S}\right〉=S\_{n} \vec{n} . $$

Thus, by calculating the average spin components as half sums of the maximum and minimum values of the corresponding spin components:

$$\left〈S\_{x}\right〉=\frac{(S\_{x})\_{min}+⁡(S\_{x})\_{max}}{2} , \left〈S\_{y}\right〉=\frac{(S\_{y})\_{min}+⁡(S\_{y})\_{max}}{2} , \left〈S\_{z}\right〉=\frac{(S\_{z})\_{min}+⁡(S\_{z})\_{max}}{2} ,$$

we get the  axis components:

$$n\_{x}=\pm \frac{\left〈S\_{x}\right〉}{S\_{n}} , n\_{y}=\pm \frac{\left〈S\_{y}\right〉}{S\_{n}} , n\_{z}=\pm \frac{\left〈S\_{z}\right〉}{S\_{n}} , S\_{n}=\sqrt{\left〈S\_{x}\right〉^{2}+\left〈S\_{y}\right〉^{2}+\left〈S\_{z}\right〉^{2}} .$$

The sign of the $\vec{n}$ vector is determined from the condition that the spin vector rotates about $\vec{n}$ counterclockwise. Calculations of the proton and deuteron precession axes at the interaction point give:

$$\vec{n}\_{IP}^{prot}=\left( 0.330,-0.020, 0.944 \right), \vec{n}\_{IP}^{deut}=\left( -0.536, 0.0015, 0.844\right).$$

If a particle is launched along the closed orbit with its initial spin direction being along $\vec{n}\_{IP}$ at the interaction point, then the polarization will be stable from turn to turn of the particle, which is completely confirmed by the calculations presented in Fig. 12.

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| **Figure 12a:** Stable proton polarization at the interaction point in a non-ideal collider lattice with the 3D spin rotators off. | **Figure 12b:** Stable deuteron polarization at the interaction point in a non-ideal collider lattice with the 3D spin rotators off. |

*Compensation of the depolarization caused by imperfections*

Below we consider compensation of the coherent resonance strength component using protons as example. The deuteron case can be considered similarly.

The calculation of the coherent resonance strength component in the collider ring shows that its value for protons is $ω\_{coh}^{prot}≈9.14⋅10^{-3}$. This means that using a 3D rotator with a spin tune of 10-2 to control the proton polarization already becomes, at least, inconvenient, since, during a spin manipulation process, one should always make a “correction” of the spin field for the coherent resonance strength component. Besides, the coherent component grows with increase in energy along with the fields required for its compensation. Nevertheless, the solenoid fields of the control 3D rotator can be left at the same level if one compensates the coherent resonance strength component using a second 3D rotator with static field located in the opposite straight (see Fig. 8).

To determine the direction of the precession axis induced by the coherent resonance strength component near the 2nd 3D rotator, Fig. 13a shows graphs of the proton spin components versus the number of particle turns in the collider ring with the random quadrupole shifts according to the diagrams presented in Fig. 8 and 9. The particle is launched from the interaction point along the closed orbit with longitudinal spin. The spin is observed near the 2nd 3D rotator in a section opposite to the interaction point. The graph yields that the direction of the spin precession axis $\vec{n}\_{2}$ near the second 3D rotator is

$$\vec{n}\_{2}=\left( -0.845, -0.001, -0.534\right).$$

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| **(a)** | **(b)** |
| **Figure 13:** Proton spin components in a section opposite to the interaction point in a non-ideal collider lattice with the 3D rotators off before (a) and after (b) compensation of the coherent resonance strength component. |

Figure 4.13b shows the spin components after compensation of the coherent resonance strength component. The parameters of the compensating 3D rotator were chosen as

$$\vec{n}\_{comp}=\vec{n}\_{2}=\left( -0.845, -0.001, -0.534\right), ν\_{comp}=-ω\_{coh}=-9.14 10^{-3}.$$

The graph in Fig. 13b yields that, after compensation, the coherent resonance strength component, became $2.1⋅10^{-4}$, i.e. decreased practically to the value of the incoherent resonance strength components.

Since we set the 3D rotator parameters using formulae derived in the linear approximation in the spin tune $ν$, the accuracy of compensation in an ideal collider lattice is determined by the square of the spin tune $ν^{2}\~10^{-4}$. One can further improve the compensation by specifying the 3D rotator parameters up to the second order including the non-commutativity of the spin rotations about the different axes in the 3D rotator modules. One should also analyze the effect on the 3D rotator of additional fields arising inside the rotator due to random quadrupole misalignments.

Figure 14a shows a graph of the spin component evolution in a non-deal collider lattice when setting vertical proton polarization at the interaction point with compensation of the coherent resonance strength component. The parameters of the control 3D rotator are: *ny*=1, νsol=0.01. The beam momentum is 100 GeV/c. The particle is launched along the closed orbit with vertical spin. For comparison, Fig. 14b shows a similar graph without compensation of the coherent resonance strength component.

The provided example shows that a non-ideal collider with compensation of the coherent resonance strength component becomes equivalent to an ideal one in terms of polarization control.

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| **(a)** | **(b)** |
| **Figure 14:** Setting vertical polarization in a non-ideal collider lattice with **(a)** and without **(b)** compensation of the coherent resonance strength component. |